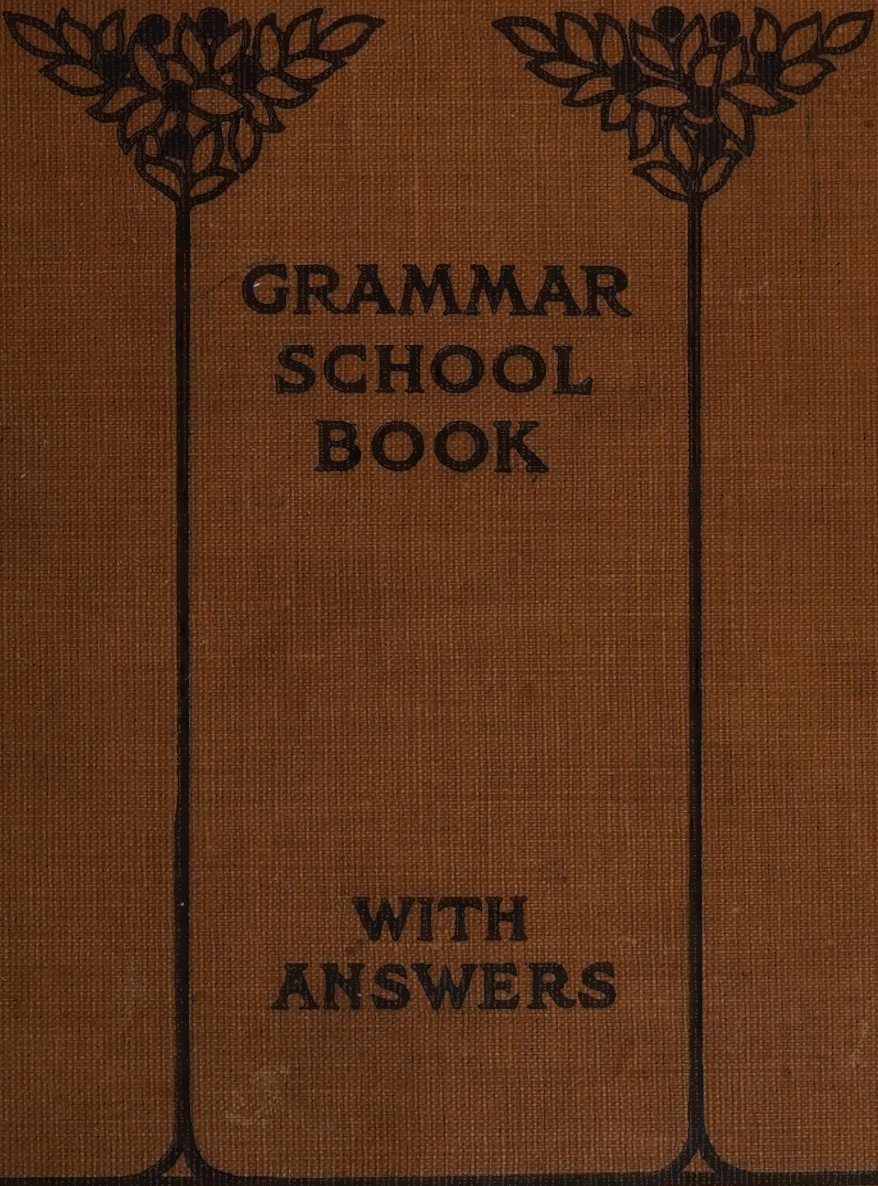
THE APPLETON ARITHMETICS



GRAMMAR SCHOOL BOOK

WITH ANSWERS

THE APPLETON ARITHMETICS

BY

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PREFACE

THE present volume of THE APPLETON ARITHMETICS covers the four years of work usually taught in the fifth, sixth, seventh, and eighth grades.

Like the PRIMARY BOOK, this text is for the *pupils'* use, and as such possesses certain well-defined features. Important definitions, principles, and processes are introduced inductively by means of a series of questions and illustrations labeled "Preparatory." In this way a natural and easy approach to the subject is provided for the pupil, without, however, placing hard and fast limitations on the teacher's method. When a principle or process has been developed and stated, it is put into practice through easy Oral Exercises. These are followed by Written Exercises, involving applications that contain more difficult calculations.

In addition to giving proper attention to the culture value peculiar to the exact reasoning of school mathematics, this book fully recognizes the utility of arithmetic. The problem material is rich in data drawn from the pupil's experience. School subjects like manual training, domestic art, geography, nature study, and drawing have furnished many problems; and the everyday occupations and industries of the average community are made to contribute their share of applications. Finally, the larger interests of the community, such as production, transportation, communication, and government, have not been neglected as sources for problems of interest and value.

In all of this, arithmetic is made the subject of study and the core of the instruction, without, however, sacrificing the pupil to abstract number drill; and the whole subject is classified and labeled in harmony with a logical development.

The two parts of the present volume are designed to cover the work of the fifth and sixth years, and that of the seventh and eighth years, respectively. Part I begins with a brief review of the work of the primary grades, and then provides a course that covers the essentials of elementary arithmetic. Part II reviews and summarizes in a formal way the theory of arithmetic, furnishes more difficult drill, extends its applications into advanced business arithmetic, and introduces such algebra, geometry, and measurement as prepare for high-school work.

Other features that the teacher may appreciate are the frequent and systematic reviews; the oral and written work, properly labeled; the well-balanced abstract and concrete drill; the classified tables of measurement, and the attempt to supplement and strengthen the text by careful typography and effective illustration.

THE AUTHORS.

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GRAMMAR-SCHOOL ARITHMETIC

REVIEW AND PREPARATION

NOTATION AND NUMERATION

ORAL EXERCISES

1. Name the ten figures commonly used in writing numbers.

2. Name the symbols used in writing numbers to 1000 by the Roman system.

3. How many places in each period in the common system of notation? Name the three places in units' period. Name the first three groups or periods.

4. How many of any place equal 1 of the next higher place?

5. In writing whole numbers, what is used to separate the periods?

6. In writing decimal numbers, what is used to show where the decimal begins?

Read:

7. 425,026.
8. 486.01.
9. 802,004.
10. 1,525.6.
11. 1,630,200.
12. 9,990.95.
13. MCM.
14. DCV.
15. CIV.
16. XLIX.
17. XCV.
18. MCMIX.

WRITTEN EXERCISES

Write in figures:

1. Two hundred sixty-five thousand, one hundred twenty-five.

2. Five hundred twenty thousand, seventy-five.

3. Eight hundred one thousand, five hundred two.

4. Three million, five hundred thousand, nine.

5. Six hundred twenty-eight, and thirty-five hundredths.

6. Twenty-four hundred, and twenty-four hundredths.

Write in Roman notation:

7. 76.
8. 1000.
9. 1908.
10. 43.
11. 500.
12. 1776.
13. 17.
14. 125.
15. 1910.
16. 25.
17. 100.
18. 1620.
19. 1854.
20. 1265.
21. 1492.

ADDITION AND SUBTRACTION

TABLE 1

$$\begin{array}{ccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8} & \underline{9} \end{array}$$

TABLE 2

$$\begin{array}{cccccccc} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ \underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8} & \underline{9} \end{array}$$

1. State the sum in each case above; then write a similar table, using 3 as the top number. Similarly, write a table using 4 as the top number. Similarly, 5, 6, 7, 8, 9, as top numbers.

2. How many sums in Table 1? In Table 2? In each of the tables you have made? In the whole nine tables?

ORAL EXERCISES

1. Read the signs: +, −, =.

2. When several numbers are added into one sum, what is each number called?

3. In adding numbers of two or more figures, the sum of a column often exceeds ten. How do we proceed with the work in this case?

4. Since 8 and 9 are 17, seventeen is how many more than eight? Seventeen is how many more than nine?

5. In subtraction what is the number called from which we subtract? The number that we subtract?

6. Which is the larger number, the subtrahend or the minuend?

7. State a way of testing the work of subtraction.

Add rapidly:

1.	**2.**	**3.**	**4.**	**5.**	**6.**
26	92	45	4	39	50
35	29	32	30	49	96
24	10	26	98	59	23
17	49	19	76	46	44
30	3	40	19	76	25

7.	**8.**	**9.**	**10.**
$46.33	$146.25	$99.36	139.06
18.96	87.75	77.46	800.05

Subtract rapidly:

11.	$176.82	**13.**	$65.29	**15.**	$234.00	**17.**	$480.72
	77.46		46.33		146.25		166.99
12.	145 in.	**14.**	86.5 yd.	**16.**	139.8 gal.	**18.**	184.5 mi.
	97 in.		17.9 yd.		98.9 gal.		67.8 mi.

MULTIPLICATION

ORAL EXERCISES

1. Read the sign ×. Read 3 × 4 in two ways.

2. What is the result of multiplication called? What are the numbers that form a product called?

3. Read the factors in each product:

$3 \times 4 = 12.$ $8 \times 8 = 64.$ $2 \times 3 \times 5 = 30.$

4. There are 5 rows of 6 desks each. Would 6 rows of 5 desks each seat any more pupils? Any less?

5. In this problem which number is the multiplicand? The multiplier? The product?

6. What is a short way of finding 36 + 36 + 36 + 36?

$$\begin{array}{r} 45 \text{ ft.} \\ \underline{23} \\ 135 \\ \underline{90} \\ 1035 \text{ ft.} \end{array}$$

7. When the multiplier contains more than one figure, how are the partial products written?

8. How is the complete product found?

WRITTEN EXERCISES

1. The numbers in the top row of this table are the multiples of 2 from 1 × 2 to 12 × 2. The numbers in the second row are multiples of what number?

2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36

2. Complete the table by writing the multiples of 4, 5, 6, and so on to 12.

3. Find the sum 45 + 45 + 45 + 45 + 45 by multiplication.

4. How many hours in 7 full days?

5. How many days in 52 weeks?

Multiply:

6. 136 8	**9.** 205 9	**12.** 710 7	**15.** 2714 15	**18.** $40.25 8
7. 813 29	**10.** 645 43	**13.** 708 25	**16.** 1802 97	**19.** $56.33 9
8. 960 40	**11.** 876 25	**14.** 943 77	**17.** 1669 97	**20.** $325.18 68

DIVISION

ORAL EXERCISES

1. What does the sign ÷ mean?

2. Read the expression, 35 ÷ 7 = 5.

3. In 56 ÷ 8 = 7, which term is the dividend? The divisor? The quotient?

4. How does the product of the divisor and the quotient compare with the dividend?

5. In division we have the product and one factor given, and are asked to find the other factor. In 75 ÷ 3 = (?) which term is the product? Which is the one factor given? What is the other factor?

6. How many 8's are there in 72? How many times is 8 contained in 72?

7. How many 9's in 45? How many times is 9 contained in 45?

8. 24 is how many more than 8? 24 is how many times 8?

9. Similarly, compare 36 with 12 in two ways.

10. Compare 48 with 8 in two ways.

11. What is the test of the work in division?

WRITTEN EXERCISES

Divide:

1. $9\overline{)729}$

2. $11\overline{)1210}$

3. $7\overline{)\$48.30}$

4. $13\overline{)270}$

5. $22\overline{)778}$

6. $8\overline{)728}$

7. $12\overline{)132}$

8. $9\overline{)\$82.26}$

9. $17\overline{)359}$

10. $36\overline{)4893}$

11. $7\overline{)483}$

12. $12\overline{)4812}$

13. $15\overline{)\$90.15}$

14. $21\overline{)463}$

15. $25\overline{)1758}$

16. $6\overline{)336}$

17. $15\overline{)2250}$

18. $5\overline{)\$7.50}$

19. $35\overline{)7065}$

20. $48\overline{)9624}$

Find:

21. $\frac{1}{8}$ of 568.

22. $\frac{3}{5}$ of 4095.

23. $\frac{1}{12}$ of 1728.

24. $\frac{4}{15}$ of 3045.

25. $\frac{1}{15}$ of 2265.

26. $\frac{8}{25}$ of 625.

27. $\frac{1}{11}$ of 14641.

28. $\frac{7}{18}$ of 7236.

COMMON AND DECIMAL FRACTIONS

ORAL EXERCISES

1. What is a fraction? Name the numerator and the denominator in $\frac{5}{8}$.

2. What is an improper fraction? Name one.

3. What is a proper fraction? Name one.

4. Which is the larger: $\frac{1}{2}$ or $\frac{1}{3}$? $\frac{2}{3}$ or $\frac{3}{4}$? $\frac{1}{6}$ or $\frac{1}{5}$? $\frac{2}{3}$ or $\frac{1}{2}$? $\frac{3}{4}$ or $\frac{5}{6}$?

5. $\frac{6}{8}$ is how many fourths? $\frac{5}{15}$ is how many thirds?

6. How is a fraction reduced to lower terms?

7. By what must both numerator and denominator of $\frac{3}{4}$ be multiplied to make $\frac{9}{12}$?

8. Change $\frac{2}{3}$ to an equal fraction whose denominator is 6.

9. How must fractions be expressed before they can be added or subtracted?

10. In decimal fractions what is the first place at the right of the decimal point? What is the second place?

WRITTEN EXERCISES

Add:

1. $\frac{1}{3}+\frac{3}{4}$.
2. $\frac{2}{3}+\frac{5}{6}$.
3. $\frac{4}{5}+\frac{2}{3}$.
4. $\frac{3}{4}+\frac{2}{3}$.
5. $\frac{2}{5}+\frac{1}{2}$.
6. $\frac{5}{8}+\frac{1}{2}$.
7. $\frac{3}{4}+\frac{5}{6}$.
8. $\frac{3}{5}+\frac{1}{3}$.

Subtract:

9. $\frac{5}{6}-\frac{1}{2}$.
10. $\frac{2}{5}-\frac{1}{6}$.
11. $\frac{5}{6}-\frac{3}{8}$.
12. $\frac{3}{4}-\frac{2}{3}$.
13. $\frac{4}{5}-\frac{1}{2}$.
14. $\frac{7}{8}-\frac{3}{4}$.
15. $\frac{2}{3}-\frac{3}{5}$.
16. $\frac{5}{6}-\frac{4}{5}$.

Solve:

17. $\frac{1}{2}$ day and $\frac{1}{3}$ day are what part of a day?

18. $\frac{2}{3}$ hr. and $\frac{5}{8}$ hr. are what part of an hour?

19. A man owned $\frac{1}{5}$ of a factory and purchased $\frac{1}{6}$ more. What fraction did he then own?

20. A man owned $\frac{2}{3}$ of a factory and sold an amount equal to $\frac{1}{2}$ of the whole factory. What part of the property did he still own?

21. A man's will divided $\frac{8}{9}$ of his property equally among 4 heirs. What part of the whole property did each receive?

Multiply:

22. $\frac{2}{3}$ by 3.
23. $\frac{3}{4}$ by 9.
24. $3\times\frac{1}{2}$.
25. $6\times\frac{9}{10}$.
26. 2 by $\frac{5}{6}$.
27. 8 by $\frac{8}{9}$.
28. $\frac{7}{8}\times 3$.
29. $\frac{3}{7}\times 4$.
30. 8 by $\frac{4}{5}$.
31. 6 by $\frac{2}{3}$.
32. $\frac{6}{7}\times 3$.
33. $5\times\frac{8}{9}$.

34. One-half of two-thirds of a day is how many hours?

35. How many are two-thirds of five-sixths of 36?

Divide:

36. $\frac{4}{5}\div 4$.
37. $\frac{6}{7}\div 2$.
38. $\frac{9}{10}\div 3$.
39. $\frac{5}{6}\div 4$.
40. $\frac{8}{9}\div 4$.
41. $\frac{14}{16}\div 7$.
42. $\frac{2}{5}\div 3$.
43. $\frac{10}{11}\div 5$.

Add:

44. $\begin{array}{r}8.65\\ \underline{7.32}\end{array}$ **45.** $\begin{array}{r}409.6\\ \underline{178.9}\end{array}$ **46.** $\begin{array}{r}110.46\\ \underline{49.73}\end{array}$ **47.** $\begin{array}{r}84.51\\ \underline{47.09}\end{array}$

Subtract:

48. $\begin{array}{r}99.8\\ \underline{65.7}\end{array}$ **49.** $\begin{array}{r}40.73\\ \underline{9.88}\end{array}$ **50.** $\begin{array}{r}176.1\\ \underline{48.7}\end{array}$ **51.** $\begin{array}{r}100.01\\ \underline{98.76}\end{array}$

Multiply:

52. $\begin{array}{r}3.6\\ \underline{45}\end{array}$ **53.** $\begin{array}{r}1.26\\ \underline{34}\end{array}$ **54.** $\begin{array}{r}4.58\\ \underline{96}\end{array}$ **55.** $\begin{array}{r}7.03\\ \underline{89}\end{array}$

Divide:

56. $15\overline{)2.25}$ **57.** $23\overline{)4.60}$ **58.** $12\overline{)49.2}$ **59.** $9\overline{)85.59}$

60. State the decimal form for each:

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{2}{5}$

Solve:

61. The distance from city A to city B is 135.5 mi. and from city B to city C is 79.9 mi. How many miles is it from A to C by way of B?

62. A farmer bought 40.15 acres of land and sold 29.65 acres. How many acres had he left?

63. A square is 14.3 yd. on a side. How many yards is it around the square?

DENOMINATE NUMBERS

ORAL EXERCISES

1. The foot is a *unit of measure* for finding length. Name two other units of measure for finding length.

2. Name two units of measure for measuring liquid.

3. Name two units of area for measuring surfaces.

4. Name three units for measuring grains and vegetables.

5. Name two units for measuring volume.

6. Name four units for measuring time.

7. Name three units for measuring weight.

8. Name five coins for measuring value.

WRITTEN EXERCISES

Copy, supplying the missing numbers:

1. ( ) in. = 1 ft.
( ) ft. = 1 yd.
( ) ft. = 1 rd.
( ) in. = 1 yd.

2. ( ) qt. = 1 pk.
( ) pk. = 1 bu.
( ) qt. = 1 bu.
( ) qt. = $\frac{1}{2}$ bu.

3. ( ) sq. in. = 1 sq. ft.
( ) sq. ft. = 1 sq. yd.
( ) sq. rd. = 1 acre.

4. ( ) cu. in. = 1 cu. ft.
( ) cu. ft. = 1 cu. yd.
( ) cu. ft. = $\frac{2}{3}$ cu. yd.

5. ( ) pt. = 1 qt.
( ) qt. = 1 gal.
( ) pt. = 1 gal.
( ) pt. = $\frac{3}{4}$ gal.

6. ( ) cts. = 1 dime.
( ) dimes = 1 dollar.
( ) cts. = 1 dollar.
( ) 5-cent pieces = $1.

7. ( ) oz. = 1 lb.
( ) oz. = $\frac{5}{8}$ lb.
( ) lb. = 1 T.

8. ( ) hr. = 1 da.
( ) da. = 1 wk.
( ) min. = 1 hr.

Add:

9. 10 ft. 9 in.
8 ft. 11 in.

10. 7 rd. 15 ft.
5 rd. 10 ft.

11. 7 gal. 3 qt.
4 gal. 2 qt.

12. 20 min. 18 sec.
17 min. 50 sec.

13. 8 bu. 3 pk.
7 bu. 3 pk.

14. 12 wk. 5 da.
37 wk. 6 da.

Subtract:

15. 40 lb. 12 oz.
17 lb. 10 oz.

16. 7 gal. 2 qt.
4 gal. 3 qt.

17. 37 wk. 2 da.
12 wk. 5 da.

BUSINESS ARITHMETIC

1. What is a bill? A price list?

2. Find the value of each item and the total of this bill:

Sacks and Co. *2/27/1909.*
Importers and Retailers,
San Francisco, Cal.

Sold to John Smith
620 Market St.

5 yd. lace	@	$.45 a yd.				
15 handkerc'fs	@	.25 each				
8 yd. muslin	@	.16 a yd.				
12 yd. silk	@	.90 a yd.				

---

3. At 25 cts. per yard, how many yards can be bought for a dollar?

4. What part of a dollar is: 25¢? 50¢? 20¢? 40¢? 75¢?

5. When 40 bu. of potatoes cost $30, what will 17 bu. cost at the same rate?

PROBLEMS

1. The usual weight of a bale of hops is 200 lb. How many bales are there in 1 ton? In a car-load of 10 tons?

2. 2,000 roots are required to plant an acre of hops. How many are required to plant 25 acres? What is the cost of planting 25 acres at $1 per 1000 plants?

3. The wire trellis to support the vines costs $75 an acre. What is the cost for 25 acres?

4. Picking the hops costs at the rate of 1 ct. a pound, and 3½ lb. of green hops make 1 lb. of dry hops. What is the cost of picking enough to make 1 ton of dry hops?

5. One cord of willow wood costing $5 will dry 1000 lb. of hops. What is the cost of wood enough to dry 100 bales of 200 lb. each?

6. A common kit of manual-training tools costs about $6. How much does it cost to supply 25 boys with these tools?

7. A garden in the form of a rectangle 30 yd. wide and 40 yd. long is to be represented on the blackboard. Name a convenient length to represent 1 yd.

8. What is the distance around the garden mentioned in Exercise 7? What is the distance around the drawing?

9. If an inch in a drawing represents 40 ft., how many feet does $\frac{1}{4}$ of an inch represent? $\frac{1}{2}$ of an inch? How many do 3 inches represent? 3½ inches?

10. In such a drawing what length represents 20 feet? 10 ft.? 30 ft.? 60 ft.? 75 ft.?

11. How many hours are there in a week?

12. How many minutes are there in a working day of 8 hr.?

13. 4 min. are how many seconds? Are what part of an hour?

14. 3 months and 15 days are what part of a year?

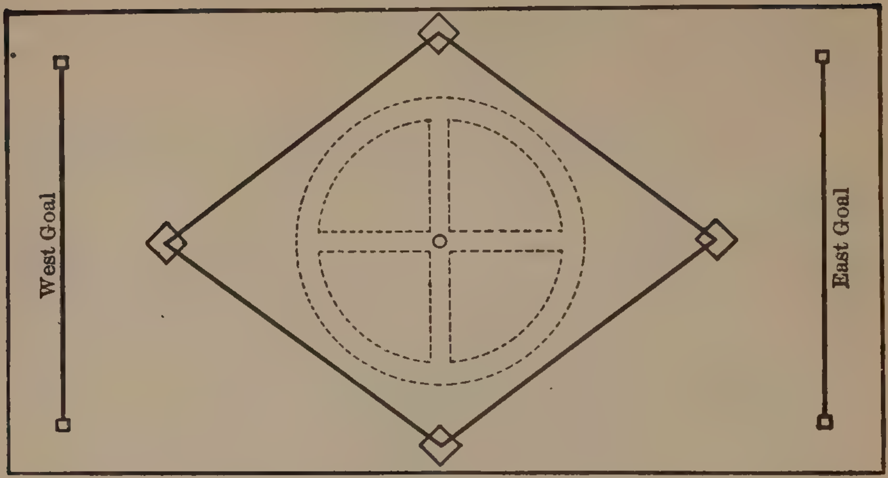
15. At 20 cts. a dozen for buttons, how much must I pay for a gross?

16. A case of 300 oranges brought 45 cts. a dozen. What were the oranges worth?

17. Into how many compartments, 1 ft. square, could I divide a box 8 ft. long and 6 ft. wide?

18. 8 pk. 6 qt. of berries will fill how many pint baskets?

19. A farm of 120 acres was worth $2 a sq. rd. How much was the farm worth?



20. In this drawing of a playground 1 inch represents 20 ft. Find the length of each side. What is the distance around the playground?

21. What is the length of each goal? What is the distance between the goals? How far are they from the ends of the playground? Measure other distances.

22. What is the area of the playground?

23. If your school has a playground, estimate or measure its length and breadth. Select a convenient scale and make a drawing of the playground.

24. Estimate or measure the length and breadth of the school-room. Make to scale a drawing of the room.

25. Find the size and position of each of the desks and locate them in your drawing. Do the same for windows and doors.

26. In your village or city what is the length of a block? How many blocks are there to the mile?

27. What is the cost of 108 stamped envelopes at 26 cts. a dozen?

28. What is the cost of 25 two-cent stamps, 2 dozen stamped envelopes bearing 2-cent postage (Exc. 27), and 50 postal cards?

29. If a postman has in his sack 200 pieces of mail averaging 2 oz. in weight, what is the weight of the mail?

30. A certain post-office has 4 mail-carriers, and each carrier has a route with 1000 addresses. If each carrier delivers 1 piece for every 5 addresses, how many pieces does each distribute at one delivery? How many pieces do the 4 carriers distribute?

31. There are two deliveries a day. How many pieces of mail are delivered in all per day?

32. If the postage averages 2 cts. per piece, how many dollars' worth of stamps are canceled on 3,200 pieces?

33. 15 money-orders were sold, for which the post-office received $2.25; also 480 stamped envelopes at 26 cts. a dozen; 300 single postal cards, and 200 with return card attached. How much did the office receive from these sources?

34. The four carriers are paid at the rate of $2 a day each, the postmaster at the rate of $5, and 2 clerks at the rate of $1.75 each. What does the labor in this post-office cost the Government per month of 30 days?

35. There are 120 lock-boxes in a post-office, $\frac{4}{5}$ of which are rented at 25 cts. a month each. What is the income from these boxes in a year?

36. Make and solve 3 problems, using the data of the table:

SALARIES PER YEAR.		RECEIPTS.	
Postmaster	$2,000	Stamps..................	$5,000
Assistant................	1,500	Money-orders	500
2 clerks...........each at	800	Cards....................	400
4 carriers.........each at	725	400 boxes.........each at	2

37. Make problems about the postal service in your city.

38. How much did the farmer receive who sold these crops: Hay, $75; wheat, $50.75; corn, $100; oats, $96.50; cabbage, $136.42; apples, $236.25?

39. The following are the ingredients of a fertilizer for vineyards: phosphate, ground bone, and potash in equal amounts. If an acre requires 900 lb. of fertilizer, how many pounds of each ingredient are used on an acre?

40. How many tons of such fertilizer are needed for a 5-acre vineyard?

41. A fertilizer for growing strawberries consists of 1 part nitrate of soda, 2 parts phosphate, 2 parts ground bone, and 2 parts potash. Thus, $\frac{1}{7}$ is nitrate of soda, $\frac{2}{7}$ is phosphate, and so on. How many pounds of each ingredient are there in 700 lb. of fertilizer?

42. $1\frac{3}{4}$ tons of fertilizer will fertilize 5 acres of strawberries. How many pounds are used per acre?

43. If a cow gives 450 lb. of milk in a month and each 100 lb. contains 4 lb. of fat, how much fat is there in the month's product? What part of the milk is fat?

44. According to Exercise 43, how many pounds of milk would a dairy of 20 cows produce in a month? If $\frac{4}{5}$ of this amount remains after removing the cream, what is the weight of the month's product of cream?

45. A car loaded with coal weighs about 60,000 lb. How many tons does such a car with its load weigh?

46. A large battleship weighs 20,000 tons. How many carloads of coal like the one mentioned in Exercise 45 would weigh as much as a large battleship?

47. A cubic foot of water weighs 1000 oz. How many pounds in this?

48. A tank 10 ft. long, 3 ft. wide, and 2 ft. deep is filled with water. According to Exercise 47, what is the weight of the water in the tank?

49. Similarly, what is the weight of the water in a tank 25 ft. long, 16 ft. wide, and 10 ft. deep when half full of water?

PART I

CHAPTER I

PROCESSES WITH INTEGERS

NOTATION AND NUMERATION

1. Whole numbers are called **integers.** Integers are expressed by places and periods as follows:

1. Ten units make one ten.
2. Ten tens make one hundred.
3. Ten hundreds make one thousand.
4. Ten thousands make 1 ten-thousand.
5. Ten ten-thousands make 1 hundred-thousand.

2. Similarly, 10 hundred-thousands are grouped to make 1 **million;** 10 millions are grouped into 1 **ten-million;** and 10 ten-millions are grouped into 1 **hundred-million.**

	Hundred-Millions	Ten-Millions	Millions		Hundred-Thousands	Ten-Thousands	Thousands		Hundreds	Tens	Units
	6	7	5	,	3	2	1	,	5	3	6
Periods:	Millions				Thousands				Units		

The table above shows the names of the first nine places or **orders** and the first three **periods.**

The period at the left in a number may not be complete. It may contain only one or two figures, as in 6,218 or in 65,421,233. How many must the other periods contain? Read the number in the table.

3. Each period, beginning at the left, is read as if it were a number by itself, the name of the period being added except in the case of units' period, for which it is understood.

ORAL EXERCISES

Read:

1. 4,000,000.
2. 9,001,430.
3. 4,100,000.
4. 8,600,020.
5. 7,326,000.
6. 8,625,365.
7. Read, from the table below, the numbers of people speaking various languages in each century:

PEOPLE.	19TH CENTURY.	20TH CENTURY.
English	20,520,000	111,100,000
French	31,450,000	51,200,000
German	30,320,000	75,200,000
Italian	15,070,000	33,400,000
Spanish	26,190,000	42,800,000
Russian	30,770,000	75,000,000

WRITTEN EXERCISES

Write in figures:

1. Eight millions.
2. Eighty-five millions.
3. Four million two hundred seventy-five thousand.
4. Sixty-two million, three hundred eight thousand, two hundred seventy-nine.
5. Five million, six hundred ninety-eight thousand, ten.
6. Two million, thirty-one thousand, one hundred seven.
7. Twenty million, four hundred thousand, two.

ADDITION

4. Counting two or more numbers into a single number is called **addition.** The sign of addition is +.

The numbers added are called **addends.**

The result is called the **sum.**

5. In adding pairs of two-figure numbers it is shorter to add the tens of one addend to the whole of the other.

Thus, in adding 25 and 76, we think "76 plus 20 are 96, and 5 are 101."

$$\begin{array}{r} 76 \\ \underline{25} \\ 101 \end{array}$$

ORAL EXERCISES

Add the following as indicated above:

1. 46	**3.** 46	**5.** 86	**7.** 40	**9.** 48	**11.** 98
35	71	45	61	90	89
2. 93	**4.** 36	**6.** 39	**8.** 57	**10.** 29	**12.** 96
28	53	94	75	32	84

6. When the addends are small two-figure numbers and one-figure numbers, add both columns at once.

Thus, in Exercise 1 below, think "21, 28, 33, 44," and so on.

ORAL EXERCISES

Add rapidly upward; then add downward:

1. 9	**2.** 12	**3.** 10	**4.** 13	**5.** 22
10	5	11	9	8
6	10	8	7	11
11	7	9	12	1
5	6	12	10	7
7	11	5	4	10
21	15	13	9	9

6.
20
10
11
7
6
5
10
8

7.
30
10
16
6
4
9
7
8

8.
5
11
5
4
6
11
10
30

9.
15
19
11
31
7
5
10
23

10.
50
8
2
1
9
9
11
8

Add, grouping by 10 where possible:

Thus, in Exercise 11, think "6 and 10, or 16; 16 and 10, or 26; 36; 40; 50; 57."

11.
$$\begin{array}{r}7\\ \left.\begin{array}{r}8\\2\end{array}\right\}\\ 4\\ \left.\begin{array}{r}5\\5\end{array}\right\}\\ \left.\begin{array}{r}7\\3\end{array}\right\}\\ \left.\begin{array}{r}8\\2\end{array}\right\}\\ 6\end{array}$$

12.
1
9
7
3
8
2
5
3
7

13.
8
9
7
3
2
8
6
1
9
8
7

14.
4
6
5
5
2
3
5
8
2

15.
9
9
2
4
6
5
3
7
8
2
9

16.
8
2
4
4
2
7
3
9
1

17.
8
3
8
5
7
7
1
9
4
6
5

18.
4
6
3
1
6
2
7
1
5

19.
6
3
3
3
3
1
1
1
5
5
5

20.
7
1
2
3
4
3
5
4
1

WRITTEN EXERCISES

	AREA IN SQ. MI.	POPULATION.		AREA IN SQ. MI.	POPULATION.
Alabama	52,250	2,250,000	New H'pshire	9,305	443,700
Alaska	590,884	125,000	New Jersey	7,815	2,294,413
Arizona	113,020	185,000	New Mexico	122,580	302,000
Arkansas	53,850	1,750,000	New York	49,170	8,476,427
California	158,360	2,000,000	N. Carolina	52,250	2,100,000
Colorado	103,925	800,000	N. Dakota	70,795	475,000
Connecticut	4,990	1,010,000	Ohio	41,060	4,557,000
Delaware	2,050	190,000	Oklahoma	70,430	1,408,732
Dist. of Col.	70	330,000	Oregon	96,030	550,000
Florida	58,680	650,000	Pennsylvania	45,215	6,900,000
Georgia	59,475	2,600,000	Rhode Island	1,250	502,302
Hawaii	6,449	154,001	S. Carolina	30,570	1,474,735
Idaho	84,800	300,000	S. Dakota	77,650	490,000
Illinois	56,650	5,590,000	Tennessee	42,050	2,220,000
Indiana	36,350	2,678,492	Texas	265,780	3,600,000
Iowa	56,025	2,216,068	Utah	84,970	350,000
Kansas	82,080	1,680,000	Vermont	9,565	340,000
Kentucky	40,400	2,435,000	Virginia	42,450	2,042,388
Louisiana	48,720	1,700,000	Washington	69,180	900,000
Maine	33,040	731,760	W. Virginia	24,780	1,200,000
Maryland	12,210	1,441,602	Wisconsin	56.040	2,275,000
Massachusetts	8,315	3,173,487	Wyoming	97,890	117,500
Michigan	58,915	2,655,463			
Minnesota	83,365	2,200,000	ISLANDS.		
Mississippi	46,810	1,750,000	Guam (Ladrones)	150	8,661
Missouri	69,415	3,885,989	Philippine Is.	140,000	7,635,426
Montana	146,080	275,000	Porto Rico	3,600	953,243
Nebraska	77,510	1,225,000	Samoan Is.	79	5,800
Nevada	110,700	65,000			

1. The estimated population of the United States, excluding the islands, is 89,066,059. What is the total *including* them?

Find from the table the total area and population of the states bordering on:

2. The Atlantic Ocean.

3. The Gulf of Mexico.

4. The Great Lakes.

5. The Pacific Ocean.

6. The Mississippi River.

7. Your own State.

7. $4 and $7 are **like numbers,** because they are made up of the same *units*, dollars; similarly 3 qt. and 5 qt. are like numbers; but 3 qt. and 1 pt. are **unlike numbers.**

In order to add 4 gal. and 3 qt., how must 4 gal. be expressed?

8. *Only like numbers can be added or subtracted.*

Hence, in writing for addition numbers expressing dollars and cents, they should be so arranged that the decimal points stand in a column; cents then fall in columns with cents, and dollars with dollars.

9. In adding several long columns, write the sum of each column separately and add the partial sums, as in the example at the right.

$$\begin{array}{r}
\$96.50 \\
84.30 \\
106.35 \\
99.67 \\
208.95 \\
706.52 \\
500.08 \\
\hline
27 \\
3\ 1 \\
39 \\
26 \\
15 \\
\hline
\$1802.37
\end{array}$$

WRITTEN EXERCISES

Add:

The correct results of these four problems should be obtained in 3 minutes or less.

1. $145.35	**2.** $108.16	**3.** $208.25	**4.** $463.20
86.25	94.15	196.40	892.17
90.07	83.95	307.65	101.05
120.33	204.76	99.77	40.15
45.10	607.25	44.96	69.90
17.98	594.33	863.52	128.77

SUBTRACTION

10. Finding how much one number exceeds another is called **subtraction.** The sign of subtraction is – .

The result of subtraction is called the **difference** or **remainder.**

In subtraction the larger number is called the **minuend,** and the smaller the **subtrahend.**

11. Subtraction is most rapidly done by thinking what number added to the smaller number will make the larger.

563
198
365

Thus, in the problem at the left, we think "8 and **5** are 13" and write 5; "9 and **6** are 15" and write 6; "1 and **3** are 4" and write 3.

Test: 198 + 365 = 563.

ORAL EXERCISES

Subtract rapidly:

1. 89 46	**3.** 87 29	**5.** 409 161	**7.** 969 492	**9.** 765 291
2. 63 36	**4.** 45 19	**6.** 980 39	**8.** 738 370	**10.** 464 252

11. 80 in. 26 in.	**14.** 59 ft. 32 ft.	**17.** 65 lb. 46 lb.	**20.** 40 da. 26 da.
12. $7.63 2.48	**15.** $8.25 3.65	**18.** $1.93 .76	**21.** $2.45 .75
13. $10.05 3.05	**16.** $12.20 8.40	**19.** $19.75 6.95	**22.** $25.36 15.98

WRITTEN EXERCISES

Subtract:

Find how many of these you can solve correctly in 5 minutes.

1. $463.20 188.29	**3.** $635.89 263.74	**5.** 89735 28679	**7.** 46893 25635
2. $523.42 263.98	**4.** $998.65 325.87	**6.** 32577 8929	**8.** 73548 34767

9. $673.19	**11.** $727.65	**13.** 46969	**15.** 85927
87.64	423.84	15797	9876
10. $325.50	**12.** $827.88	**14.** 798354	**16.** 72555
169.75	525.97	259899	34969

Solve:

17. The cost of a city dwelling and site was $500,000; the site cost $175,500. What was the cost of the building alone?

18. In building a street the cost of grading and paving was $159,650; of grading alone, $45,650. What did the paving cost?

19. The elementary schools of a certain city had 45,268 pupils, of which 19,829 were in the Primary schools. How many were in the Grammar schools?

20. The area of New York State is 49,170 sq. mi., and that of Texas 265,780 sq. mi. Find the difference in their areas.

21. Alaska contains 590,884 sq. mi., and Texas 265,780 sq. mi. Alaska is how many square miles larger than Texas?

MAKING CHANGE

12. In making change we add to the amount of the purchase enough to equal the amount of money offered in payment.

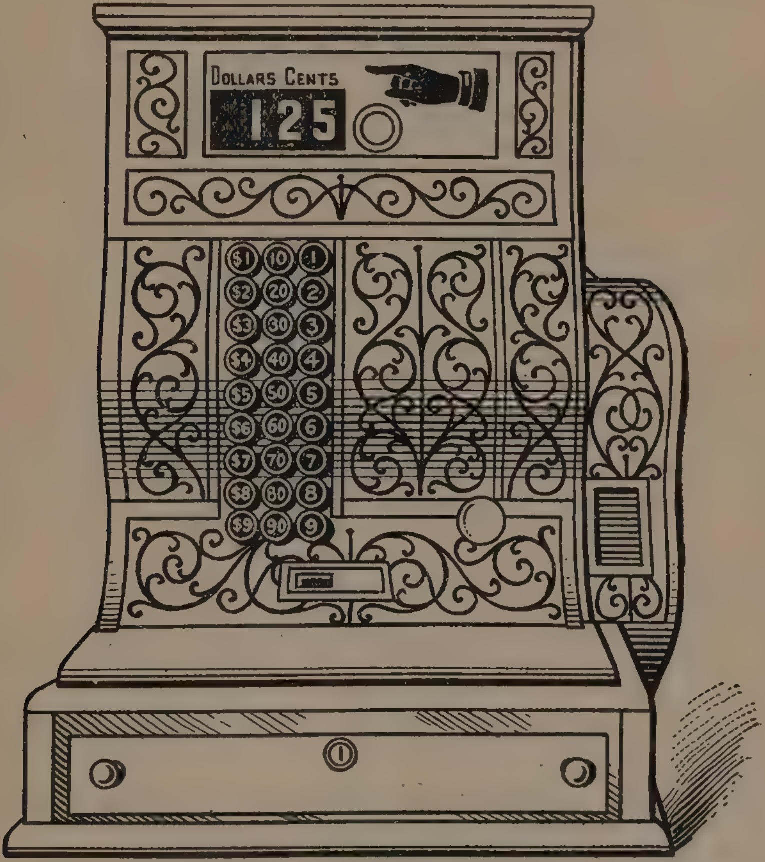
Thus, a customer bought a book for $1.20 and offered a two-dollar bill. The clerk said, as he gave the customer a 5-cent piece, a quarter, and a 50-cent piece, "One twenty-five, fifty, two dollars."

ORAL EXERCISES

What change was given in each of the following cases?

	PURCHASE	AMT. REC'D		PURCHASE	AMT. REC'D
1.	$2.25	$5.00	**4.**	$ .58	$1.00
2.	1.98	5.00	**5.**	.63	2.00
3.	.97	2.00	**6.**	.38	1.00

	PURCHASE	AMT. REC'D
7.	$2.45	$4.00
8.	3.75	5.00
9.	4.80	5.00
10.	5.75	10.00
11.	7.68	10.00
12.	.63	2.00
13.	$1.37	$5.00
14.	2.48	3.00
15.	3.70	4.00
16.	8.90	10.00
17.	9.01	10.00
18.	10.05	12.00



The picture shows a cash register. To record a purchase of $1.25, the clerk presses the key marked $1, then the key marked 20 in the next column, and the key marked 5 in the last column at the right. The machine prints this amount on a slip of paper within.

ORAL EXERCISES

Read the amount of money shown by the cash register when each of the following sets of keys is pressed:

	DOLLARS	CENTS	CENTS		DOLLARS	CENTS	CENTS
1.	1	30	5	**4.**	1	20	9
2.	2	50	5	**5.**	2		5
3.	3	10	6	**6.**	3	20	

WRITTEN EXERCISES

1. Write the numbers in the Oral Exercises above, using the dollar sign and the decimal point.

What keys would indicate:

2. Two dollars and thirty-five cents?

3. Three dollars and twenty-three cents?

4. Five dollars and seventy-five cents?

5. $1.18? **6.** $3.29? **7.** $9.99? **8.** $8.53?

9. If no key is pressed in the dollar column, this part of the register shows 0 dollars; similarly, zeros register no dimes and cents. What keys are pressed to indicate $0.17, or $.17? $.75? $1.05? $2.20? $.08?

What would be the total amount registered:

10. By pressing the 1-dollar key twice and the 5-dime key 5 times?

11. By pressing the 3-dollar key once, the 1-dime key twice, and the 5-cent key once?

12. By pressing the 7-dollar key six times, the 4-dime key three times and the 8-cent key five times?

13. What amount would be registered by pressing all of the keys in the right hand column?

14. What amount would be registered by pressing all of the keys in the three columns?

MULTIPLICATION

13. The process of taking one number as many times as an addend as there are units in another is called **multiplication.**

$$\left.\begin{array}{r} 36 \\ 36 \\ \underline{36} \\ 108 \end{array}\right\} = \left\{\begin{array}{r} 36 \\ \underline{3} \\ 108 \end{array}\right.$$

Thus, in the example at the left, 3 times 36 means 36 + 36 + 36, or 36 taken 3 times as an addend.

The number multiplied is called the **multiplicand.**

The number by which the multiplicand is multiplied is called the **multiplier.**

The result of multiplication is called the **product.**

$$\begin{array}{rl} 85 & \text{Multiplicand} \\ \underline{12} & \text{Multiplier} \\ 170 & \\ \underline{85} & \\ 1020 & \text{Product.} \end{array}$$

14. The multiplier and multiplicand are **factors** of the product.

15. The sign × is read "times" or "multiplied by."

Thus, 4 × 6 is read "4 times 6" or "4 multiplied by 6."

ORAL EXERCISES

Read, and supply the results in each line:

(Thus, the first line begins "2 times 2 are 4, 2 times 3 are 6.")

1.

2 ×	2	3	5	4	7	6	9	8	1	10	12	11

2.

3 ×	2	3	5	4	7	6	9	8	1	10	12	11

3.

4 ×	2	3	5	4	7	6	9	8	1	10	12	11

4.

5 ×	2	3	5	4	7	6	9	8	1	10	12	11

16. 2, 4, 6, 8, are called **multiples** of 2. 3, 6, 9, 12, are called multiples of 3, and so on.

ORAL EXERCISES

1. Name the multiples of 3 from 3 to 36.
2. Name the multiples of 4 from 4 to 48.
3. Name the multiples of 5 from 5 to 60.
4. Name the multiples of 10 from 10 to 120.

WRITTEN EXERCISES

1–7. Write tables similar to those on page 25, using 6, 7, 8, 9, 10, 11, 12 as the numbers at the left.

17. Numbers that relate to particular objects or measures are called **concrete numbers.** Those that do not are called **abstract numbers.**

Thus, $46, 8 cts., 2 books, 1.5 ft. are concrete numbers, while 46, 8, 2, 1.5 are abstract numbers.

18. *The multiplier must be an abstract number.*

19. *The product is of the same kind as the multiplicand.*

20. Example: What is the cost of 8 yd. of muslin at 12 cts. per yard?

Solution: $8 \times 12¢ = 96¢$.

The multiplier is 8, not 8 yd. It would be meaningless to say 8 yd. times 12 cts. The product is *cents* because the multiplicand is *cents*.

WRITTEN EXERCISES

Multiply:

Find how many problems you can do correctly in five minutes.

1.	645	**2.**	763	**3.**	496	**4.**	290	**5.**	975
	18		29		87		15		49

6. 863	**7.** 1296	**8.** 4089	**9.** 5201	**10.** 4631
247	834	735	910	208

11. 908 × 476. **12.** 826 × 542. **13.** 6309 × 569.

Solve:

14. How many tons of coal in 123 car loads of 45 tons each?

15. How many yards of wire in 215 rolls of 125 yd. each?

16. How many miles will an automobile travel in 26 days, if it runs 165 mi. a day?

17. A wall was built of 672 blocks of stone, each weighing 985 lb. What was the weight of the wall?

Multiplication by 10 and by 100

21. If any integer is multiplied by 10, the result is called a multiple of 10. What is the figure at the right in a multiple of 10?

22. *Any integer is multiplied by* 10 *by adding a zero at the right.*

1. To multiply by 20 is to multiply by 2 and annex how many zeros?

2. To multiply by 30 is to multiply by 3 and annex how many zeros?

23. Name some multiples of 100. Multiples of 100 have what two figures at the right?

24. *Any integer is multiplied by* 100 *by adding two zeros at the right.*

1. To multiply by 200 is to multiply by 2 and annex how many zeros?

2. To multiply by 300 is to multiply by 3 and annex how many zeros?

ORAL EXERCISES

1. Multiply by 10: 7; 8; 10; 9; 15; 12; 25.

2. State how to multiply any integer by 30. By 40. By 60. By 90. By 100.

3. Multiply by 100: 8; 5; 6; 10; 15; 40; 25.

4. State how to multiply any integer by 300. By 400. By 700. By 900. By 500.

WRITTEN EXERCISES

1. Multiply the following by 20: 17; 15; 9; 12; 48.

2. Multiply the following by 40: 15; 17; 20; 86; 54.

3. Multiply the following by 100: 9; 15; 17; 48; 125.

4. Multiply the following by 300: 25; 15; 9; 20; 40.

Solve:

5. When each acre of land yields 20 tons of cabbage, how many tons will 17 acres yield? 25 acres? 13 acres? 28 acres?

6. When land produces 40 bu. of rye per acre, how many bushels will 19 acres produce? 24 acres? 33 acres? 65 acres?

7. If each village block is 300 ft. long, how far do 9 blocks extend? 15 blocks? 29 blocks?

Multipliers Containing Zeros

25. When the multiplier contains zeros, the work is shortened. Thus:

The full work would be:		*In practice we use only:*	
(1)	(2)	(1)	(2)
869	624	869	624
230	203	230	203
000	1872	26070	1872
2607	000	1738	1248
1738	1248	199870	126672
199870	126672		

WRITTEN EXERCISES

Multiply:

1. $673 × 105.
2. 178 gal. × 207.
3. 763 qt. × 408.
4. 48 min. × 500.
5. $876 × 200.
6. 144 in. × 120.
7. 902 lb. × 420.
8. 1240 lb. × 505.
9. 624 ft. × 300.
10. 308 yd. × 201.
11. 777 hr. × 1001.
12. 630 T. × 420.

Multiplication of Dollars and Cents

26. When the multiplicand expresses dollars and cents, the product will have two places denoting cents.

Thus, in these examples the two figures at the right in each product indicate the number of cents in the result; for, as soon as the cents in the multiplicand have been multiplied, the cents in the product have been found. The decimal point separates the dollars from the cents.

(1)
$4.25
4
$17.00

(2)
$4.86
19
43 74
48 6
$92.34

WRITTEN EXERCISES

Multiply:

1. $4.96
15

2. $11.25
40

3. $8.05
76

4. $73.25
5

5. $9.81
77

6. $18.65
36

7. $20.08
206

8. $14.27
19

9. $112.75
62

10. 98¢ = $.98
17 17

11. 48¢ = $.48
20 20

12. 79¢ = $.79
218 218

Solve:

13. What is the cost of 468 bu. of apples at $1.35 a bushel?

14. What is the cost of 96 Dictionaries at $9.80 each?

15. How much does a man earn in 18 months at $245 a month?

Powers

27. PREPARATORY.

1. What is the area of a square 2 in. on a side?

2. What is the area of a square 3 ft. on a side? What is such a square called?

3. What is the area of a square flower-bed 4 ft. on a side?

4. How did we find the areas of these squares?

5. Compare the factors in each product:

$2 \times 2 = 4.$ $3 \times 3 = 9.$ $4 \times 4 = 16.$

28. The product of two equal factors is called the **square** of the repeated factor, or its **second power.**

Thus, in Exercise 5, above, 4 is the square (or second power) of 2; 9 is the square (or second power) of 3; and 16 is the square (or second power) of 4. The square of 2 (or 4) may be written 2^2; the square of 3 (or 9) may be written 3^2. Similarly, $25 = 5^2$; $100 = 10^2$; and so on.

29. The product of three equal factors is called the **cube or third power** of the repeated factor.

Thus, $2 \times 2 \times 2$ is the cube or third power of 2, and is written 2^3. Similarly, $3 \times 3 \times 3 = 3^3$, and so on.

ORAL EXERCISES

1. 25 is the square of what number? 36 is the second power of what number? 49 is the second power of what number?

2. 81 is the second power of what number? 64 is the square of what number? 100 is the square of what number?

3. State orally the squares of all integers from 1 to 10 inclusive.

4. 27 is the cube of what number? 64 is the cube of what number? 1000 is the third power of what number?

5. What is the cube of 4? Of 5? Of 3? Of 2? Of 10?

WRITTEN EXERCISES

Indicate each of the following by the second power of a number:

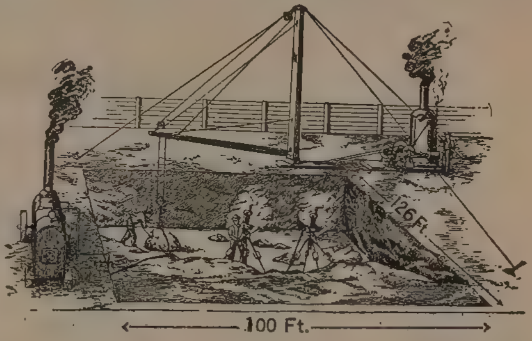
1. The number of days in seven weeks.
2. The number of things in 12 dozen.
3. The number of square feet in a square yard.
4. The number of square inches in a square foot.
5. The number of tens in one hundred.

Solve:

1. A certain building has a floor space 90 ft. wide and 115 ft. long. How many square feet in this space?

2. A building has a cellar 9 ft. deep as shown in the picture. How many cubic feet does the cellar contain?

3. It was necessary to drill and blast. 5 drills were used, each costing $5 per day to operate. It took 70 days to do the drilling. What did this part of the work cost?


126 Ft.
100 Ft.

4. A steam boiler was used to supply steam for the drill and for the derrick. Estimating the cost of labor in this service to be $12 per day, what did it cost for 70 days?

5. The boiler required 2 tons of coke a day, costing $2.50 per ton. What did the coke cost for 70 days?

6. 15 men were employed at $1.75 per day for each man. What did the labor cost for the 70 days?

7. 10 wagons each drawing 6 loads of stone daily were used for 70 days. How many loads did they take away in all?

8. A team and driver cost the contractor $6 a day. How much did the services of 10 teams for 70 days cost?

DIVISION

30. The ease with which we can divide numbers depends upon our knowledge of multiplication.

Thus, to divide 48 by 6 we must know how many 6's in 48.

ORAL EXERCISES

Divide rapidly:

Thus, 6 divided by 3 is 2; say merely "2."

1. $3\overline{)\,6,\quad 9,\quad 15,\quad 21,\quad 12,\quad 24,\quad 18,\quad 30,\quad 27}$

2. $4\overline{)12,\quad 8,\quad 20,\quad 32,\quad 24,\quad 40,\quad 28,\quad 36,\quad 16}$

3. $5\overline{)10,\quad 25,\quad 35,\quad 20,\quad 40,\quad 15,\quad 30,\quad 50,\quad 45}$

4. $6\overline{)24,\quad 36,\quad 12,\quad 48,\quad 18,\quad 60,\quad 42,\quad 54,\quad 30}$

WRITTEN EXERCISES

1—3. Make tables, similar to the above, for 7. For 8. For 9.

4. $$\begin{array}{r}2\text{ tens}+3\text{ units}\\ 3\overline{)6\text{ tens}+9\text{ units}}\end{array}=\begin{array}{r}20+3\\ 3\overline{)60+9}\end{array}=\begin{array}{r}(\quad)?\\ 3\overline{)69}\end{array}$$

Copy, and divide:

5. $3\overline{)129}$
6. $4\overline{)168}$
7. $5\overline{)205}$
8. $8\overline{)648}$
9. $9\overline{)549}$
10. $6\overline{)426}$
11. $8\overline{)728}$
12. $6\overline{)486}$
13. $4\overline{)808}$
14. $11\overline{)110}$
15. $7\overline{)567}$
16. $9\overline{)909}$
17. $7\overline{)427}$
18. $5\overline{)250}$
19. $10\overline{)200}$

31. The process of finding how many times one number is contained in another, or of separating a number into equal parts, is called **division**.

32. The number divided is called the **dividend**.

The number by which the dividend is divided is called the **divisor**.

The result of division is called the **quotient**.

33. Example: Divide 912 by 12.

$$\begin{array}{rr} & 76 \text{ Quotient} \\ \text{Divisor } 12 & \overline{)912} \text{ Dividend} \end{array}$$

12 is not contained in 9, but is contained in 91. There are 7 twelves in 91, and 7 remains undivided. Then, 7 tens and the 2 units make 72. 12 is contained 6 times in 72.

Test: $12 \times 76 = 912$.

34. One number is not always contained an exact number of times in another.

The number left undivided, because it is smaller than the divisor, is called the **remainder.**

Thus, 7 is contained 6 times in 45, with 3 over. 3 is the remainder.

It is customary to indicate the division of the remainder by the use of a fraction.

Thus, the division of 45 by 7 is expressed by $\begin{array}{r} 6\frac{3}{7} \\ 7\overline{)45} \end{array}$

WRITTEN EXERCISES

Divide, and indicate by a fraction the division of whatever remainder there may be:

1. $5\overline{)195}$ **3.** $5\overline{)206}$ **5.** $8\overline{)712}$ **7.** $9\overline{)828}$ **9.** $4\overline{)1288}$

2. $7\overline{)504}$ **4.** $8\overline{)128}$ **6.** $6\overline{)384}$ **8.** $6\overline{)820}$ **10.** $9\overline{)4572}$

Solve:

11. Find $\frac{1}{25}$ of \$450. Also $\frac{1}{7}$ of \$504.

12. Find $\frac{1}{6}$ of 516 ft. Also $\frac{1}{15}$ of 900 mi.

13. Find $\frac{1}{16}$ of 144 sq. in. Also $\frac{1}{12}$ of 1,728 cu. in.

14. A man earning \$9 per week received \$108. How many weeks had he worked?

15. A stack of 2,250 books has 15 shelves each holding the same number of books. How many books on a shelf?

16. 1,730 plants were set 24 in each row. How many rows were there? How many plants were left over?

35. The sign ÷ is read "divided by."

36. The dividend is the product of the divisor and the quotient. Consequently, the work of division may be tested by seeing if *the divisor* × *the quotient equals the dividend.*

Thus, 128 ÷ 4 = 32 is correct, because 4 × 32 = 128.

37. What is the quotient of 50 ft. divided by 10 ft.?

Dividend		Divisor		Quotient
50 ft.	÷	10 ft.	=	5

38. *If the dividend and divisor are concrete numbers, the quotient is an abstract number.*

39. *If both the dividend and the divisor are concrete, they must be like numbers.*

40. What is the quotient of 45 oz. divided by 5?

Dividend		Divisor		Quotient
45 oz.	÷	5	=	9 oz.

41. *If the dividend is a concrete number and the divisor an abstract number, the quotient is a concrete number like the dividend.*

WRITTEN EXERCISES

1. How many gallons in 27 quarts?

Solution: One gal. = 4 qt.
4 qt.) 27 qt.
6 times with 3 qt. over.
6 gal. and 3 qt. = $6\frac{3}{4}$ gal.

2. How many yards in 38 ft.?

3. How many bushels in 42 pk.?

4. How many gallons in a barrel containing 125 qt.?

5. How many weeks in 365 da.?

6. How many feet in 256 in.?

7. How many pounds in 320 oz.?

Division by 10 and by 100

42. PREPARATORY.

1. Divide 240 by 10.

2. Divide 240 by 20. How does the result differ from the answer to Exercise 1 above?

43. Since multiplying an integer by 10 adds a zero at the right,

An integer ending in zero is divided by 10 *by removing that zero.*

An integer ending in 0 may be divided by 20 by removing the zero and dividing by 2.

ORAL EXERCISES

1. Divide the following numbers by 10: 20; 700; 60; 50; 150; 290.

2. Divide the following numbers by 20: 40; 140; 80; 280; 400; 1200.

3. How may an integer ending in 0 be divided by 30? 70?

4. An orchard contained 600 trees in 30 equal rows. How many trees in a row?

5. A farmer carted to market 480 bu. of potatoes in loads of 30 bu. each. How many loads did he draw?

44. *An integer ending in two zeros may be divided by* 100 *by removing two zeros from the right.*

1. Divide 1800 by 100, then divide the result by 2.

2. Divide 1800 by 200. The results are the same.

An integer ending in two zeros is divided by 200 by removing the two zeros and dividing by 2.

Similarly, an integer ending in two zeros is divided by 300 by removing the two zeros and dividing by 3.

ORAL EXERCISES

1. Divide by 100: 300; 500; 1,200; 3,000; 9,900.

2. Divide by 200: 400; 10,000; 800; 900; 1,500.

3. How may an integer ending in two zeros be divided by 300? By 500? By 600? By 800? By 900?

4. A regiment of 1000 soldiers was drawn up in ranks of 200 each. How many ranks were there?

5. 4,900 bu. of grain were shipped in carloads of 700 bu. each. How many cars were used?

45. PREPARATORY.

1. If the units' digit of an integer is subtracted from it, the result ends in zero and is divisible by 10.

Thus, 127 – 7 = 120, a number divisible by 10.

2. If the number formed by the units' and tens' digits of an integer is subtracted from it, the result ends in two zeros and is divisible by 100.

Thus, 1438 – 38 = 1400, a number divisible by 100.

46. *To divide any integer by 10 cut off the units' digit, and consider it the remainder.*

EXAMPLE: Divide 389 by 10.

$$\begin{array}{r@{\,)\,}r|l} & 38 & \frac{9}{10} \\ \cline{2-3} 10 & 38 & 9 \end{array}$$

38, the part at the left, is the quotient; and 9, the part at the right, is the remainder.

47. *To divide any integer by 100 cut off the units' and tens' digits, and consider them the remainder.*

EXAMPLE: Divide 5,348 by 100.

$$\begin{array}{r@{\,)\,}r|l} & 53 & \frac{48}{100} = 53\frac{12}{25} \\ \cline{2-3} 100 & 53 & 48 \end{array}$$

53, the part at the left, is the quotient; and 48, the part at the right, is the remainder.

WRITTEN EXERCISES

1. Divide by 10: 46; 129; 385; 621; 1,286; 1,955.

2. Divide by 100: 2,936; 945; 186; 625; 1,006; 4,609.

3. A man paid $658 in 10 equal payments. How many dollars in each payment?

Long Division

48. Division by numbers larger than 12 is usually performed by a process called **long division.**

Example: Divide 40,397 by 197.

$$\begin{array}{r} 5 \\ 200 \\ 197\overline{)40397} \\ \underline{39400} \\ 997 \\ \underline{985} \\ 12 \end{array}$$

197 is not contained in 40, but is contained twice in 403. Then 197 is contained 200 times in 40,300. Subtracting 200 × 197 or 39,400 from the dividend, there remains 997.

We find how many times 197 is contained in 997 by noticing how many times 99 contains 19. 197 is contained 5 times in 997. Subtracting 985 from 997, there remains 12. The division ends here because the remainder 12 is less than the divisor 197.

$$\begin{array}{r} 205 \\ 197\overline{)40397} \\ \underline{394} \\ 997 \\ \underline{985} \\ 12 \end{array}$$

In actual work a shorter form is used. Thus, 197 is contained twice in 403, hence the first figure of the quotient is 2. Subtract 2 × 197 from 403. The remainder is 9. Bring down the next figure of the dividend, making 99. 197 is not contained in 99, hence the next figure of the quotient is 0. Bring down the last figure of the dividend, making 997. 197 is contained 5 times in 997, hence the next figure of the quotient is 5. Subtract 5 × 197 from 997, and the remainder is 12.

Test: 205 × 197 + 12 = 40,397.

The first figure obtained in the quotient should be placed over the last figure of the part of the dividend used to find it. Thus, in the example above, 2, in the quotient, is placed over 3 in the dividend.

WRITTEN EXERCISES

Divide and test:

A rapid worker can solve five of these problems correctly in five minutes.

1. 2,090 ÷ 55.
2. 1,625 ÷ 25.
3. 1,308 ÷ 19.
4. 57,602 ÷ 92.
5. 10,556 ÷ 28.
6. 2,672 ÷ 66.
7. 23,100 ÷ 77.
8. 10,935 ÷ 63.
9. 66,888 ÷ 223.
10. 33,534 ÷ 46.
11. 306,954 ÷ 137.
12. 816,124 ÷ 365.
13. 693,777 ÷ 707.
14. 459,678 ÷ 208.
15. 539,024 ÷ 532.

Solve:

16. A lot of Oriental rugs at $65 each cost a dealer $8,320. How many did he buy?

17. A farm cost $14,400 at $75 per acre. How many acres did the farm contain?

18. $300,625 of prize money was divided equally among 125 sailors. How many dollars did each receive?

19. Rhode Island has a population of 430,000 and an area of 1,250 sq. mi. How many inhabitants has the State per acre?

20. The District of Columbia has an area of 70 sq. mi. and a population of 278,740. How many inhabitants is this per square mile?

21. The total number of station agents employed by the railroads of the United States in a recent year was 32,283; the number per 100 mi. of road was 17. How many miles of railroad in the United States?

22. Some places on the earth are carried by its rotation through 25,000 mi. in 24 hr. How many miles is this per hour? Per minute?

23. Coal is weighed at the mines by the gross ton, 2,240 lb. How many gross tons in 311,360 lb.?

24. An automobile traveled 1,980 mi. in 18 da. What was the rate in miles per day?

Division of Dollars and Cents

49. Example: Divide \$196.42 into 14 equal parts.

$$\begin{array}{r} \$14.03 \\ 14\overline{)\$196.42} \\ \underline{14} \\ 56 \\ \underline{56} \\ 42 \\ \underline{42} \end{array}$$

If the quotient is placed over the dividend as in this solution and as explained in Sec. 48, we shall have found the dollars of the quotient when we have divided the dollars of the dividend; hence the decimal point of the quotient will come directly over the decimal point of the dividend.

WRITTEN EXERCISES

Divide:

1. \$225.60 ÷ 15.

2. \$156.25 ÷ 25.

3. \$168.08 ÷ 48.

4. \$330.34 ÷ 46.

5. \$105.56 ÷ 28.

6. \$216.32 ÷ 16.

7. \$169.65 ÷ 13.

8. \$20.90 ÷ 55.

9. \$643.06 ÷ 79.

Solve:

10. 21 wagons, including transportation, cost a dealer \$632.31. What did they cost him per wagon?

11. A stock dealer bought 31 Mexican horses, which cost him delivered \$837.31. How much was this per horse?

12. A house cost its builder \$4,650; its floor area was 1,500 sq. ft. How much did it cost per square foot of floor space?

13. A broker sold 105 bales of cotton for \$5,853.75. How many dollars did he receive per bale?

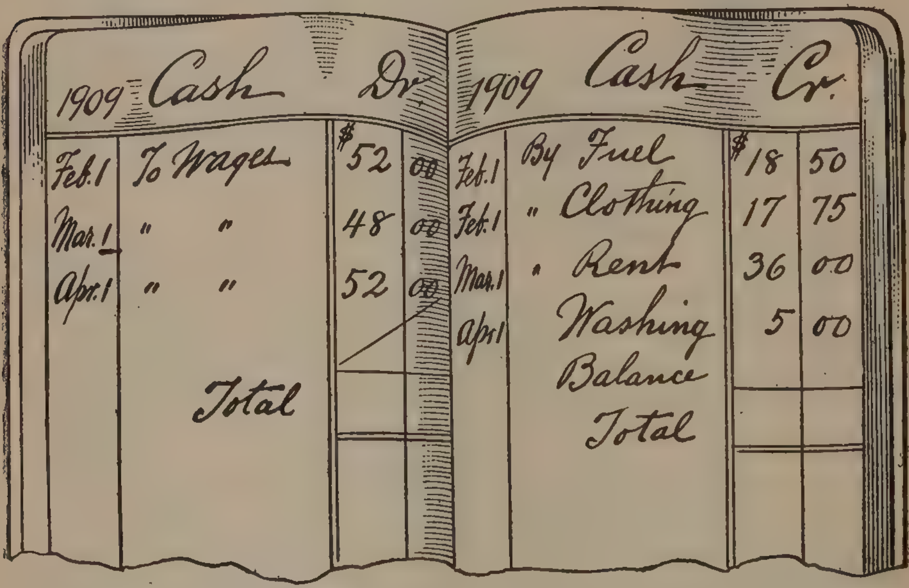
14. A real estate agent sold 43 lots for \$5,439.50. How much was this per lot?

15. If a man's salary is \$2,250 a year, how much does he earn per week?

16. A fund of \$25,180.75 was applied to the relief of 18,000 persons. How much was this per person?

CASH ACCOUNTS

50. It is desirable to keep an account of money received and paid out. Such a record is called a **cash account.**



1909	Cash Dr.			1909	Cash Cr.		
Feb. 1	To Wages	$52	00	Feb. 1	By Fuel	$18	50
Mar. 1	" "	48	00	Feb. 1	" Clothing	17	75
Apr. 1	" "	52	00	Mar. 1	" Rent	36	00
				Apr. 1	Washing	5	00
					Balance		
	Total				Total		

Dr. stands for debtor, and *Cr.* for creditor. *Cash* may be thought of as a person holding the cash, hence as a debtor to the person keeping the account for all money received and as creditor for all money paid out.

51. Finding how much must be added to one side of an account to make the footings of the two sides equal is called **balancing** the account.

The double lines under the columns show that the account has been balanced to date, and that the amount on hand, if such there be, is the same as the balance on the credit side.

1. On which side of the account are entered the amounts of money, or cash, received? On which side are entered the amounts of money, or cash, paid out?

2. In the account on page 40, on what items was money received from January 1 to April 1?

3. What is the total of the debit side? Of the credit side?

4. How much should be entered opposite the word *balance* to make the footing of the credit side also $152?

WRITTEN EXERCISES

Find the balance in each of these accounts:

1. Mr. Minor had on hand, March 1, $5. Earned to March 20, $25. March 31, sold a watch for $15. March 10, purchased a pair of shoes for $4. March 30, paid for board, $10.

2. Received, May 1, $25.25. Received, May 2, $50. Bought, May 5, a suit of clothes for $14.50. Paid board, May 20, $9. Incidentals to May 30, $5.75. Received May 31, a month's wages, $62.

3. Balance on hand, July 1, $335. Received from rents, July 3, $75. Paid for board, July 15, $35. Paid for laundry, July 18, $3.35. Paid for incidentals, July 20, $28.87.

4. On hand, May 1, $25. Received for wages, May 15, $5. Paid for board, May 16, $7.50. Paid note, May 20, $25. Received for wages, May 31, $40.

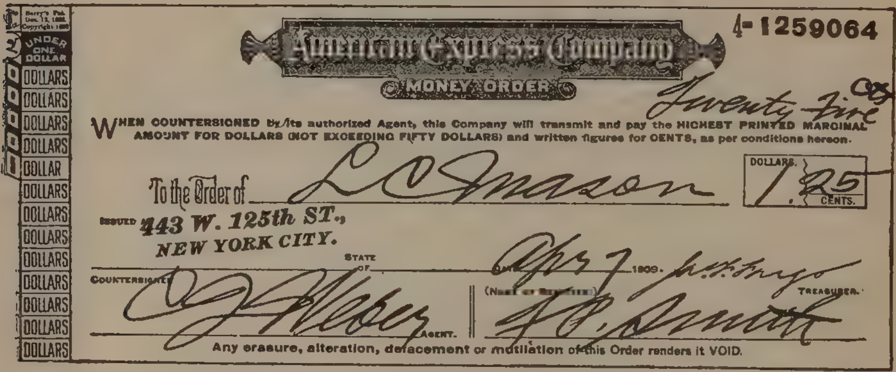
5. On hand, July 1, $365. Paid for labor, July 3, $12.45. Paid for groceries, July 5, $18.35. Received interest, July 8, $175. Paid for dry-goods, July 15, $67.75. Paid taxes, July 23, $95.75.

6. On hand, Apr. 1, $25. Paid rent, Apr. 2, $15.50. Paid labor, Apr. 15, $17.75. Paid for merchandise, Apr. 25, $500.66. Received by sale of merchandise to Apr. 28, $600.25.

7. Balance on hand, Jan. 1, $500. Received from rents, March 1, $50; July 1, $75; Dec. 1, $125. Received from interest, Nov. 1, $205. Paid grocery bills, Apr. 1, $45.75; July 1, $60; Dec. 5, $75. Paid dry-goods bill, Sept. 1, $100.75. Paid for incidentals to Dec. 31, $125.35.

MONEY-ORDERS

52. Money may be paid by means of postal **money-orders,** which are obtainable at post-offices, or by means of express money-orders obtainable at the offices of the express companies.



4-1259064

American Express Company

MONEY ORDER

Twenty-five cts

WHEN COUNTERSIGNED by its authorized Agent, this Company will transmit and pay the HIGHEST PRINTED MARGINAL AMOUNT FOR DOLLARS (NOT EXCEEDING FIFTY DOLLARS) and written figures for CENTS, as per conditions hereon.

To the Order of L C Mason

DOLLARS 1. 25 CENTS.

ISSUED 443 W. 125th ST., NEW YORK CITY.

STATE OF Apr 7 1909. J. C. Fargo TREASURER.

COUNTERSIGNED AGENT.

Any erasure, alteration, defacement or mutilation of this Order renders it VOID.

The fees for postal money-orders, payable in the United States (including Hawaii, Porto Rico and the Philippines), in Canada, and in Cuba, are:

For orders for sums not exceeding	$2.50,	3¢
" " over $2.50 and not "	5.00,	5¢
" " " 5.00 " " "	10.00,	8¢
" " " 10.00 " " "	20.00,	10¢
" " " 20.00 " " "	30.00,	12¢
" " " 30.00 " " "	40.00,	15¢
" " " 40.00 " " "	50.00,	18¢
" " " 50.00 " " "	60.00,	20¢
" " " 60.00 " " "	75.00,	25¢
" " " 75.00 " " "	100.00,	30¢

WRITTEN EXERCISES

1. Carl sent for a camera which cost $15; a tennis-racket, $3.75; and a rifle, $25. What was the fee for the money-order sent in payment? What was the total cost?

2. What is the fee for a money-order for $1.75? For $2.50? For $2.55? For $4.20? For $6? For $18?

3. A lady ordered a rug for $25; 20 yd. of carpet at $1.32 a yard; and a couch cover for $8.75. What did she pay for all? What was the total cost of the money-order?

4. A musician ordered a mandolin for $8; a guitar for $7.50; a violin for $55.75; and a banjo for $12. What was the total cost of the money-order?

5. Mr. Mason bought of Ward and Company 1 box of soap at $4; 1 bbl. of flour, $3.50; and 50 lb. of dried meat at 9 cts. a pound. What was the cost of a money-order to pay for these goods?

6. Mrs. Martin bought of Thomas Smith 3 yd. of lace at $.65 a yard; 1 doz. handkerchiefs for $4.75; 4 yd. of silk at $1.15 a yard; and a table-cloth for $8. What was the cost of the money-order to pay for these goods?

7. Mr. Carr bought of Milton Woodward & Co. a harness for $25; a saddle for $21.85; a horse-blanket for $3.75; and a set of sleigh-bells for $2.75. What did he pay for all? What was the total cost of the money-order?

State the cost of a money-order with which to pay each of the following bills:

8. Bought 25 suits of clothing at $9.50 each and 15 overcoats at $16.25 each.

9. Bought 7 copies of Eliot's "Silas Marner" at 35 cts. a copy, 12 copies of Milton's "Paradise Lost," at $1.25 a copy, and 15 Coleridge's "Ancient Mariner" at 40 cts. a copy.

10. Bought 40 lb. of nails at 3 cts. a pound, 25 yd. of wire netting at 95 cts. a yard, and 3 axes at $1.50 each.

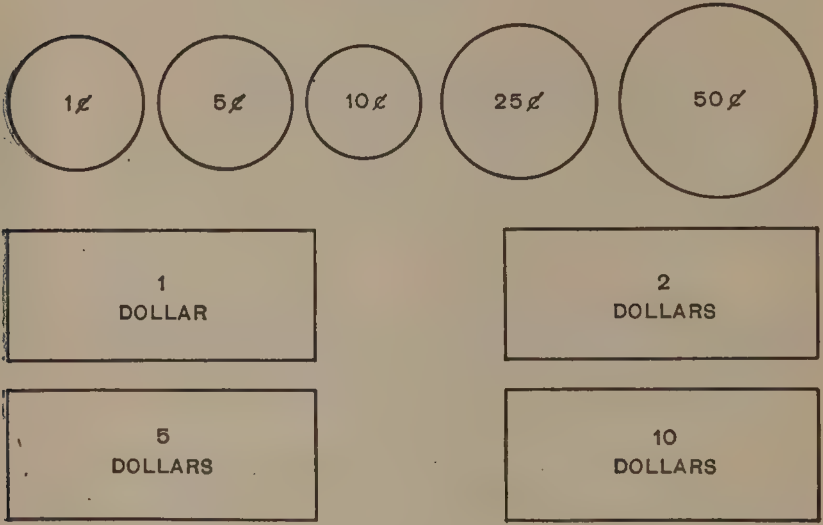
11. Bought 3 sleds for $2.50 each, and 5 pairs of skates at $1.75 each.

12. Bought 25 copies of *Appleton's Magazine* at 15 cts. a copy and 38 copies of the *Saturday Evening Post* at 5 cts. a copy.

KEEPING STORE

53. Prepare and mark paper forms similar to these:

MONEY



ARTICLES FOR SALE

a	b	c	d
HAT 75¢	SKATES $ 1.25	READER 30¢	CANDY 35¢
e	f	g	h
1 TON COAL $ 6.75	1 SK. FLOUR $ 1.20	1 BU. POTATOES 60¢	1 LB. SUGAR 5¢
i	j	k	l
1 LOAF BREAD 5¢	1 CAKE 40¢	1 DOZ. ORANGES 30¢	WAFERS 10¢

As many sets may be made as are needed for the work; other tickets representing articles for sale may be made.

1. Some pupils may act as salesmen and others as customers. Let each customer take at least 3 articles and pay for them in money, offering the correct amount.

2. Let each customer take 3 articles and pay with the smallest single piece of money whose value exceeds his purchase. Require the salesman to make the correct change.

ORAL EXERCISES

1. Carl, acting as buyer, purchased the three articles in the first column on page 44; that is, those shown by tickets *a*, *e*, *i*. How much did they cost?

2. Susie purchased those in the second column; that is, *b*, *f*, *j*. How much did she pay for them?

3. Myron purchased *d*, *h*, *l*. How much did they cost?

Find the cost of:

4. *a*, *b*, *c*, *d*.

5. *a*, *b*, *c*, *f*.

6. *c*, *d*, *g*, *h*.

7. *e*, *f*, *g*, *h*.

8. *e*, *f*, *i*, *j*.

9. *b*, *c*, *f*, *g*.

10. *i*, *j*, *k*, *l*.

11. *g*, *h*, *k*, *l*.

12. *f*, *g*, *j*, *h*.

WRITTEN EXERCISES

1. Write the names of the articles and prices indicated in Exercises 4 and 5 above. Find their total cost.

Find the total cost after writing the names and prices of the articles indicated in the exercises above:

2. 4 and 6.

3. 8 and 9.

4. 10 and 12.

5. 5, 8, and 11.

6. 7, 8, and 9.

7. 5 and 6.

8. 7 and 9.

9. 11 and 12.

10. 6, 9, and 12.

11. 10, 11, and 12.

12. 7 and 8.

13. 10 and 11.

14. 4, 7, and 10.

15. 4, 5, and 6.

16. 6, 8, and 10.

17. Write a bill with your own name as purchaser, and another pupil's name as dealer, for the items indicated in Oral Exercise 11. (See page 10 for a correct form of bill.)

18. Similarly, write a bill for the items indicated in Written Exercise 4. Also 5. Also 11. Also 16.

POSTAGE PROBLEMS

54. *Domestic Postage:* The following are the rates to all parts of the United States, including Hawaii, Porto Rico, and the Philippine Islands. The same rates apply to England, Canada, Mexico, and Cuba.

First-Class Matter: Letters or sealed matter 2 cents an ounce or fraction thereof. Postal cards 1 cent each; with paid reply card, 2 cents each.

Second-Class Matter: Newspapers and other periodical publications, when sent by publishers or news-agents, 1 cent per pound or fraction thereof; when sent by others, 1 cent for each four ounces or fractional part thereof.

1. What is the postage on a letter weighing $\frac{3}{5}$ oz.? $2\frac{1}{2}$ oz.? $4\frac{1}{4}$ oz.? $1\frac{3}{4}$ oz.? $1\frac{3}{8}$ oz.? 3 oz.? $6\frac{1}{2}$ oz.?

2. A publisher sends 15,000 copies of his paper by mail. Each paper with its wrapper weighs $4\frac{1}{2}$ oz. He pays for the total weight at 1 ct. per lb. What is his postage bill?

3. One of the subscribers remails his copy to a friend. How much postage must he pay?

4. Anna found that one of the monthly magazines weighed $13\frac{1}{4}$ oz. If 125,000 copies of it were sent by mail, what was the postage bill of the publisher for that month?

5. What is the greatest weight a newspaper can have that requires 3 cts. postage when remailed?

6. How many ounces or less does a letter weigh on which the postage is 6 cts.?

7. A manuscript weighs 1 lb. $1\frac{1}{4}$ oz. What is the postage on it at letter rate?

8. What is the cost of a gross of postal cards with paid reply cards? What is the cost of 500?

9. What is the postage on 10,000 copies of a 4-oz. newspaper when sent by the publisher?

FACTORS AND MULTIPLES

Divisibility

55. PREPARATORY.

1. Name the multiples of 2 from 2 to 30. If each of these multiples is divided by 2, what is the remainder?

These numbers are said to be *divisible* by 2.

2. Name the multiples of 3 from 3 to 30. If each of these multiples is divided by 3, what is the remainder?

These numbers are said to be *divisible* by 3.

56. An integer which can be divided by another integer without a remainder is said to be **divisible** by the other integer.

57. Integers divisible by 2 are called **even numbers,** and all other integers are called **odd numbers.**

58. EASY TESTS OF DIVISIBILITY:

1. An integer is divisible by 2 if the units' figure is even.

2. An integer is divisible by 4 if the 2 figures at the right represent a number divisible by 4.

Thus, 1,524 is divisible by 4, because 24 is divisible by 4.

3. An integer is divisible by 3 if the sum of its digits is divisible by 3.

Thus, 24,120 is divisible by 3, because $2 + 4 + 1 + 2 + 0 = 9$ is divisible by 3.

4. An integer is divisible by 5 if the units' figure is 0 or 5.

5. An integer is divisible by 8 if the last three figures at the right represent a number divisible by 8.

Thus, 56,816 is divisible by 8, because 816 is divisible by 8.

6. An integer is divisible by 9 if the sum of its digits is divisible by 9.

Thus, 24,129 is divisible by 9, because $2 + 4 + 1 + 2 + 9 = 18$ is divisible by 9.

Factors and Divisors

59. An integer divisible only by itself and one is called a **prime number.** Numbers not prime are **composite numbers.**

Thus, 2, 3, 5, 7, are prime numbers; 4, 6, 8, 14, are composite.

60. The numbers that, multiplied together, form a product are called the **factors** of the product. Every factor of a number is a divisor of it.

Thus, in $2 \times 5 \times 4 = 40$, 2, 5, and 4 are factors and divisors of 40.

61. Factors, or divisors, that are prime numbers are called **prime factors** or **prime divisors.**

62. Finding the prime factors or divisors of a number is called **factoring** the number.

Example: Find the prime factors of 4,158.

The last digit, 8, is even; hence one divisor is 2.

The quotient 2,079 is divisible by 3, since $2 + 0 + 7 + 9 = 18$, which is divisible by 3.

$$\begin{array}{r} 2\underline{)4158} \\ 3\underline{)2079} \\ 3\underline{)\ 693} \\ 3\underline{)\ 231} \\ 7\underline{)\ \ 77} \\ 11 \end{array}$$

The next quotient 693 is divisible by 3 for the same reason; so also is the next.

The next quotient, 77, is evidently a multiple of 7; therefore the prime factors of 4,158 are 2, 3, 3, 3, 7, and 11.

Test: $2 \times 3 \times 3 \times 3 \times 7 \times 11 = 4{,}158$.

After dividing by 2, the quotient is divisible by 9 because $2 + 0 + 7 + 9 = 18$, is divisible by 9. The work might be shortened by dividing by 9 instead of by 3 and writing 3×3 for the divisor 9 in the result.

WRITTEN EXERCISES

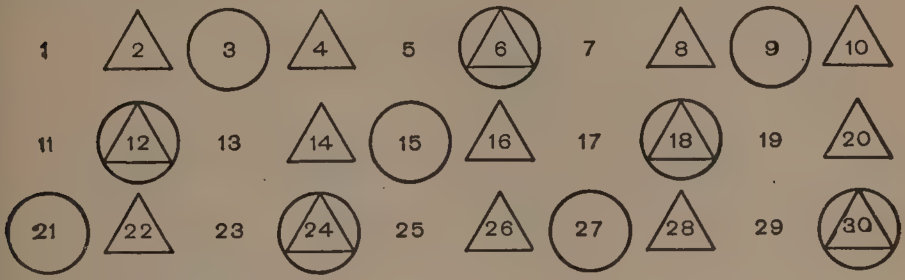
Find the prime factors of:

1. 210.
2. 2,240.
3. 2,427.
4. 5,390.
5. 288.
6. 1,625.
7. 1,485.
8. 3,645.
9. 625.
10. 2,178.
11. 1,000.
12. 2,121.
13. 1,182.
14. 6,160.
15. 6,250.

Common Divisors

63. PREPARATORY.

1. Find the numbers in the table that are divisible by 2. Read those divisible by 3.



2. By what form are all the numbers divisible by 2 enclosed?

3. By what form are all the numbers divisible by 3 enclosed?

4. How are all the numbers divisible by both 2 and 3 enclosed?

64. Two numbers that are divisible by the same number are said to have a **common divisor.**

ORAL EXERCISES

1. Name a common divisor of all the numbers in the table. Of the numbers enclosed by a triangle. By a circle.

2. Name a common divisor of the numbers enclosed by both a triangle and a circle.

WRITTEN EXERCISES

1. Write on the board in ten rows the numbers from 1 to 100. Cross in white all the even numbers; in red all the numbers divisible by 3; in yellow all the numbers divisible by 5.

2. What kind of numbers are those not crossed? Are there any odd numbers crossed?

3. What digits occupy units' place in all numbers divisible by 5?

4. With what colors are the numbers divisible by 2 and by 3 crossed? Read these numbers. Read the multiples of 6 from the table.

5. Read the numbers not divisible by 2, by 3, and by 5.

6. What common divisor have all even numbers? All multiples of 3? Of 5? Of 12?

Find a common divisor of each pair of numbers:

7. 18, 27.
8. 45, 30.
9. 24, 36.
10. 22, 55.
11. 16, 40.
12. 21, 56.
13. 15, 35.
14. 39, 26.

15. Name the fractions of the following list in which the numerator and the denominator have a common divisor:

$\frac{2}{4}$ $\frac{3}{5}$ $\frac{4}{6}$ $\frac{5}{8}$ $\frac{3}{8}$ $\frac{6}{8}$ $\frac{5}{10}$. $\frac{4}{100}$ $\frac{25}{100}$ $\frac{17}{25}$ $\frac{9}{10}$.

65. The largest number that will divide each of two or more numbers is called their **greatest common divisor.** This name is abbreviated by using **g. c. d.**

66. *To find the greatest common divisor of several numbers, first separate the numbers into their prime factors; then multiply together all the common factors, taking any repeated factor as many times as it is found in the number where it occurs least often.*

EXAMPLE: Find the greatest common divisor of 36 and 56.

Solution: Since, $36 = 3 \times 3 \times 2 \times 2$,
and $56 = 7 \times 2 \times 2 \times 2$,
the greatest common divisor of 36 and 56 is 2×2 or 4.

WRITTEN EXERCISES

Find the greatest common divisor of each set:

1. 18, 45, 1089.
2. 88, 121, 2244.
3. 48, 210, 288.
4. 1024, 1728.
5. 3645, 5390.
6. 2412, 6600.
7. 625, 1800.
8. 2121, 1484.
9. 144, 1728.

Find the greatest common divisor of the numerator and denominator of each fraction:

10. $\frac{210}{288}$. **12.** $\frac{121}{2244}$. **14.** $\frac{147}{189}$. **16.** $\frac{1000}{6250}$.

11. $\frac{1484}{2828}$. **13.** $\frac{1024}{7168}$. **15.** $\frac{1182}{2400}$. **17.** $\frac{216}{6160}$.

Multiples

67. The result of multiplying a number by an integer is **a multiple** of the number.

Thus, in 3 × 7 = 21 the result, 21, is a multiple of 3.

68. Preparatory.

1. Write the multiples of 3 from 3 to 39. Of 4 from 4 to 40.
2. What numbers are found in both sets?

69. An integer that is a multiple of other integers is called **a common multiple** of those integers.

Thus, 21 is a common multiple of 3 and 7.

ORAL EXERCISES

1. What are the common multiples of 3 and 4 as far as 48? Which is the smallest of these multiples?

2. What are the common multiples of 4 and 5 as far as 60? Which is the smallest of these multiples?

3. Some boys formed a military company and found that they could march 3, 4, or 5 abreast in full ranks. What number is a multiple of 3, 4, and 5? What is the least number that is a multiple of 3, 4, and 5? What is the least number of boys that could have been in the company?

70. The smallest common multiple of two or more numbers is called their **least common multiple.** This name is abbreviated by using **l. c. m.**

ORAL EXERCISES

1. What is the least common multiple of 4 and 6? Of 5 and 6? Of 3 and 5? Name common multiples for each set.

Find by inspection the least common multiple of:

2. 6, 9.
3. 4, 6.
4. 4, 8.
5. 8, 10.
6. 3, 4, 6.
7. 3, 5, 10.
8. 4, 8, 10.
9. 5, 6, 15.

71. *To find the least common multiple, factor the given numbers and multiply together all the different prime factors. If any factor occurs more than once in any one of the numbers, take it as many times as it is found in the number where it occurs most often.*

72. Example: Find the least common multiple of 12, 21, 39.

$24 = 2 \times 2 \times 2 \times 3.$
$28 = 2 \times 2 \times 7.$
$39 = 3 \times 13.$

The prime factors are 2, 3, 7, and 13. The factor 2 is repeated in two of the numbers. It occurs most often in the first, namely, 3 times; hence it must occur 3 times in the least common multiple.

Therefore, the l. c. m. equals $2 \times 2 \times 2 \times 3 \times 7 \times 13 = 2184$.

WRITTEN EXERCISES

Find, by factoring, the least common multiple of:

1. 36, 42.
2. 27, 24.
3. 14, 22.
4. 18, 28.
5. 14, 28.
6. 22, 32.
7. 12, 18, 27.
8. 25, 60, 10.
9. 32, 48, 72.

10. A florist when decorating a hall found that he had just enough roses to enable him to put either 12, 15, or 18 roses into each vase and have no roses left. How many roses did he have?

11. Harry could spend all of his money by buying tops at 10 cts. each or at 15 cts. each, or at 25 cts. each. How much money did he have?

WRITTEN EXERCISES

Find the l. c. m. of the denominators in each set:

1. $\frac{1}{10}, \frac{7}{12}, \frac{5}{16}$.
2. $\frac{4}{15}, \frac{7}{30}, \frac{11}{75}$.
3. $\frac{7}{18}, \frac{8}{27}, \frac{13}{99}$.
4. $\frac{5}{32}, \frac{7}{48}, \frac{11}{64}$.
5. $\frac{1}{18}, \frac{11}{36}, \frac{19}{45}$.
6. $\frac{1}{21}, \frac{2}{49}, \frac{8}{63}$.
7. $\frac{9}{25}, \frac{8}{15}, \frac{1}{10}$.
8. $\frac{1}{11}, \frac{1}{77}, \frac{1}{121}$.

Cancellation

73. PREPARATORY.

1. How many times are 2 *apples* contained in 4 *apples*?
2. How many times are 2 *threes* contained in 4 *threes*?
3. How many times are 2 × 3 contained in 4 × 3?
4. How many times is 2 × *any number* contained in 4 × *any number?*

Since dividing 4 × 3 by 2 × 3 is the same as dividing 4 by 2, we may cancel, or cross, the 3 in the dividend and in the divisor without changing the quotient. When written in fractional form the work appears thus:

$$\frac{4 \times 3}{2 \times 3} = \frac{4 \times \cancel{3}}{2 \times \cancel{3}} = \frac{4}{2} = 2.$$

74. *The same factors can be canceled from the divisor and the dividend without changing the quotient.*

EXAMPLE: $$\frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{7}} \times \overset{1}{\cancel{11}}}{5 \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{11}} \times 13} = \frac{1}{5 \times 13} = \frac{1}{65}.$$

WRITTEN EXERCISES

Write in the above form and divide by cancellation:

1. 12 × 28 × 77 divided by 8 × 24 × 154.
2. 25 × 18 × 11 divided by 5 × 11 × 3.
3. 35 × 63 × 121 divided by 15 × 77 × 11.
4. 900 × 360 × 55 divided by 12 × 5 × 2 × 11.

REVIEW

ORAL EXERCISES

1. What are the names of the first four orders in a whole number?

2. What is the name of the seventh order in a whole number? What is the name of the third period?

3. It costs 25 cts. to telephone from a certain city to another, 3 minutes being allowed for conversation; 10 cts. is charged for each additional minute. What does it cost to telephone for 5 minutes between those cities?

4. The cost of a telegram of 10 words (or less) between Syracuse and Rochester is 25 cts., and 2 cts. for each word above 10. (This rate is expressed thus: 25–2.) State the cost to send between these places the following message:

Can not come this week; business prevents; arrive New York City Tuesday; will write.

5. What would it cost to send a 12-word message from Chicago to New York City at 40–3? A 14-word message?

6. State the cost of sending, at the rate given, a 15-word day message from Chicago to each of the following cities:

PLACE.	RATE.		PLACE.	RATE.	
	Day.	*Night.*		*Day.*	*Night.*
San Francisco...	75–5	75–5	New Orleans......	50–3	30–2
Detroit.........	25–2	25–1	Galveston, Texas..	60–4	40–3
New York......	40–3	30–2	Kansas City, Mo..	30–2	25–1

NOTE.—Address and signature are free.

7. What would be the cost of sending a 15-word night message to each?

8. What is the cost of a 25-word day message to each?

WRITTEN EXERCISES

Write:

1. Three million, two hundred seventy thousand, sixty-five.

2. Twenty-five million, ninety thousand, one hundred twenty.

3. Eighty-seven million, eighty-seven thousand, forty-nine.

4. Twenty million, forty thousand, one hundred forty.

5. In Roman notation: 19; 26; 35; 39; 43; 56; 49; 100; 97; 63; 45; 90; 75.

6. The squares of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. In what different figures do these squares end? Then, in what figures may the squares of integers end?

Solve:

7. What is the g. c. d. of 156 and 1728? Of 144 and 256?

8. What is the l. c. m. of 121, 77, 14? Of 14, 196, and 1794?

9. Simplify by cancellation $\frac{18 \times 49 \times 121}{12 \times 90 \times 77 \times 54}$.

10. Simplify by cancellation $\frac{24 \times 38 \times 95}{6 \times 15 \times 361}$.

11. A merchant gained $1,325.75 the first year, $2,195.50 the second year, and lost $989.85 the third year. How many dollars did he gain in the three years?

12. A ranchman paid $565 for sheep, $1,250 for cattle, and $863 for horses. How much had he left out of $3,000?

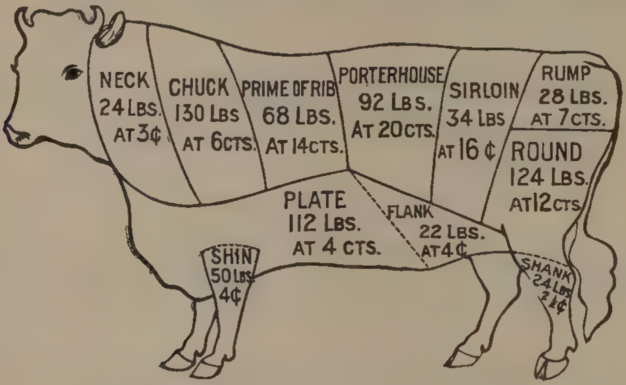
13. A landowner received $25,673 from the sale of lands and $2,565 for rent, and expended $565 for repairs. How much had he left?

14. A conductor received on one trip $9.45 in fares. Of this amount, $\frac{5}{9}$ was in nickel coins and the rest in dimes. How many dimes did he receive?

15. A man started in business with $2,765.25 in cash and a stock of goods worth $5,850.25. At the end of the first year he had $3,650.50 in cash and a stock of goods worth $4,985. Did he gain or lose, and how much?

16. A man bought a horse and carriage for $300.50. He sold the horse for $125 at a loss of $50.50. What was the cost of the carriage?

17. The diagram shows the different kinds of meats as obtained by a standard method of cutting beef. The figures are approximate. Find the total weight of beef in one animal as shown in the diagram.



18. Find the value of each kind of beef, and the total value of one animal.

19. Which kind contains the greatest number of pounds? The least? Which has the greatest value per pound? The greatest total value?

20. The following table gives the parts into which dressed mutton is cut for market, and the average price of each kind. Find the number of pounds in an animal and the total value.

Leg	22.5 lb., 10¢.	Rib	14.5 lb., 9¢.
Loin	17.5 lb., 9¢.	Chuck	19.8 lb., 1¾¢.

21. Answer questions about mutton similar to those in Exercise 19.

FOOD AND PRODUCTS.	ROSE.	NORA.
Food consumed:	*Pounds.*	*Pounds.*
Clover hay	3,873.10	3,547.50
Corn silage and green corn	8,107.50	8,082.00
Rape	2,482.00	2,482.00
Cowpeas	750.00	750.00
Corn meal	1,692.70	1,653.50
Wheat bran	1,256.50	1,158.00
Ground oats	670.10	567.60
Gluten meal	516.50	573.00
Old process linseed-meal	848.00	785.30
Milk produced	11,329.00	7,759.40
Butter fat produced	564.82	298.64

22. The table shows the foods consumed by two cows, and the milk and butter produced by them in one year. Find the total number of pounds of food consumed by each animal in the year.

23. Find the difference in the amounts of milk produced. In the amounts of butter produced.

24. If 10 quarts of milk weighs 23 lb., how many quarts did each cow produce in a year?

25. What is the total value of the milk produced by each cow at 5 cts. a quart? Of the butter at 16 cts. a pound?

26. If $\frac{3}{4}$ lb. of butter is lost per 100 lb. of milk by setting in pans instead of using a separator, how many pounds of butter are lost by setting 800 lb. of milk in pans? What is the money loss, if butter is worth 20 cts. a pound?

27. What would be the loss from 1 ton of milk? If this loss is $\frac{1}{6}$ of the whole product, how many pounds in the whole?

28. A good cow averages 5,000 lb. of milk a year. How many pounds of butter are lost by setting the milk in pans? What would be the amount of loss when butter is worth 32 cts. a lb.?

29. The coinage of gold at the United States mints in 1880 was $62,308,279; in a recent year it was $99,272,942.50. How much more was coined in the latter year than in 1880?

30. If the wages of 10,000 men in the employ of a certain company are cut 10 cts. a day, how many dollars will the company save in wages in a year of 300 working-days?

31. The Canadian power canal at Sault de Ste. Marie furnishes 20,000 horse-power. The canal on the Michigan side furnishes 2½ times as much. How many horse-power does the latter furnish?

32. In a profit-sharing concern each employee earning $500 a year receives a dividend equal to ⅛ of his wages. How much does he receive from the company in a year?

33. There were 150,000 automobiles in use in the United States in a recent year. Their total value was $245,400,000. If the cars were of equal value what was the value of each?

34. In this country fifty companies, whose average capital is $300,000, manufacture automobiles. What is the total capital of these companies?

35. In a recent year 1,540 automobiles were imported from foreign countries, ¾ of which were from France. How many came from other countries?

36. There are 1,200 Sunday schools in France and 38 times as many in Great Britain and Ireland. How many are there in the latter country?

37. South America has 350 Sunday schools averaging 425 pupils each. How many pupils in all of these schools?

38. A large power station has four electric dynamos making 7,000 horse-power each and six making 11,500 horse-power each. How many horse-power does the station furnish?

39. A coast line company has 8 steamers running out of New York, having an average capacity of 965 passengers. What is their total capacity?

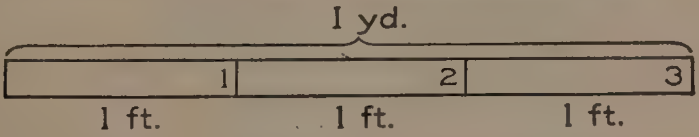
CHAPTER II

COMMON FRACTIONS

75. One or more of the equal parts of a unit, or of a quantity thought of as a whole thing, is called a **fraction.**

76. PREPARATORY.

1. How many feet in a yard? When the yard measure is divided into feet, how many spaces are marked off?



2. What part of the whole is one of these spaces? What parts are two of them? Three of them?

3. When a quantity is divided into 3 equal parts, what are the parts called? In the fractions $\frac{1}{3}$ and $\frac{2}{3}$, which numbers show that the fractions express thirds?

77. The number that names the kind of parts expressed by the fraction is called the **denominator** of the fraction.

The number that tells how many parts are expressed by a fraction is called the **numerator.**

The numerator and the denominator are called the **terms** of the fraction.

ORAL EXERCISES

1. In $\frac{7}{8}$ what is the numerator? What is the denominator?

2. Answer the same questions for $\frac{5}{16}$. For $\frac{8}{9}$. For $\frac{6}{3}$. For $\frac{7}{10}$. For $\frac{20}{10}$.

Read:

3. $\frac{12}{25}$. **5.** $\frac{43}{64}$. **7.** $\frac{17}{45}$. **9.** $\frac{1}{102}$. **11.** $\frac{84}{197}$.

4. $\frac{5}{411}$. **6.** $\frac{5}{500}$. **8.** $\frac{11}{216}$. **10.** $\frac{24}{1001}$. **12.** $\frac{200}{1212}$.

WRITTEN EXERCISES

Write in figures:

1. Twenty-five thirty-sixths.
2. Seventeen eighty-fifths.
3. Thirty one-hundred-firsts.
4. Three hundred thousandths.
5. Five five-hundredths.
6. Two hundred one three-hundred-firsts.
7. Two hundred five three-thousand-five-hundred-tenths.
8. Fifty-nine and nineteen ninety-firsts.

Write in words:

9. $\frac{32}{95}$. **12.** $\frac{17}{108}$. **15.** $\frac{123}{155}$. **18.** $50\frac{5}{562}$.

10. $\frac{1}{1001}$. **13.** $\frac{41}{144}$. **16.** $\frac{423}{2621}$. **19.** $\frac{500}{5012}$.

11. $38\frac{19}{25}$. **14.** $\frac{201}{501}$. **17.** $\frac{4}{400}$. **20.** $190\frac{190}{2000}$.

78. A fraction with the numerator 1 is called a **unit fraction.**

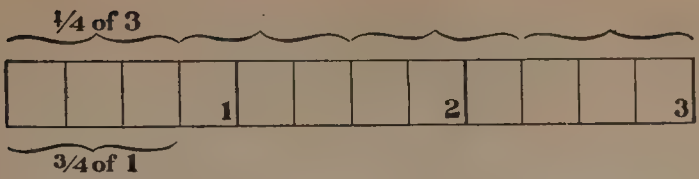
Thus, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{25}$, $\frac{1}{206}$ are unit fractions. Each expresses *one* part of a whole thing.

The unit fraction in $\frac{3}{10}$ is $\frac{1}{10}$; in $\frac{5}{7}$ is $\frac{1}{7}$; in $\frac{19}{100}$ is $\frac{1}{100}$, and so on.

79. Fractions with the same denominator are said to be **expressed in the same unit.**

80. Preparatory.

How many fourths are there in 3 in.? $\frac{3}{4}$ of an inch is what part of 3 in.? Point this out in the figure at the top of page 61.



$\frac{3}{4}$ of an inch = $\frac{1}{4}$ of 3 inches, or 3 inches divided by 4.

81. *A fraction means the quotient obtained by dividing the numerator by the denominator.*

Thus, $\frac{14}{7}$ means 14 divided into 7 equal parts, or 14 ÷ 7, which is 2.

Similarly, $\frac{1}{2}$ means 1 divided into 2 equal parts, or 1 ÷ 2, which is $\frac{1}{2}$.

82. A fraction equal to, or greater than, 1 is called an **improper fraction.** A fraction less than 1 is called a **proper fraction.**

Thus, $\frac{2}{2}$, $\frac{14}{7}$, $\frac{11}{3}$ are improper fractions; and $\frac{2}{3}$, $\frac{1}{2}$, $\frac{9}{16}$ are proper fractions.

83. PREPARATORY.

1. Perform the division of 13 ÷ 6, indicating the division of the remainder.

Thus, $\frac{13}{6} = 2\frac{1}{6}$.

2. Perform the division of 15 ÷ 8, indicating the division of the remainder.

Thus, $\frac{15}{8} = 1\frac{7}{8}$.

84. A number that is expressed by a whole number and a fraction is called a **mixed number.**

Thus, $2\frac{1}{6}$ and $1\frac{7}{8}$ are mixed numbers.

ORAL EXERCISES

1. In a proper fraction, which is larger, the numerator or the denominator? In an improper fraction?

Point out the improper fractions :

2. $\frac{3}{5}$.
3. $\frac{4}{9}$.
4. $\frac{21}{21}$.
5. $\frac{42}{14}$.
6. $\frac{11}{3}$.
7. $\frac{9}{4}$.
8. $\frac{24}{17}$.
9. $\frac{25}{23}$.
10. $\frac{12}{4}$.
11. $\frac{12}{17}$.
12. $\frac{16}{23}$.
13. $\frac{23}{24}$.
14. $\frac{1\text{[illegible]}}{5}$.
15. $\frac{9}{20}$.
16. $\frac{36}{9}$.
17. $\frac{8}{17}$.

18. State the whole number or mixed number equal to each improper fraction in the list above.

Express as mixed numbers :

19. $\frac{14}{3}$.
20. $\frac{12}{9}$.
21. $\frac{14}{5}$.
22. $\frac{11}{3}$.
23. $\frac{12}{11}$.
24. $\frac{11}{2}$.
25. $\frac{23}{4}$.
26. $\frac{28}{15}$.
27. $\frac{35}{17}$.
28. $\frac{21}{16}$.
29. $\frac{5}{3}$.
30. $\frac{81}{10}$.
31. $\frac{43}{40}$.
32. $\frac{27}{20}$.
33. $\frac{19}{13}$.
34. $\frac{9}{5}$.
35. $\frac{125}{100}$.
36. $\frac{15}{7}$.
37. $\frac{16}{9}$.
38. $\frac{25}{12}$.

85. A mixed number may be expressed as an improper fraction.

EXAMPLE: Express $7\frac{3}{4}$ as an improper fraction.

$$7 \times \frac{4}{4} = \frac{28}{4}.$$
$$\frac{28}{4} + \frac{3}{4} = \frac{31}{4}.$$

1 equals 4 fourths, therefore 7 equals 7 × 4 fourths, or 28 fourths. 28 fourths + 3 fourths = 31 fourths.

WRITTEN EXERCISES

Express as an improper fraction :

1. $2\frac{3}{5}$.
2. $4\frac{1}{7}$.
3. $27\frac{2}{3}$.
4. $1\frac{1}{12}$.
5. $7\frac{2}{3}$.
6. $6\frac{4}{5}$.
7. $1\frac{1}{4}$.
8. $9\frac{2}{3}$.
9. $41\frac{4}{5}$.
10. $1\frac{7}{20}$.
11. $1\frac{5}{16}$.
12. $1\frac{6}{13}$.
13. $3\frac{1}{3}$.
14. $6\frac{4}{9}$.
15. $12\frac{1}{2}$.
16. $2\frac{7}{8}$.
17. $5\frac{8}{9}$.
18. $7\frac{3}{10}$.
19. $1\frac{4}{5}$.
20. $2\frac{2}{3}$.
21. $8\frac{1}{2}$.
22. $12\frac{3}{4}$.
23. $16\frac{2}{5}$.
24. $18\frac{2}{3}$.
25. $5\frac{3}{8}$.
26. $9\frac{4}{5}$.
27. $19\frac{2}{3}$.
28. $3\frac{1}{8}$.
29. $2\frac{1}{17}$.
30. $40\frac{1}{2}$.

COMPARISON

86. PREPARATORY.

1. Which is greater, 7 yd. or 5 yd.?

2. Which is greater, $\frac{7}{3}$ or $\frac{5}{3}$? $\frac{6}{7}$ or $\frac{4}{7}$? $\frac{3}{8}$ or $\frac{5}{8}$?

3. If two unequal fractions have equal denominators, which fraction is the greater?

4. Which fraction is greater, $\frac{1}{4}$ or $\frac{1}{5}$? Why? $\frac{1}{6}$ or $\frac{1}{3}$? Why?

5. Which is less, $\frac{1}{7}$ or $\frac{1}{8}$? Why? $\frac{2}{9}$ or $\frac{2}{10}$? Why?

6. If two unequal fractions have equal numerators, which fraction is the greater?

WRITTEN EXERCISES

Arrange the numbers in each set according to size, placing the least at the top:

1.	**2.**	**3.**	**4.**	**5.**
$\frac{8}{3}$	$\frac{10}{4}$	$\frac{7}{3}$	$\frac{9}{4}$	$\frac{7}{8}$
$\frac{8}{12}$	$\frac{10}{2}$	$\frac{7}{5}$	$\frac{9}{5}$	$\frac{3}{8}$
$\frac{8}{9}$	$\frac{10}{10}$	$\frac{7}{2}$	$\frac{9}{16}$	$\frac{5}{8}$
$\frac{8}{8}$	$\frac{10}{13}$	$\frac{7}{9}$	$\frac{9}{20}$	$\frac{11}{8}$
$\frac{8}{7}$	$\frac{10}{11}$	.7	.9	$\frac{8}{8}$

Solve:

6. The areas of Denmark, Venezuela, England, and Germany are respectively $\frac{10}{15}$, $\frac{10}{3}$, $\frac{10}{20}$, and $\frac{10}{6}$ of that of the British Isles. Arrange the names of these countries in order according to their areas.

7. Of all the cotton goods produced in the United States in 1900, about $\frac{1}{3}$ was made in Massachusetts, $\frac{1}{11}$ in South Carolina, $\frac{1}{14}$ in Pennsylvania, $\frac{1}{19}$ in Georgia, $\frac{1}{13}$ in Rhode Island, $\frac{1}{20}$ in Connecticut, $\frac{1}{12}$ in North Carolina, and $\frac{1}{15}$ in New Hampshire. Arrange the names of these states in order according to the amount of cotton goods produced.

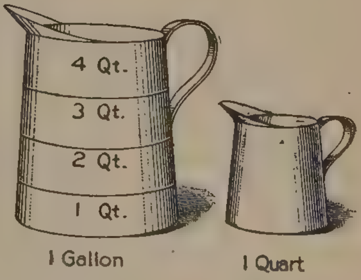
8. Of all the boots and shoes made in the United States in 1900, $\frac{9}{100}$ were made in New Hampshire, $\frac{9}{20}$ in Massachusetts, and $\frac{9}{99}$ in New York. Arrange the names of these states in order according to the quantity of boots and shoes manufactured.

REDUCTION

87. Changing the form of a fraction without changing its value is called **reduction.**

Reduction to Higher or Lower Terms

88. PREPARATORY.


4 Qt.
3 Qt.
2 Qt.
1 Qt.
1 Gallon
1 Quart

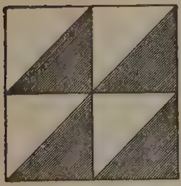
1. How many quarts in a gallon?

2. One quart is what part of a gallon?

3. Two quarts are how many fourths of a gallon?

4. How many quarts in $\frac{1}{2}$ of a gallon?

5. Is there any difference between $\frac{1}{2}$ gallon and $\frac{2}{4}$ gallon?



6. Each triangular tile is what part of this whole square of tiling? Two tiles are how many eighths of the square?

7. How many tiles in $\frac{1}{4}$ of the square?

8. Compare $\frac{1}{4}$ of the square with $\frac{2}{8}$ of it.

9. Similarly, compare $\frac{4}{8}$ of the square with $\frac{2}{4}$ of it. Also with $\frac{1}{2}$ of it.

$$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}.$$

10. If both terms of a fraction are multiplied by two, what is the effect on the value of the fraction? If both terms are divided by 2, what is the effect on the value of the fraction?

89. We have seen (page 61) that a fraction denotes division, and we know that a dividend and its divisor may be divided by the same number without changing the quotient.

Likewise, both dividend and divisor may be multiplied by the same number without changing the quotient. Hence:

90. *Both terms of a fraction may be divided by any number without changing the value of the fraction.*

Thus, $\left.\frac{12 \div 6}{18 \div 6}\right\} = \frac{2}{3}$.

91. *Both terms of a fraction may be multiplied by the same number without changing the value of the fraction.*

Thus, $\left.\frac{2 \times 6}{5 \times 6}\right\} = \frac{12}{30}$.

ORAL EXERCISES

Reduce these fractions to equal fractions as indicated:

1. $\frac{12}{16}$ to fourths.
2. $\frac{8}{24}$ to thirds.
3. $\frac{16}{20}$ to fifths.
4. $\frac{3}{30}$ to tenths.
5. $\frac{9}{54}$ to sixths.
6. $\frac{15}{30}$ to halves.
7. $\frac{9}{27}$ to thirds.
8. $\frac{18}{24}$ to fourths.
9. $\frac{20}{24}$ to sixths.
10. $\frac{9}{30}$ to tenths.
11. $\frac{14}{49}$ to sevenths.
12. $\frac{30}{36}$ to sixths.
13. $\frac{16}{32}$ to halves.
14. $\frac{4}{20}$ to fifths.
15. $\frac{16}{24}$ to thirds.
16. $\frac{1}{2}$ to eighths.
17. $\frac{2}{3}$ to twelfths.
18. $\frac{2}{3}$ to eighteenths.
19. $\frac{1}{4}$ to twelfths.
20. $\frac{3}{4}$ to eighths.
21. $\frac{3}{4}$ to twentieths.
22. $\frac{2}{5}$ to twentieths.
23. $\frac{3}{5}$ to fifteenths.
24. $\frac{5}{6}$ to twelfths.
25. $\frac{5}{6}$ to twenty-fourths.
26. $\frac{2}{5}$ to twenty-fifths.
27. $\frac{1}{6}$ to thirty-sixths.
28. $\frac{4}{7}$ to forty-ninths.
29. $\frac{3}{7}$ to twenty-firsts.
30. $\frac{5}{8}$ to thirty-seconds.

31. $\frac{12}{60}$ to fifths.

32. $\frac{14}{35}$ to fifths.

33. $\frac{11}{44}$ to fourths.

34. $\frac{4}{9}$ to eighty-firsts.

35. $\frac{3}{11}$ to seventy-sevenths.

36. $\frac{7}{12}$ to ninety-sixths.

92. If the terms of a fraction have no common factors except 1, the fraction is said to be in its **lowest terms**.

93. Example: Reduce $\frac{42}{252}$ to lowest terms.

1st Solution: *2d Solution:*

$$\frac{2)42}{2)252} = \frac{3)21}{3)126} = \frac{7)7}{7)42} = \frac{1}{6} \quad \text{or} \quad \frac{\overset{\overset{\overset{1}{\cancel{7}}}{\cancel{21}}}{\cancel{42}}}{\underset{\underset{\underset{6}{\cancel{42}}}{\cancel{126}}}{\cancel{252}}} = \frac{1}{6}$$

The form at the left is neater in appearance and shows the steps more plainly.

94. *To reduce a fraction to its lowest terms, cancel from the numerator and denominator all common factors.*

If the greatest common divisor of the terms is canceled, the fraction is reduced at once to its lowest terms.

WRITTEN EXERCISES

Reduce to lowest terms:

1. $\frac{30}{75}$.
2. $\frac{64}{216}$.
3. $\frac{16}{256}$.
4. $\frac{54}{702}$.
5. $\frac{14}{168}$.
6. $\frac{26}{169}$.
7. $\frac{252}{756}$.
8. $\frac{648}{1944}$.
9. $\frac{720}{900}$.
10. $\frac{15}{215}$.
11. $\frac{56}{147}$.
12. $\frac{44}{198}$.
13. $\frac{49}{686}$.
14. $\frac{35}{280}$.
15. $\frac{81}{225}$.
16. $\frac{162}{432}$.
17. $\frac{175}{4500}$.
18. $\frac{52}{169}$.
19. $\frac{81}{243}$.
20. $\frac{315}{369}$.
21. $\frac{35}{210}$.
22. $\frac{90}{345}$.
23. $\frac{24}{600}$.
24. $\frac{495}{585}$.
25. $\frac{324}{2700}$.
26. $\frac{813}{4065}$.
27. $\frac{625}{1000}$.
28. $\frac{96}{144}$.
29. $\frac{28}{552}$.
30. $\frac{18}{120}$.
31. $\frac{16}{256}$.
32. $\frac{76}{266}$.
33. $\frac{120}{210}$.
34. $\frac{225}{2475}$.
35. $\frac{141}{1269}$.

Reduction to a Common Denominator

95. Examples:

1. The market gardeners of New Jersey owned $\frac{3}{35}$ of the farms in that state, and the small farmers $\frac{3}{5}$ of them. What part did they both own?

Add $\frac{3}{35}$ and $\frac{3}{5}$.

Solution: $\frac{3}{5} = \frac{21}{35}$. How is this found?

$$\frac{21}{35} + \frac{3}{35} = \frac{24}{35}.$$

They both owned $\frac{24}{35}$ of the farms.

2. What part of the New Jersey farms were owned by other farmers?

Subtract $\frac{24}{35}$ from 1.

Solution: $1 = \frac{35}{35}$. Why express 1 by $\frac{35}{35}$?

$$\frac{35}{35} - \frac{24}{35} = \frac{11}{35}.$$

Other farmers owned $\frac{11}{35}$ of the farms.

3. One-fifth of a farm was pasture, $\frac{2}{7}$ woodland, $\frac{6}{35}$ meadow, and the rest was planted with crops. What part of the whole farm was planted with crops?

Add $\frac{1}{5}$, $\frac{2}{7}$, and $\frac{6}{35}$.

Solution:

$$\begin{array}{rcll} \frac{1}{5} &=& \frac{7}{35} & \quad 7 \\ \frac{2}{7} &=& \frac{10}{35} & \quad 10 \\ \frac{6}{35} &=& \frac{6}{35} & \quad \underline{\ \ 6} \\ &&& \quad 23 \end{array}$$

$$\frac{7}{35} + \frac{10}{35} + \frac{6}{35} = \frac{23}{35}.$$

$$\frac{35}{35} - \frac{23}{35} = \frac{12}{35}$$

$\frac{12}{35}$ of the whole farm was planted with crops.

Before adding or subtracting the fractions in the above exercises, how were the fractions charged?

96. When two or more fractions have the same denominator, this denominator is called their **common denominator.**

WRITTEN EXERCISES

Change the fractions of each exercise to equal fractions having the common denominator given:

FRACTIONS.	COMMON DENOMINATOR.	FRACTIONS.	COMMON DENOMINATOR.
1. $\frac{5}{16}, \frac{2}{32}$.	64.	**4.** $\frac{1}{4}, \frac{3}{8}, \frac{7}{2}$.	16.
2. $\frac{1}{3}, \frac{4}{7}, \frac{5}{2}$.	42.	**5.** $\frac{5}{9}, \frac{6}{3}, \frac{11}{6}$.	18.
3. $\frac{12}{7}, \frac{2}{2}, \frac{1}{4}$.	28.	**6.** $\frac{3}{5}, \frac{9}{10}, \frac{12}{4}$.	40.

7. From Exercises 1–6 select those exercises in which the new denominator is the least number that could be so used.

8. What is the least common multiple of the denominators given in each exercise?

97. The least common multiple of the given denominators is called the **least common denominator** of the fractions. The abbreviation for least common denominator is **l. c. d.**

ORAL EXERCISES

Find the least common denominator of:

1. $\frac{2}{3}, \frac{4}{5}$.
2. $\frac{2}{9}, \frac{7}{8}$.
3. $\frac{5}{9}, \frac{7}{12}$.
4. $\frac{5}{9}, \frac{5}{18}$.
5. $\frac{7}{9}, \frac{11}{18}$.
6. $\frac{5}{8}, \frac{3}{4}$.
7. $\frac{3}{10}, \frac{7}{8}$.
8. $\frac{3}{8}, \frac{3}{4}$.
9. $\frac{5}{8}, \frac{7}{12}$.
10. $\frac{9}{16}, \frac{7}{8}$.
11. $\frac{4}{7}, \frac{3}{5}$.
12. $\frac{7}{9}, \frac{1}{6}$.
13. $\frac{3}{5}, \frac{9}{10}$.
14. $\frac{3}{20}; \frac{7}{10}$.
15. $\frac{4}{5}, \frac{11}{20}$.
16. $\frac{5}{6}, \frac{8}{9}$.
17. $\frac{5}{7}, \frac{2}{3}$.
18. $\frac{3}{8}, \frac{5}{24}$.
19. $\frac{1}{4}, \frac{5}{16}$.
20. $\frac{3}{7}, \frac{9}{14}$.

Change to equal fractions with the least common denominator:

21. $\frac{3}{8}, \frac{5}{6}, \frac{1}{4}$.
22. $\frac{3}{2}, \frac{2}{5}, \frac{7}{10}$.
23. $\frac{1}{3}, \frac{5}{6}, \frac{7}{12}$.
24. $\frac{1}{2}, \frac{2}{7}, \frac{3}{14}$.
25. $\frac{3}{4}, \frac{3}{8}, \frac{5}{16}$.
26. $\frac{8}{5}, \frac{3}{2}, \frac{3}{10}$.
27. $\frac{2}{7}, \frac{1}{4}, \frac{5}{2}$.
28. $\frac{1}{5}, \frac{3}{10}, \frac{7}{20}$.
29. $\frac{1}{2}, \frac{1}{4}, \frac{5}{16}$.
30. $\frac{1}{3}, \frac{1}{6}, \frac{11}{24}$.
31. $\frac{3}{4}, \frac{2}{3}, \frac{7}{6}$.
32. $\frac{3}{5}, \frac{2}{3}, \frac{1}{15}$.
33. $\frac{1}{4}, \frac{7}{8}, \frac{9}{16}$.
34. $\frac{2}{3}, \frac{5}{6}, \frac{11}{12}$.
35. $\frac{1}{9}, \frac{2}{27}, \frac{1}{3}$.
36. $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$.
37. $\frac{1}{2}, \frac{5}{16}, \frac{11}{12}$.
38. $\frac{3}{7}, \frac{9}{14}, \frac{1}{2}$.
39. $\frac{5}{8}, \frac{3}{4}, \frac{9}{16}$.
40. $\frac{1}{5}, \frac{2}{3}, \frac{4}{15}$.

ADDITION AND SUBTRACTION

Addition

98. Preparatory.

1. How many are 3 pages and 4 pages? 3 fifths and 4 fifths?

2. What is the sum of 3 qt. and 8 qt.? Of 3 fifths and 8 fifths?

3. Add $\frac{2}{10}$ and $\frac{5}{10}$. Add $\frac{9}{100}$ and $\frac{4}{100}$. Add $\frac{3}{8}$ and $\frac{7}{8}$.

99. *To add fractions with like denominators, add their numerators, place the result over their common denominator, and simplify, if possible.*

ORAL EXERCISES

Add:

1. $\frac{11}{4}$	2. $\frac{1}{9}$	3. $\frac{3}{11}$	4. $\frac{5}{12}$	5. $\frac{3}{10}$
$\frac{15}{4}$	$\frac{9}{9}$	$\frac{2}{11}$	$\frac{11}{12}$	$\frac{9}{10}$
$\frac{3}{4}$	$\frac{10}{9}$	$\frac{4}{11}$	$\frac{7}{12}$	$\frac{7}{10}$

6. $\frac{3}{7} + \frac{8}{7}$.
8. $\frac{2}{3} + \frac{5}{3}$.
10. $\frac{1}{6} + \frac{4}{6} + \frac{3}{6}$.
7. $\frac{11}{14} + \frac{2}{14}$.
9. $\frac{3}{10} + \frac{4}{10}$.
11. $\frac{1}{6} + \frac{3}{6} + \frac{7}{6}$.

Solve:

12. A man spent $\frac{2}{9}$ of his money for shoes and $\frac{5}{9}$ for books. What part of his money did he spend for both?

13. The height and width of a rectangle are $\frac{4}{7}$ of a ft. and $\frac{2}{7}$ of a ft. What is the distance around the figure?

14. A man set out on a journey; he walked $\frac{1}{10}$ of the way, rode $\frac{3}{10}$ on the electric car, and $\frac{5}{10}$ on the steam railway. What part of his journey had he then traveled?

15. What part of a journey is traveled in an hour and a half by covering $\frac{2}{9}$ of it in the first half hour and $\frac{1}{9}$ in each 20 minutes following?

100. *To add fractions with unlike denominators, reduce them to equal fractions having the least common denominator and proceed as with fractions of like denominators.*

Example: Add $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{8}$.

We must first change the fractions to equal fractions with the least common denominator. In this case the least common denominator is 24.

It is convenient in adding several fractions to arrange the work as here shown:

$$\begin{array}{l} \frac{2}{3} = \frac{16}{24} \\ \frac{1}{4} = \frac{6}{24} \\ \frac{5}{8} = \frac{15}{24} \\ \hline \qquad \frac{37}{24} \end{array}$$

WRITTEN EXERCISES

Change to equal fractions with least common denominator and add:

1. $\frac{5}{6} + \frac{7}{8}$.
2. $\frac{1}{6} + \frac{11}{18}$.
3. $\frac{5}{16} + \frac{5}{8}$.
4. $\frac{8}{9} + \frac{7}{18}$.
5. $\frac{5}{7} + \frac{4}{35}$.
6. $\frac{8}{9} + \frac{9}{12}$.
7. $\frac{4}{5} + \frac{9}{20}$.
8. $\frac{9}{10} + \frac{14}{15}$.
9. $\frac{7}{9} + \frac{3}{5}$.
10. $\frac{5}{12} + \frac{5}{18}$.
11. $\frac{3}{5} + \frac{5}{12}$.
12. $\frac{7}{8} + \frac{9}{16}$.
13. $\frac{8}{9} + \frac{17}{18}$.
14. $\frac{4}{5} + \frac{9}{10}$.
15. $\frac{2}{3} + \frac{9}{16}$.

16. $\begin{array}{c} \frac{3}{4} \\ \frac{5}{8} \\ \frac{1}{6} \\ \hline \end{array}$ 18. $\begin{array}{c} \frac{8}{9} \\ \frac{5}{6} \\ \frac{4}{15} \\ \hline \end{array}$ 20. $\begin{array}{c} \frac{3}{5} \\ \frac{11}{10} \\ \frac{7}{8} \\ \hline \end{array}$ 22. $\begin{array}{c} \frac{7}{12} \\ \frac{11}{8} \\ \frac{2}{3} \\ \hline \end{array}$ 24. $\begin{array}{c} \frac{5}{6} \\ \frac{3}{8} \\ \frac{9}{14} \\ \hline \end{array}$ 26. $\begin{array}{c} \frac{3}{10} \\ \frac{4}{5} \\ \frac{4}{9} \\ \hline \end{array}$

17. $\begin{array}{c} \frac{1}{4} \\ \frac{2}{5} \\ \frac{7}{10} \\ \hline \end{array}$ 19. $\begin{array}{c} \frac{2}{3} \\ \frac{1}{1} \\ \frac{5}{6} \\ \hline \end{array}$ 21. $\begin{array}{c} \frac{4}{9} \\ \frac{5}{12} \\ \frac{11}{18} \\ \hline \end{array}$ 23. $\begin{array}{c} \frac{3}{4} \\ \frac{13}{10} \\ \frac{4}{5} \\ \hline \end{array}$ 25. $\begin{array}{c} \frac{1}{2} \\ \frac{3}{4} \\ \frac{7}{8} \\ \hline \end{array}$ 27. $\begin{array}{c} \frac{5}{9} \\ \frac{3}{4} \\ \frac{5}{6} \\ \hline \end{array}$

Solve:

28. A man sold $\frac{2}{3}$ of an acre at one time, $\frac{1}{2}$ of an acre at another, and $\frac{3}{4}$ at another. How many acres did he sell in all?

29. A farmer plowed $\frac{1}{8}$ of a field one day, $\frac{1}{10}$ of it the next, and $\frac{3}{20}$ of it the next. What part did he plow in the 3 days?

30. If a farmer laid $\frac{1}{8}$ of a mile of drain-tile one year, $\frac{3}{4}$ of a mile the second year, and $\frac{5}{8}$ of a mile the third year, how many miles did he lay in three years?

31. A rectangle is $\frac{1}{6}$ of a foot on one side and $\frac{3}{5}$ of a foot on another. What is the sum of its four sides?

32. Mary used $\frac{2}{3}$ of a yard of string to tie a package and $\frac{3}{16}$ of a yard to fasten on the shipping tag. What part of a yard did she use for both?

33. James had solved $\frac{1}{12}$ of his problems in the afternoon and $\frac{5}{6}$ of them in the evening. What part of the whole number had he solved?

34. A tailor used $\frac{1}{5}$ of a pattern of cloth for a vest and $\frac{5}{8}$ of it for a coat. What part did he use for the coat and vest?

35. A man planted $\frac{3}{16}$ of his farm with corn, $\frac{3}{32}$ of it with potatoes, and $\frac{3}{64}$ with cabbage. What part of his farm did he plant?

36. A broker invested $\frac{3}{8}$ of his money in stocks, $\frac{4}{9}$ in bonds, and $\frac{2}{27}$ in mortgages. What part of his money did he invest in all?

Subtraction

101. Preparatory.

1. How many are 12 books less 4 books?
2. How many are 12 sevenths less 4 sevenths?
3. How many are 12 hundredths less 4 hundredths?

102. *To subtract fractions with like denominators, subtract their numerators, place the result over the common denominator, and simplify, if possible.*

ORAL EXERCISES

Subtract:

1.	2.	3.	4.	5.
$\frac{13}{8}$	$\frac{12}{15}$	$\frac{16}{18}$	$\frac{13}{10}$	$\frac{26}{100}$
$\frac{6}{8}$	$\frac{7}{15}$	$\frac{11}{18}$	$\frac{4}{10}$	$\frac{6}{100}$

103. *To subtract fractions with unlike denominators, reduce them to equal fractions having the least common denominator, and proceed as with fractions having like denominators.*

Example: If from $\frac{7}{8}$ of a bolt of silk $\frac{3}{5}$ of the whole bolt are sold, what part of the whole bolt remains?

Subtract $\frac{3}{5}$ from $\frac{7}{8}$.

Solution: $\frac{7}{8} = \frac{35}{40}$; $\frac{3}{5} = \frac{24}{40}$.

$\frac{35}{40} - \frac{24}{40} = \frac{11}{40}$, the part of the bolt remaining.

WRITTEN EXERCISES

Reduce to least common denominator and subtract:

$$\begin{array}{llllll}
\textbf{1.}\ \frac{17}{3} & \textbf{2.}\ \frac{11}{3} & \textbf{3.}\ \frac{16}{25} & \textbf{4.}\ \frac{11}{8} & \textbf{5.}\ \frac{15}{16} & \textbf{6.}\ \frac{7}{8} \\
\quad\ \underline{\frac{11}{12}} & \quad\ \underline{\frac{15}{7}} & \quad\ \underline{\frac{3}{5}} & \quad\ \underline{\frac{5}{4}} & \quad\ \underline{\frac{3}{8}} & \quad\ \underline{\frac{2}{3}}
\end{array}$$

7. $\frac{9}{10} - \frac{4}{5}$.

8. $\frac{5}{8} - \frac{5}{16}$.

9. $\frac{4}{5} - \frac{3}{25}$.

10. $\frac{11}{18} - \frac{5}{11}$.

11. $\frac{11}{16} - \frac{9}{32}$.

12. $\frac{11}{18} - \frac{1}{6}$.

13. $\frac{14}{15} - \frac{9}{10}$.

14. $\frac{5}{12} - \frac{5}{18}$.

15. $\frac{13}{10} - \frac{3}{4}$.

16. $\frac{2}{1} - \frac{9}{24}$.

17. $\frac{7}{8} - \frac{9}{16}$.

18. $\frac{7}{9} - \frac{3}{4}$.

19. $\frac{2}{3} - \frac{9}{16}$.

20. $\frac{7}{10} - \frac{2}{5}$.

21. $\frac{11}{8} - \frac{7}{12}$.

22. $\frac{17}{18} - \frac{8}{9}$.

23. $\frac{7}{10} - \frac{7}{12}$.

24. $\frac{16}{25} - \frac{2}{5}$.

25. $\frac{5}{7} - \frac{4}{35}$.

26. $\frac{3}{4} - \frac{5}{8}$.

Solve:

27. From a bolt of cloth a merchant sold $\frac{1}{5}$ at one time, $\frac{1}{3}$ at another, and $\frac{1}{7}$ at another. How much did he sell in all?

28. Mary had $\frac{5}{6}$ of a yard of ribbon and used $\frac{5}{24}$ of a yard in tying a package. How much had she left?

29. A tank contained $\frac{7}{8}$ of a gallon of oil, and $\frac{9}{16}$ of a gallon leaked out. How much remained in the tank?

30. A man spent $\frac{1}{6}$ of his salary for rent, $\frac{1}{4}$ for food, $\frac{1}{10}$ for clothing, $\frac{1}{5}$ for other items. What part of his whole salary did he save?

31. A boy spent $\frac{1}{2}$ of his money for books, $\frac{1}{5}$ of it for a cap, and $\frac{1}{10}$ of it for lunch. What part did he spend?

32. Susie had $\frac{13}{16}$ yd. of cloth to make into a book bag; in cutting it she wasted $\frac{3}{32}$ of a yard. What part of a yard did she use?

33. A man invested $\frac{16}{27}$ of his money in bonds and $\frac{2}{9}$ in stocks. How much more did he invest in bonds than in stocks?

34. $\frac{7}{12}$ of a farm is meadow land, and $\frac{5}{24}$ is wooded. The meadow land is how much more than the wooded part? The two are what part of the whole farm?

35. Frank spelled $\frac{24}{25}$ of the words in a lesson, and Helen spelled $\frac{47}{50}$ of the words in the same lesson. Who spelled the more words, and what part of the lesson more?

36. Roy gathered $\frac{13}{16}$ of a bushel of nuts, and Myron gathered $\frac{25}{32}$ of a bushel. Which gathered the more and what part of a bushel more?

Addition and Subtraction of Mixed Numbers

104. Examples:

1. A boy traveled $1\frac{1}{2}$ hr. by train and $8\frac{3}{4}$ hr. by boat. How many hours did he travel in all?

Solution:

$$\begin{array}{rl} 1\tfrac{1}{2}\text{ hr.} = & 1\tfrac{2}{4}\text{ hr.} \\ 8\tfrac{3}{4}\text{ hr.} = & \underline{8\tfrac{3}{4}\text{ hr.}} \\ & 9\tfrac{5}{4}\text{ hr.} = 10\tfrac{1}{4}\text{ hr.} \end{array}$$

2. A chest of tea contains $62\frac{5}{16}$ lb., and $12\frac{1}{4}$ lb. are sold. How many pounds are left?

Solution:

$$\begin{array}{rl} 62\tfrac{5}{16}\text{ lb.} = & 62\tfrac{5}{16}\text{ lb.} \\ 12\tfrac{1}{4}\text{ lb.} = & \underline{12\tfrac{4}{16}\text{ lb.}} \\ & 50\tfrac{1}{16}\text{ lb.} \end{array}$$

105. *In adding or subtracting mixed numbers, perform the operations with the integers and the fractions separately and unite the results.*

WRITTEN EXERCISES

Add, and express the sums as mixed or whole numbers:

1. $\begin{array}{r}\frac{8}{5}\\ \frac{14}{5}\\ \hline\end{array}$

2. $\begin{array}{r}\frac{3}{4}\\ \frac{6}{4}\\ \hline\end{array}$

3. $\begin{array}{r}\frac{5}{8}\\ \frac{6}{8}\\ \hline\end{array}$

4. $\begin{array}{r}\frac{7}{9}\\ \frac{2}{9}\\ \hline\end{array}$

5. $\begin{array}{r}\frac{7}{10}\\ \frac{13}{10}\\ \hline\end{array}$

6. $\begin{array}{r}\frac{12}{11}\\ \frac{16}{11}\\ \hline\end{array}$

7. $\begin{array}{r}\frac{5}{13}\\ \frac{9}{13}\\ \hline\end{array}$

8. $\begin{array}{r}\frac{13}{15}\\ \frac{28}{15}\\ \hline\end{array}$

Add.

9. $\begin{array}{r}3\frac{2}{3}\\ 5\frac{3}{4}\\ \hline\end{array}$

10. $\begin{array}{r}3\frac{4}{5}\\ 2\frac{5}{10}\\ \hline\end{array}$

11. $\begin{array}{r}50\frac{3}{4}\\ 17\frac{5}{12}\\ \hline\end{array}$

12. $\begin{array}{r}6\frac{1}{5}\\ 7\frac{2}{3}\\ \hline\end{array}$

13. $\begin{array}{r}1\frac{1}{4}\\ 1\frac{11}{15}\\ \hline\end{array}$

14. $\begin{array}{r}63\frac{7}{12}\\ 21\frac{3}{10}\\ \hline\end{array}$

15. $\begin{array}{r}4\frac{5}{12}\\ 2\frac{1}{2}\\ \hline\end{array}$

16. $\begin{array}{r}7\frac{3}{8}\\ 4\frac{1}{12}\\ \hline\end{array}$

17. $\begin{array}{r}84\frac{17}{20}\\ 49\frac{9}{16}\\ \hline\end{array}$

18. $\begin{array}{r}2\frac{1}{8}\\ 5\frac{2}{3}\\ \hline\end{array}$

19. $\begin{array}{r}2\frac{3}{8}\\ 6\frac{13}{16}\\ \hline\end{array}$

20. $\begin{array}{r}61\frac{5}{18}\\ 27\frac{4}{15}\\ \hline\end{array}$

21. $\begin{array}{r}4\frac{1}{3}\\ 3\frac{1}{4}\\ \hline\end{array}$

22. $\begin{array}{r}6\frac{5}{9}\\ 2\frac{5}{12}\\ \hline\end{array}$

23. $\begin{array}{r}7\frac{7}{8}\\ 2\frac{1}{2}\\ \hline\end{array}$

24. $\begin{array}{r}2\frac{1}{2}\\ 7\frac{3}{4}\\ \hline\end{array}$

25. $\begin{array}{r}12\frac{1}{3}\\ 15\frac{3}{4}\\ \hline\end{array}$

26. $\begin{array}{r}98\frac{3}{16}\\ 15\frac{11}{24}\\ \hline\end{array}$

Subtract:

27. $\begin{array}{r}48\frac{3}{4}\\ 14\frac{1}{2}\\ \hline\end{array}$

28. $\begin{array}{r}27\frac{5}{8}\\ 19\frac{7}{16}\\ \hline\end{array}$

29. $\begin{array}{r}53\frac{3}{8}\\ 11\frac{1}{10}\\ \hline\end{array}$

30. $\begin{array}{r}18\frac{7}{12}\\ 10\frac{1}{3}\\ \hline\end{array}$

31. $\begin{array}{r}44\frac{7}{8}\\ 29\frac{3}{4}\\ \hline\end{array}$

32. $\begin{array}{r}40\frac{7}{8}\\ 20\frac{3}{16}\\ \hline\end{array}$

33. $\begin{array}{r}91\frac{13}{16}\\ 47\frac{1}{3}\\ \hline\end{array}$

34. $\begin{array}{r}36\frac{8}{9}\\ 18\frac{2}{3}\\ \hline\end{array}$

35. $\begin{array}{r}34\frac{11}{12}\\ 18\frac{3}{10}\\ \hline\end{array}$

36. $\begin{array}{r}48\frac{11}{12}\\ 27\frac{5}{6}\\ \hline\end{array}$

Solve:

37. A man walked $11\frac{1}{4}$ mi. on Monday, and $6\frac{7}{14}$ mi. on Tuesday. How many miles did he walk? The next day he went back 10 mi. How far was he then from the starting point?

38. William is $11\frac{3}{4}$ yr. old, and Thomas is $9\frac{5}{12}$ yr. old. William is how much older than Thomas? How many years will have passed before Thomas is 30 yr. old? How old will William be then?

39. A gold dollar contains $23\frac{11}{50}$ grains of gold, and $2\frac{58}{100}$ grains of alloy. What is the weight of the gold dollar?

40. The half-eagle or 5-dollar gold piece contains $92\frac{22}{25}$ more grains of gold than the gold dollar. How much gold does the half-eagle contain?

106. *In subtracting mixed numbers, if the fraction in the minuend is smaller than that in the subtrahend, supply 1 from the integral part.*

EXAMPLES:

1. From a jar containing $18\frac{1}{2}$ lb. of butter, $7\frac{3}{4}$ lb. are sold. How many pounds remain in the jar?

Solution:
$$\begin{array}{lcccr} 18\frac{1}{2}\text{ lb.} & = & 18\frac{2}{4}\text{ lb.} & = & 17\frac{6}{4}\text{ lb.} \\ 7\frac{3}{4}\text{ lb.} & & & = & \underline{7\frac{3}{4}\text{ lb.}} \\ & & & & 10\frac{3}{4}\text{ lb.} \end{array}$$

2. A park contains 63 acres, of which $13\frac{3}{16}$ acres are given to lakes and lagoons. How many acres are dry land?

63 acres = $62\frac{16}{16}$ acres. Why?

Solution:
$$\begin{array}{lcr} 63 & = & 62\frac{16}{16} \\ \underline{13\frac{3}{16}} & & \underline{13\frac{3}{16}} \\ & & 49\frac{13}{16} \end{array}$$

WRITTEN EXERCISES

Subtract:

1. $\begin{array}{r} 16\frac{2}{5} \\ \underline{7\frac{9}{10}} \end{array}$

2. $\begin{array}{r} 12\frac{1}{2} \\ \underline{\frac{8}{3}} \end{array}$

3. $\begin{array}{r} 6\frac{2}{3} \\ \underline{3\frac{2}{5}} \end{array}$

4. $\begin{array}{r} 9 \\ \underline{5\frac{11}{12}} \end{array}$

5. $\begin{array}{r} 8\frac{1}{7} \\ \underline{4\frac{3}{4}} \end{array}$

6. $\begin{array}{r} 16\frac{1}{3} \\ \underline{\frac{17}{5}} \end{array}$

7. $\begin{array}{r} 2\frac{3}{4} \\ \underline{1\frac{5}{6}} \end{array}$

8. $\begin{array}{r} 3\frac{4}{9} \\ \underline{1\frac{1}{6}} \end{array}$

9. $\begin{array}{r} 6 \\ \underline{2\frac{9}{10}} \end{array}$

10. $\begin{array}{r} \frac{11}{4} \\ \underline{1\frac{1}{6}} \end{array}$

11. $\begin{array}{r} 17\frac{1}{4} \\ \underline{\frac{17}{4}} \end{array}$

12. $\begin{array}{r} 18\frac{1}{5} \\ \underline{8\frac{1}{17}} \end{array}$

13. $\begin{array}{r} 11\frac{3}{7} \\ \underline{9\frac{1}{2}} \end{array}$

14. $\begin{array}{r} \frac{27}{8} \\ \underline{2\frac{7}{12}} \end{array}$

15. $\begin{array}{r} 6\frac{1}{2} \\ \underline{\frac{7}{5}} \end{array}$

16. $\begin{array}{r} 105\frac{3}{8} \\ \underline{89\frac{11}{20}} \end{array}$

17. $\begin{array}{r} 25\frac{3}{14} \\ \underline{\frac{25}{14}} \end{array}$

18. $\begin{array}{r} 32\frac{1}{11} \\ \underline{\frac{16}{3}} \end{array}$

19. $16 - \frac{35}{4}$.

20. $18 - \frac{17}{45}$.

21. $26 - \frac{7}{25}$.

22. $33 - \frac{26}{15}$.

23. $10 - \frac{40}{11}$.

24. $35 - \frac{5}{14}$.

25. $45 - \frac{11}{18}$.

26. $63 - \frac{9}{64}$.

27. $75 - \frac{40}{27}$.

Solve:

28. Mary bought $12\frac{1}{2}$ yd. of cloth and used $7\frac{5}{8}$ yd. for a dress. How many yards had she left?

29. Charles walks $1\frac{1}{2}$ mi. to school, and George walks $\frac{3}{4}$ of a mile. Who walks the farther, and how much?

30. From a jar containing 20 lb. of butter, a grocer sold $3\frac{3}{4}$ lb. to one customer and $6\frac{5}{8}$ lb. to another. How many pounds are left in the jar?

31. From 20 yd. of cloth a merchant sold $10\frac{3}{4}$ yd., $5\frac{1}{2}$ yd., and $2\frac{3}{8}$ yd. How many yards had he left?

32. There are $20\frac{5}{8}$ lb. of butter in a jar which weighs $4\frac{4}{5}$ lb. What is the combined weight of the butter and the jar?

33. If, from a 5-gallon jug of sirup, $\frac{1}{3}$ is poured out at one time and $\frac{1}{5}$ of the remainder at another, how many gallons remain in the jug?

34. A man bought at different times $20\frac{2}{3}$ acres of land, $50\frac{3}{4}$ acres, and $17\frac{5}{8}$ acres. How many acres did he buy in all?

35. A boy walked $\frac{1}{2}$ of a mile in one day, $\frac{3}{4}$ of a mile the next day, and $1\frac{1}{2}$ mi. the next day. How far did he walk in the three days?

36. A boy took a bicycle trip into the country to visit his uncle. He was gone $6\frac{3}{4}$ hr., $2\frac{5}{6}$ of which he spent with his uncle, and $2\frac{1}{12}$ hr. in riding back. How many hours did he spend in riding out?

37. Mr. Smith's farm contains $102\frac{5}{8}$ acres; Mr. Brown's farm contains $10\frac{1}{4}$ acres more than Mr. Smith's and $26\frac{5}{16}$ less than Mr. Green's farm. How many acres are there in Mr. Brown's farm? In Mr. Green's? In all three together?

38. A school playground was $18\frac{3}{4}$ yd. wide and $27\frac{2}{3}$ yd. long. What was the distance around it?

39. A fishing boat went out from port $10\frac{3}{16}$ mi.; when it had sailed $7\frac{9}{20}$ mi. on its return course it met a squall. How many miles was it then from port?

MULTIPLICATION

Multiplication of a Fraction by an Integer

107. PREPARATORY.

1. 3 × 2 cents = ( ) cents?
2. 3 × 2 fifths = ( ) fifths?
3. $3 \times \frac{2}{5} = \frac{(\ )}{5}$?
4. $3 \times \frac{2}{3} = \frac{(\ )}{3}$?

108. *To multiply a fraction by an integer, multiply the numerator of the fraction by the integer, and place the result over the denominator.*

EXAMPLE: Find $12 \times \frac{7}{15}$.

Solution: $12 \times \frac{7}{15} = \frac{12 \times 7}{15} = \frac{\overset{4}{\cancel{12}} \times 7}{\underset{5}{\cancel{15}}} = \frac{28}{5} = 5\frac{3}{5}$.

After indicating the multiplication of the numerator, we reduce the fraction to its lowest terms by dividing both terms by the same number. Thus, 3 divides 12 and 15; hence these are canceled and replaced by the quotients 4 and 5. Then we reduce the result to a mixed number.

In case the denominator of the fraction is divisible by the multiplier, divide the denominator instead of multiplying the numerator.

Thus, $6 \times \frac{5}{18} = \frac{5}{18 \div 6} = \frac{5}{3} = 1\frac{2}{3}$.

ORAL EXERCISES

Find:

1. $6 \times \frac{2}{3}$.
2. $9 \times \frac{5}{18}$.
3. $40 \times \frac{1}{5}$.
4. $15 \times \frac{1}{30}$.
5. $42 \times \frac{3}{7}$.
6. $8 \times \frac{3}{4}$.
7. $7 \times \frac{9}{14}$.
8. $25 \times \frac{3}{25}$.
9. $13 \times \frac{5}{26}$.
10. $21 \times \frac{5}{7}$.
11. $7 \times \frac{5}{49}$.
12. $3 \times \frac{5}{30}$.
13. $10 \times \frac{9}{30}$.
14. $9 \times \frac{4}{27}$.
15. $64 \times \frac{5}{8}$.
16. $5 \times \frac{3}{25}$.
17. $8 \times \frac{7}{16}$.
18. $12 \times \frac{25}{36}$.
19. $16 \times \frac{5}{8}$.
20. $45 \times \frac{7}{15}$.

WRITTEN EXERCISES

Find:

1. $27 \times \frac{15}{9}$.
2. $16 \times \frac{13}{76}$.
3. $56 \times \frac{25}{96}$.
4. $20 \times \frac{63}{5}$.
5. $20 \times \frac{7}{50}$.
6. $18 \times \frac{23}{36}$.
7. $25 \times \frac{38}{100}$.
8. $70 \times \frac{35}{49}$.
9. $80 \times \frac{17}{120}$.
10. $22 \times \frac{35}{144}$.
11. $40 \times \frac{9}{64}$.
12. $25 \times \frac{29}{75}$.
13. $84 \times \frac{13}{12}$.
14. $90 \times \frac{8}{15}$.
15. $84 \times \frac{10}{1728}$.
16. $126 \times \frac{34}{42}$.
17. $36 \times \frac{49}{72}$.
18. $120 \times \frac{11}{12}$.
19. $125 \times \frac{114}{625}$.
20. $243 \times \frac{17}{81}$.

Solve:

21. What floor width can be covered by 8 strips of carpet $\frac{3}{4}$ yd. wide?

22. A fence was built solidly of 184 boards placed vertically, each $\frac{7}{24}$ ft. wide. How long was the fence?

23. If Alice can cut out an apron in $\frac{5}{12}$ of an hour, how long will it take her to cut out a dozen aprons?

24. Express by a fraction the cost of one drum at $9 per dozen. Find the cost of 20 drums.

Plan: Cost of one drum: $\frac{\$9}{12}$. Cost of 20 drums: $\frac{20 \times \$9}{12} = (\quad)$?

25. Express by a fraction the cost of 1 bracelet at $11.65 per dozen. Indicate the cost of 16 bracelets. Find the value by canceling.

As above, indicate and find the cost of:

26. 675 dolls at $26 per gross.
27. 15 doz. candles at $2 per gross.
28. 20 doz. steel pens at 66¢ per gross.
29. 168 scarf pins at $9 per hundred.
30. 18 handkerchiefs at $2 per dozen.
31. 600 drawing pads at $17 per gross.
32. 32 nickel watches at $14 per dozen.
33. 21 bottles of ammonia at 88¢ per dozen.
34. 78 toy steam engines at $65 per hundred.

Multiplication of an Integer by a Fraction

109. PREPARATORY.

1. What is the cost of $\frac{1}{2}$ of a pound of butter at 20 cts. a pound? What is the cost of 2 lb.?

For $\frac{1}{2}$ lb. the cost is $\frac{1}{2}$ *of* 20 cts.
For 2 lb., it is 2 *times* 20 cts.

The words *times* and *of* have been used here with the same meaning.

When the multiplier is a proper fraction, the symbol × has the same meaning as "of."

Thus, $\frac{2}{3} \times 30$ means $\frac{2}{3}$ of 30.

2. What is the meaning of $\frac{3}{4} \times 16$? $\frac{7}{8} \times 36$? $\frac{2}{3} \times \frac{3}{7}$?

110. *To multiply an integer by a fraction, multiply the integer by the numerator of the fraction and place the result over the denominator.*

EXAMPLE: Find $\frac{3}{4} \times 16$.

Solution: $\frac{3}{4} \times 16 = \frac{3 \times \overset{4}{\cancel{16}}}{\cancel{4}} = 3 \times 4 = 12.$

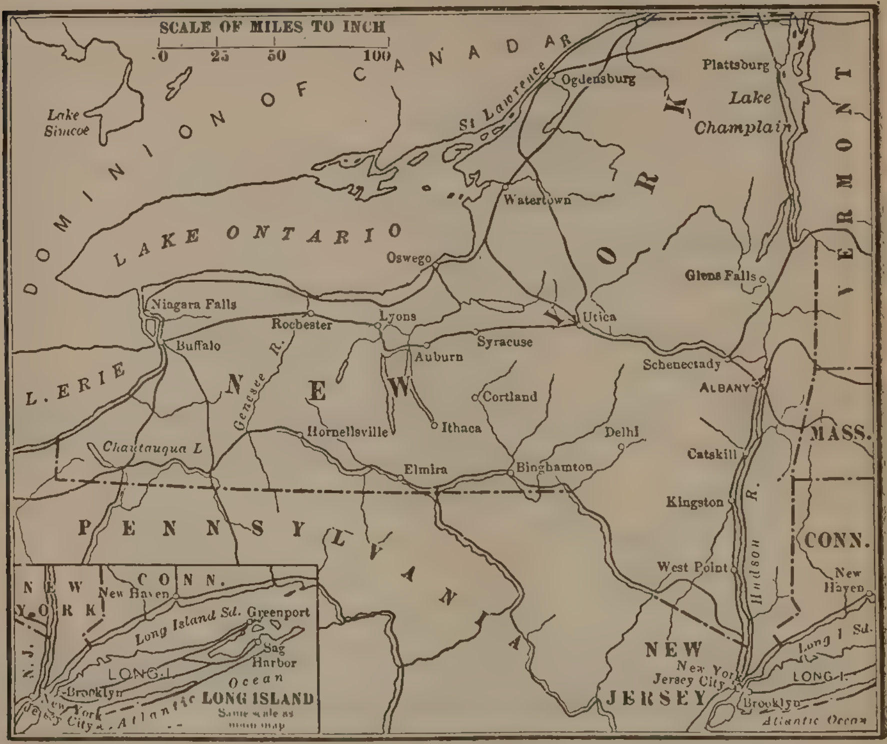
The work of multiplying an integer by a fraction is exactly the same as that of multiplying a fraction by an integer.

WRITTEN EXERCISES

Find:

1. $\frac{5}{6} \times 420$.
2. $\frac{7}{12} \times 42$.
3. $\frac{3}{14} \times 729$.
4. $\frac{5}{36} \times 840$.
5. $\frac{11}{18} \times 360$.
6. $\frac{1}{125} \times 1000$.
7. $\frac{3}{25} \times 2240$.
8. $\frac{8}{45} \times 600$.
9. $\frac{6}{7} \times 210$.
10. $\frac{5}{8} \times 196$.
11. $\frac{4}{15} \times 55$.
12. $\frac{11}{81} \times 243$.
13. $\frac{16}{25} \times 920$.
14. $\frac{1}{128} \times 64$.
15. $\frac{9}{17} \times 289$.
16. $\frac{9}{16} \times 1200$.
17. $\frac{8}{9} \times 810$.
18. $\frac{4}{5} \times 210$.
19. $\frac{9}{10} \times 320$.
20. $\frac{9}{42} \times 600$.
21. $\frac{8}{15} \times 500$.
22. $\frac{5}{12} \times 1728$.
23. $\frac{5}{21} \times 421$.
24. $\frac{5}{64} \times 1024$.

25. According to the scale of miles given on the map, to how many miles does 1 in. on the map correspond? $\frac{1}{2}$ in.? $\frac{1}{4}$ in.? $\frac{1}{8}$ in.? $\frac{1}{10}$ in.?



Find the distance within 10 miles:

26. From Buffalo to Albany.

27. From Elmira to Buffalo.

28. From Syracuse to Rochester.

29. From Syracuse to Binghamton.

30. From Albany to New York City.

31. From Binghamton to New York.

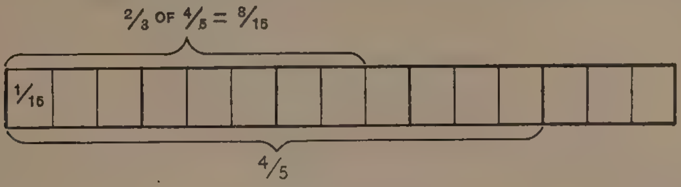
32. How many miles apart are two cities that are $1\frac{4}{10}$ in. apart on this map? If possible, find two such cities.

33. Make and solve 3 other problems about the map above.

Multiplication of a Fraction by a Fraction

111. PREPARATORY.

1. Count the number of small parts into which the figure is divided. Each division is what part of the whole figure?



2. The *heavy* lines divide the figure into how many parts? Each of these is what part of the whole figure?

3. $\frac{1}{3}$ of $\frac{1}{5}$ = ( )? $\frac{1}{3}$ of $\frac{4}{5}$ = ( )?

4. Since $\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{3 \times 5}$, what is $\frac{2}{3}$ of $\frac{4}{5}$?

5. In the diagram find $\frac{1}{3}$ of $\frac{1}{5}$. Also $\frac{1}{3}$ of $\frac{4}{5}$. Also $\frac{2}{3}$ of $\frac{4}{5}$.

WRITTEN EXERCISES

Find by diagram:

1. $\frac{3}{5}$ of $\frac{2}{3}$.
2. $\frac{2}{5}$ of $\frac{2}{3}$.
3. $\frac{2}{3}$ of $\frac{2}{3}$.
4. $\frac{3}{4}$ of $\frac{1}{2}$.
5. $\frac{3}{4}$ of $\frac{3}{4}$.
6. $\frac{2}{5}$ of $\frac{3}{4}$.

Find without diagram:

7. $\frac{3}{4}$ of $\frac{5}{8}$.
8. $\frac{4}{5}$ of $\frac{2}{5}$.
9. $\frac{1}{2}$ of $\frac{7}{8}$.
10. $\frac{2}{3}$ of $\frac{2}{3}$.
11. $\frac{4}{5}$ of $\frac{4}{5}$.
12. $\frac{3}{8}$ of $\frac{2}{3}$.
13. $\frac{5}{6}$ of $\frac{3}{4}$.
14. $\frac{5}{6}$ of $\frac{5}{6}$.
15. $\frac{1}{3}$ of $\frac{4}{7}$.

112. The results found in Exercises 1–15 are called the products of the fractions.

113. *To find the product of two fractions use the product of their numerators as the numerator of the result and the product of their denominators as the denominator of the result. Simplify this result, if possible.*

Example: Find $\frac{2}{3}$ of $\frac{3}{4}$, or, find the product of $\frac{2}{3} \times \frac{3}{4}$.

$$\textit{Solution:}\quad \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times \underset{2}{\cancel{4}}} = \frac{1}{2}.$$

The factors common to the terms are canceled as above.

WRITTEN EXERCISES

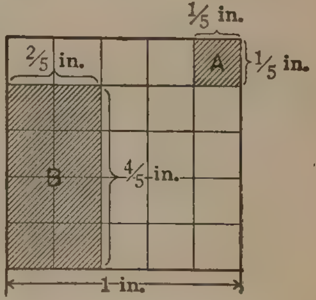
Find by canceling:

1. $\frac{9}{16}$ of $\frac{10}{21}$.
2. $\frac{10}{27}$ of $\frac{15}{7}$.
3. $\frac{18 \times 21}{7 \times 9}$.
4. $\frac{8}{33}$ of $\frac{9}{4}$.
5. $\frac{28}{5}$ of $\frac{17}{4}$.
6. $\frac{3 \times 4 \times 12}{10 \times 48}$.
7. $\frac{12}{35}$ of $\frac{14}{27}$.
8. $\frac{7}{6}$ of $\frac{8}{5}$.
9. $\frac{63 \times 72 \times 10}{10 \times 32}$.

Solve:

10. A gasoline stove uses $\frac{4}{5}$ of a quart of gasoline per hour. How many quarts does it use in $\frac{5}{6}$ of an hour?

Plan: $\frac{5}{6}$ of $\frac{4}{5}$ qt. $= \frac{5 \times 4}{6 \times 5}$ qt. $= (\quad)$ qt.



11. What is the width of the shaded square A? What is its height? Its area?

$\frac{1}{5} \times \frac{1}{5} = (\quad)$?

12. What is the width of the rectangle B? Its height? Its area?

$\frac{2}{5} \times \frac{4}{5} = (\quad)$?

13. Draw a diagram like this and shade it to show that

$$\frac{3}{5} \times \frac{4}{5} = \frac{12}{25}.$$

14. Draw another to show that

$$\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}.$$

15. Another to show that

$$\frac{4}{5} \times \frac{2}{5} = \frac{8}{25}.$$

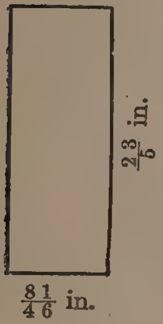
16. A street is $\frac{9}{10}$ of a mile long; $\frac{2}{3}$ of it is paved. What part of a mile is paved? Express the product and cancel the common factors.

17. A cook has $\frac{5}{6}$ of a sack of flour and uses $\frac{3}{5}$ of the flour for a baking. What part does she use? Express the product and cancel the common factors.

18. The length of the Amazon is $\frac{33}{23}$ times that of the Mississippi, and the length of the Yangtse (China) is $\frac{9}{11}$ of that of the Amazon. The length of the Yangtse is how many times that of the Mississippi?

Find the area of each of these figures:

19.



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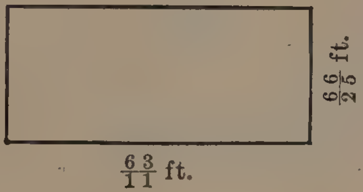

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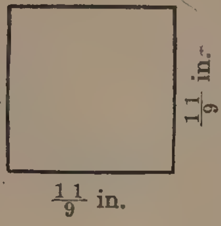

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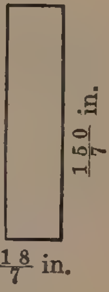


Multiplication with Mixed Numbers

114. The multiplication of an integer and a mixed number depends upon finding the product of an integer and a fraction.

EXAMPLES:

1. Find $12\frac{3}{5} \times 145$.

$$\begin{array}{rr} & 145 \\ & 12\frac{3}{5} \\ \hline \frac{3}{5} \times 145 = & 87 \\ 2 \times 145 = & 290 \\ 10 \times 145 = & 145 \\ \hline 12\frac{3}{5} \times 145 = & 1827 \end{array}$$

2. Find $12 \times 145\frac{3}{5}$.

$$\begin{array}{rr} & 145\frac{3}{5} \\ & 12 \\ \hline 12 \times \frac{3}{5} = & 7\frac{1}{5} \\ 2 \times 145 = & 290 \\ 10 \times 145 = & 145 \\ \hline 12 \times 145\frac{3}{5} = & 1747\frac{1}{5} \end{array}$$

The numbers at the left of the equality signs show how the partial products are found.

WRITTEN EXERCISES

Multiply:

1. $196 \times 11\frac{3}{4}$.
2. $120 \times 8\frac{7}{12}$.
3. $200 \times 8\frac{3}{25}$.
4. $160 \times \frac{7}{8}$.
5. $156 \times 9\frac{5}{12}$.
6. $64 \times 9\frac{5}{8}$.
7. $40 \times 67\frac{1}{2}$.
8. $35 \times 25\frac{1}{2}$.
9. $86\frac{2}{3} \times 15$.
10. $12\frac{17}{64} \times 32$.
11. $50\frac{7}{12} \times 24$.
12. $26\frac{9}{10} \times 40$.
13. $360 \times 15\frac{2}{3}$.
14. $18 \times 33\frac{1}{3}$.
15. $95 \times 14\frac{2}{5}$.
16. $144 \times 8\frac{3}{16}$.

Solve:

17. The standard weights of different kinds of fowls are:

KINDS.	WEIGHT.	KINDS.	WEIGHT.
Plymouth Rocks........	$8\frac{1}{2}$ lb.	Cochins..............	10 lb.
Wyandottes...........	$7\frac{1}{2}$ "	Langshans..........	$8\frac{1}{2}$ "
Light Brahmas........	$10\frac{3}{4}$ "	Minorcas............	$7\frac{1}{4}$ "
Dark Brahmas........	$9\frac{3}{4}$ "	Spanish.............	$7\frac{1}{4}$ "

What is the value of 5 average Plymouth Rocks of the weight given in the table, at 8 cts. a pound?

18. What would be the cost of 15 fowls of each kind given in the table, at 14 cts. a pound?

19. What is the value of a dozen broilers having an average weight of $1\frac{3}{4}$ lb. at 18 cts. a pound?

20. A tailor used $5\frac{5}{8}$ yd. of cloth for a suit. How many yards would he have used for 12 such suits?

21. What is the cost of 25 bu. of wheat at $87\frac{1}{2}$ cts. a bushel?

22. How many miles does a train travel in 14 hr., if its rate is $47\frac{3}{4}$ mi. per hour?

23. 25 adjoining lots are each $4\frac{5}{8}$ rd. long. How long are they together?

24. If, when $18\frac{3}{8}$ mi. of track are laid, $\frac{1}{3}$ of the road is completed, how long is the road?

25. What is the cost of 50 gal. of paint at $66\frac{2}{3}$ cts. a gallon, and $4\frac{1}{2}$ gal. of varnish at \$1.25 a gallon?

115. The product of two mixed numbers may be found in either of two ways.

EXAMPLE: Find $12\frac{2}{3} \times 11\frac{3}{4}$.

1st Solution:

$12\frac{2}{3} = \frac{38}{3}$.

$11\frac{3}{4} = \frac{47}{4}$.

$$\frac{\overset{19}{\cancel{38}}}{3} \times \frac{47}{\underset{2}{\cancel{4}}} = \frac{893}{6} = 148\frac{5}{6}.$$

2d Solution:

$$\begin{array}{rcr} & & 12\frac{2}{3} \\ & & \underline{11\frac{3}{4}} \\ \frac{3}{4} \times \frac{2}{3} & = & \frac{1}{2} \\ \frac{3}{4} \times 12 & = & 9 \\ 11 \times \frac{2}{3} & = & 7\frac{1}{3} \\ 11 \times 12 & = & 132 \\ \hline 11\frac{3}{4} \times 12\frac{2}{3} & = & 148\frac{5}{6}. \end{array}$$

The first solution includes multiplication of larger numbers, but is simpler.

WRITTEN EXERCISES

Multiply:

1. $3\frac{4}{5} \times 5\frac{3}{4}$.
2. $12\frac{1}{2} \times 2\frac{2}{3}$.
3. $32\frac{1}{11} \times 9\frac{1}{2}$.
4. $31\frac{1}{8} \times 3\frac{3}{4}$.
5. $26\frac{4}{5} \times 11\frac{1}{2}$.
6. $23\frac{8}{9} \times 14\frac{5}{8}$.
7. $16\frac{7}{15} \times 15\frac{2}{3}$.
8. $6\frac{1}{2} \times 2\frac{7}{12}$.
9. $18\frac{1}{5} \times 17\frac{2}{5}$.
10. $87\frac{5}{6} \times 25\frac{3}{25}$.
11. $7\frac{3}{8} \times 4\frac{1}{12}$.
12. $16\frac{2}{5} \times 7\frac{9}{10}$.
13. $81\frac{9}{10} \times 5\frac{7}{8}$.
14. $40\frac{3}{7} \times 9\frac{4}{13}$.
15. $28\frac{5}{6} \times 11\frac{3}{4}$.
16. $17\frac{6}{7} \times 12\frac{9}{14}$.
17. $92\frac{5}{32} \times 20\frac{9}{10}$.
18. $30\frac{11}{12} \times 41\frac{12}{25}$.
19. $25\frac{4}{7} \times 3\frac{3}{8}$.
20. $25\frac{3}{16} \times 48\frac{4}{5}$.
21. $6\frac{5}{9} \times 2\frac{5}{12}$.
22. $81\frac{2}{7} \times 21\frac{9}{14}$.
23. $42\frac{6}{7} \times 13\frac{8}{21}$.
24. $50\frac{9}{10} \times 25\frac{2}{5}$.
25. $75\frac{2}{3} \times 10\frac{19}{20}$.
26. $16\frac{5}{8} \times 12\frac{7}{16}$.
27. $18\frac{7}{8} \times 20\frac{5}{32}$.

Solve:

28. What is the area of a court $8\frac{5}{6}$ yd. long and $12\frac{2}{3}$ yd. wide?

29. What is the area of a floor $15\frac{3}{4}$ ft. long and $12\frac{7}{8}$ ft. wide?

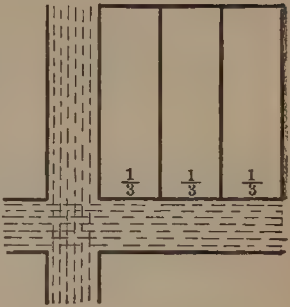
30. How far can a boatman row in $4\frac{7}{12}$ hr. at the rate of $3\frac{4}{5}$ mi. per hour?

31. A reaper harvested $10\frac{5}{6}$ acres per day. How many acres did it harvest in $5\frac{11}{12}$ days?

DIVISION

Division of Unity by any Fraction

116. Preparatory.



1. A real estate owner divided a plot so that the frontage of each part was $\frac{1}{3}$ of the frontage of the whole. Into how many lots did he divide the plot?

2. How many thirds are there in one whole thing?

3. $1 \div \frac{1}{3} = (\quad)$?

WRITTEN EXERCISES

Show by diagram that:

1. $1 \div \frac{1}{2} = 2.$

2. $1 \div \frac{1}{6} = 6.$

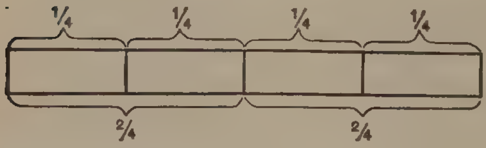
3. $1 \div \frac{1}{4} = 4.$

4. $1 \div \frac{1}{7} = 7.$

5. $1 \div \frac{1}{5} = 5.$

6. $1 \div \frac{1}{8} = 8.$

7. How many times is $\frac{1}{4}$ contained in 1? How many times is $\frac{2}{4}$ contained in 1?



Show from the diagram that $1 \div \frac{2}{4}$ is $\frac{1}{2}$ as many as $1 \div \frac{1}{4}$.

Show by diagram that:

8. $1 \div \frac{2}{6}$ is $\frac{1}{2}$ of $1 \div \frac{1}{6}$.

9. $1 \div \frac{2}{10}$ is $\frac{1}{2}$ of $1 \div \frac{1}{10}$.

10. $1 \div \frac{2}{8}$ is $\frac{1}{2}$ of $1 \div \frac{1}{8}$.

11. $1 \div \frac{3}{6}$ is $\frac{1}{3}$ of $1 \div \frac{1}{6}$.

117. From Exercise 8 we see that $1 \div \frac{2}{6} = \frac{6}{2}$; from Exercise 9, that $1 \div \frac{2}{10} = \frac{10}{2}$; from Exercise 11, that $1 \div \frac{3}{6} = \frac{6}{3}$.

118. *The division of* 1 *by any fraction results in the same fraction inverted.*

119. When the terms of a fraction are interchanged, the fraction is said to be *inverted*. Each fraction is called the **reciprocal** of the other.

Thus, $\frac{2}{3}$ becomes $\frac{3}{2}$ when inverted. $\frac{3}{2}$ is the *reciprocal* of $\frac{2}{3}$, and $\frac{2}{3}$ is the *reciprocal* of $\frac{3}{2}$.

ORAL EXERCISES

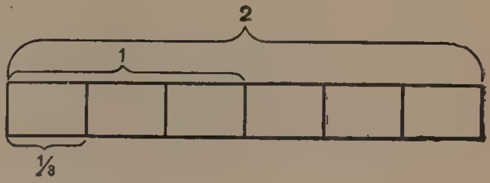
State the quotients:

1. $1 \div \frac{2}{6}$. **3.** $1 \div \frac{3}{6}$. **5.** $1 \div \frac{2}{8}$. **7.** $1 \div \frac{4}{8}$.
2. $1 \div \frac{2}{5}$. **4.** $1 \div \frac{3}{4}$. **6.** $1 \div \frac{2}{3}$. **8.** $1 \div \frac{3}{5}$.

Division of any Integer by a Fraction

120. PREPARATORY.

1. Which is the greater, the quotient of $1 \div \frac{1}{3}$ or of $2 \div \frac{1}{3}$? How many times as great? Show this from the diagram.



WRITTEN EXERCISES

Show by diagram that:

1. $3 \div \frac{1}{4}$ is 3 times as great as $1 \div \frac{1}{4}$.
2. $3 \div \frac{1}{5}$ is 3 times as great as $1 \div \frac{1}{5}$.
3. $4 \div \frac{1}{3}$ is 4 times as great as $1 \div \frac{1}{3}$.
4. $6 \div \frac{1}{4}$ is 6 times as great as $1 \div \frac{1}{4}$.

Copy and fill the blanks:

5. $1 \div \frac{2}{3} = (\quad)$. $3 \div \frac{2}{3} = (\quad)$. $2 \div \frac{2}{3} = (\quad)$.
6. $1 \div \frac{4}{5} = (\quad)$. $6 \div \frac{4}{5} = (\quad)$. $3 \div \frac{4}{5} = (\quad)$.
7. $1 \div \frac{2}{5} = (\quad)$. $4 \div \frac{2}{5} = (\quad)$. $2 \div \frac{2}{5} = (\quad)$.
8. $1 \div \frac{3}{5} = (\quad)$. $8 \div \frac{3}{5} = (\quad)$. $4 \div \frac{3}{5} = (\quad)$.

9. In Exercise 5, second column, $3 \div \frac{2}{3} = 3 \times \frac{3}{2}$; in Exercise 6, second column, $6 \div \frac{4}{5} = 6 \times \frac{5}{4}$. Write the corresponding relations for Exercises 7 and 8. Note that the divisor is inverted in the result.

10. State how the quotients were found in Exercises 5–8 above.

121. *To divide any integer by a fraction, invert the fraction and multiply.*

WRITTEN EXERCISES

Find the quotients and express them in simplest form:

1. $8 \div \frac{5}{4}$.
2. $9 \div \frac{5}{11}$.
3. $3 \div \frac{4}{5}$.
4. $18 \div \frac{5}{8}$.
5. $9 \div \frac{7}{12}$.
6. $5 \div \frac{6}{7}$.
7. $6 \div \frac{2}{7}$.
8. $90 \div \frac{9}{20}$.
9. $18 \div \frac{6}{7}$.
10. $7 \div \frac{9}{16}$.
11. $7 \div \frac{4}{5}$.
12. $84 \div \frac{12}{21}$.
13. $11 \div \frac{2}{3}$.
14. $8 \div \frac{3}{5}$.
15. $7 \div \frac{8}{9}$.
16. $65 \div \frac{13}{15}$.

Solve:

17. If one garment requires $\frac{3}{4}$ yd. of cloth, how many such garments can be cut from 12 yd.?

18. A real estate owner divided 6 acres of land into lots of $\frac{3}{8}$ of an acre each. How many lots did he have?

19. A bootblack used $\frac{2}{25}$ of a box of blacking to each pair of shoes. How many pairs did he black with 6 boxes?

20. A cook used $\frac{2}{3}$ lb. of sugar for each cake. How many cakes could she make with 10 lb. of sugar?

21. A baker used $\frac{3}{5}$ lb. of flour to a loaf of bread. How many loaves could he make from a barrel (196 lb.) of flour?

Division of a Fraction by a Fraction

122. Preparatory.

1. How is 1 divided by $\frac{3}{4}$? $\frac{1}{3}$ divided by $\frac{3}{4}$ would be what part of this?

$$1 \div \frac{3}{4} = 1 \times \frac{4}{3} = \frac{4}{3}.$$
$$\frac{1}{3} \div \frac{3}{4} = \frac{1}{3} \times \frac{4}{3} = \frac{4}{9}.$$

2. $\frac{2}{3}$ divided by $\frac{3}{4}$ is how many times $\frac{1}{3}$ divided by $\frac{3}{4}$?

$$\frac{1}{3} \div \frac{3}{4} = \frac{1}{3} \times \frac{4}{3} = \frac{4}{9}.$$
$$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}.$$

123. *To divide a fraction by a fraction, invert the divisor and multiply.*

WRITTEN EXERCISES

Find the quotients, simplifying, when necessary, by canceling:

1. $\frac{2}{3} \div \frac{3}{4}$.

2. $\frac{6}{11} \div \frac{4}{5}$.

3. $\frac{3}{7} \div \frac{3}{4}$.

4. $\frac{6}{13} \div \frac{5}{6}$.

5. $\frac{5}{6} \div \frac{7}{8}$.

6. $\frac{4}{5} \div \frac{7}{9}$.

7. $\frac{4}{5} \div \frac{9}{20}$.

8. $\frac{5}{16} \div \frac{8}{9}$.

9. $\frac{5}{8} \div \frac{2}{3}$.

10. $\frac{3}{8} \div \frac{11}{12}$.

11. $\frac{11}{12} \div \frac{5}{6}$.

12. $\frac{7}{18} \div \frac{7}{12}$.

13. $\frac{5}{8} \div \frac{9}{16}$.

14. $\frac{7}{18} \div \frac{14}{15}$.

15. $\frac{11}{32} \div \frac{22}{25}$.

16. $\frac{8}{15} \div \frac{7}{22}$.

Division of a Fraction by an Integer

124. An integer may be written in fractional form by using 1 for the denominator.

Thus, $8 = \frac{8}{1}$.

To divide a fraction by an integer, supply the denominator 1 and proceed as in dividing a fraction by a fraction.

Thus, $\frac{5}{7} \div 10 = \frac{5}{7} \div \frac{10}{1} = \frac{\cancel{5}}{7} \times \frac{1}{\underset{2}{\cancel{10}}} = \frac{1}{14}$.

WRITTEN EXERCISES

Divide:

1. $\frac{8}{9} \div 18$.

2. $\frac{9}{40} \div 36$.

3. $\frac{11}{12} \div 121$.

4. $\frac{17}{24} \div 34$.

5. $\frac{13}{20} \div 26$.

6. $\frac{7}{16} \div 245$.

7. $\frac{16}{25} \div 48$.

8. $\frac{17}{25} \div 34$.

9. $\frac{5}{12} \div 50$.

10. $\frac{9}{32} \div 27$.

11. $\frac{18}{25} \div 9$.

12. $\frac{9}{32} \div 81$.

Solve:

13. A repeating gun fired 7 shots in $\frac{11}{20}$ of a minute. What part of a minute was this per shot?

14. A man laid 125 shingles in $\frac{5}{16}$ of an hour. What part of a minute did it take per shingle?

15. A pump drew $\frac{9}{32}$ of a barrel of water in 45 seconds. How much did it draw in 1 second?

16. A boy ran $\frac{3}{16}$ of a mile in 7 minutes. How far did he run in 1 minute?

Division with Mixed Numbers

125. *To divide with mixed numbers, first reduce the mixed numbers to improper fractions, then invert the divisor, and multiply.*

EXAMPLE: Divide $4\frac{5}{8}$ by $3\frac{7}{16}$.

Solution: $4\frac{5}{8} = \frac{37}{8}$.

$3\frac{7}{16} = \frac{55}{16}$.

$$4\frac{5}{8} \div 3\frac{7}{16} = \frac{37}{8} \div \frac{55}{16} = \frac{37}{\cancel{8}} \times \frac{\overset{2}{\cancel{16}}}{55} = \frac{74}{55} = 1\frac{19}{55}.$$

Test: $\frac{74}{55} \times \frac{55}{16} = \frac{74}{16} = 4\frac{5}{8}$.

WRITTEN EXERCISES

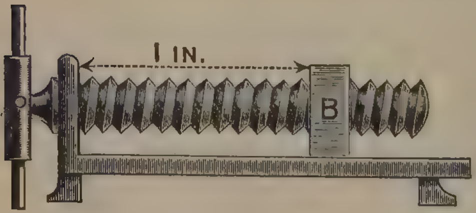
Divide and test:

1. $1\frac{5}{8} \div \frac{7}{4}$.
2. $2\frac{6}{7} \div \frac{5}{12}$.
3. $5\frac{1}{2} \div 2\frac{1}{4}$.
4. $3\frac{5}{8} \div 7\frac{4}{9}$.
5. $2\frac{1}{2} \div \frac{5}{7}$.
6. $\frac{3}{5} \div 4\frac{2}{7}$.
7. $7\frac{9}{16} \div 2\frac{1}{4}$.
8. $6\frac{9}{11} \div 8\frac{2}{3}$.
9. $6\frac{3}{5} \div 3\frac{2}{3}$.
10. $16\frac{5}{6} \div 3\frac{1}{6}$.
11. $11\frac{4}{9} \div 3\frac{2}{3}$.
12. $4\frac{8}{9} \div 17\frac{1}{3}$.

Solve:

13. How many garments requiring $\frac{4}{5}$ of a yard of cloth can be cut from $4\frac{4}{5}$ yd.?

14. How many packages of $3\frac{3}{8}$ lb. each can be made from $6\frac{3}{4}$ lb. of pepper?



15. How many threads are there in 1 inch of this screw?

When the screw is turned around once, how far is the block, B, moved? How many turns must the screw make to move B $\frac{1}{2}$ in.? $1\frac{1}{2}$ in.? $\frac{1}{4}$ in.? $1\frac{1}{4}$ in.?

16. A roll of tape contains $13\frac{1}{2}$ yd. How many pieces of $4\frac{1}{2}$ yd. each can be cut from it?

17. How many sections of $5\frac{5}{6}$ ft. each in a walk $46\frac{2}{3}$ ft. long?

To Find What Part One Number Is of Another

126. A fraction expresses what part the numerator is of the denominator.

Thus, $\frac{3}{4}$ indicates that 3 is $\frac{3}{4}$ of 4. Similarly, $\frac{8}{16}$ indicates that 8 is $\frac{8}{16}$ of 16, or $\frac{1}{2}$ of 16.

127. *To find what part one number is of another, make the first number the numerator of a fraction, the second number the denominator, and reduce the fraction to lowest terms.*

Example: 36 is what part of 144?

Solution:
$$\frac{\overset{1}{\cancel{36}}}{\underset{4}{\cancel{144}}} = \frac{1}{4}.$$

ORAL EXERCISES

1. 8 is what part of 32? Of 80? 64? 48? 72? 40? 80?

2. 9 is what part of 27? Of 45? 90? 63? 36? 72?

3. 12 is what part of 24? Of 18? 60? 36? 42? 72?

WRITTEN EXERCISES

1. 72 is what part of 144?

2. 96 is what part of 256? Of 800? Of 1,728?

3. 27 is what part of 243? Of 900? Of 1,728?

4. 15 is what part of 225? Of 400? Of 915? Of 105?

5. 18 rd. are what part of a mile?

6. 450 lb. are what part of a ton?

7. 81 cu. in. are what part of a cubic foot?

8. An investor had $650 and bought stocks with $350. What part of his money did he use?

9. There are 1,760 yd. in a mile; a balloonist was 800 yd. high. What part of a mile was he from the ground?

10. A depositor had $1,575 in the bank and drew $500. What part remained in the bank?

To Find the Whole When a Fractional Part Is Given

128. PREPARATORY.

1. If $\frac{1}{8}$ of my money is 3 cts., how many cents have I?

2. If $\frac{1}{5}$ of a class is 8 pupils, how many are in the class?

3. $\frac{8}{9}$ of the number of cubic feet in a yard is 24. How many cubic feet in a yard?

129. *To find the whole when a fractional part is given, divide the given part by the numerator of the given fraction and multiply the result by the denominator.*

EXAMPLE: $\frac{7}{15}$ of a number is 28. What is the number?

Solution: $\frac{7}{15}$ of the number is 28.
$\frac{1}{15}$ of the number $= \frac{1}{7}$ of 28, or 4.
The whole number, or $\frac{15}{15}$, $= 4 \times 15 = 60$.

Or, the problem may be indicated and worked as follows:

$$28 \div \frac{7}{15} = \frac{\overset{4}{\cancel{28}}}{1} \times \frac{15}{\cancel{7}} = \frac{60}{1} = 60.$$

WRITTEN EXERCISES

1. $\frac{3}{5}$ of a farm contained 225 acres. How many acres in the whole farm?

2. $\frac{3}{10}$ of a mile is 528 yd. How many yards in a mile?

3. $\frac{11}{71}$ of the distance from New York City to Albany is 22 mi. What is the whole distance?

4. 120 bu. in a carload of fruit spoiled in transit. This was $\frac{2}{9}$ of the carload. How many bushels were there in all?

5. A man invested $8,559, or $\frac{9}{10}$ of his money. How much was he worth?

6. The dues paid by the members of a club are $2,500 a year; this is $\frac{20}{33}$ of the whole expenses of the club. What are the whole expenses?

Complex Fractions

130. The division of one fraction by another may be indicated in fractional form.

Thus, $\frac{2}{3} \div \frac{4}{5}$ may be written $\dfrac{\frac{2}{3}}{\frac{4}{5}}$, and $4 \div 5\frac{7}{8}$ may be written $\dfrac{4}{5\frac{7}{8}}$.

131. When either one or both of the terms of a fraction contain a fraction, the expression is called a **complex fraction.**

Thus, $\dfrac{\frac{2}{3}}{\frac{4}{5}}$, $\dfrac{7\frac{1}{2}}{9}$, and $\dfrac{8}{16\frac{3}{4}}$ are complex fractions.

132. *To simplify a complex fraction, divide the numerator by the denominator.*

EXAMPLES:

1. Simplify $\dfrac{\frac{5}{8}}{\frac{3}{4}}$.

The fraction $= \frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3} = \frac{5}{6}$.

2. Simplify $\dfrac{8}{16\frac{3}{4}}$.

The fraction $= 8 \div 16\frac{3}{4} = 8 \times \frac{4}{67} = \frac{32}{67}$.

3. Simplify $\dfrac{3\frac{1}{2} \times 15}{25 \times 6\frac{1}{8}}$.

The fraction $= \dfrac{\frac{7}{2} \times 3}{5 \times 6\frac{1}{8}} = \dfrac{21}{2} \div \dfrac{245}{8} = \dfrac{21}{2} \times \dfrac{8}{245} = \dfrac{84}{245}$.

WRITTEN EXERCISES

Simplify:

1. $\dfrac{\frac{4}{5}}{\frac{8}{9}}$.

2. $\dfrac{\frac{6}{7}}{\frac{3}{14}}$.

3. $\dfrac{12}{9\frac{5}{8}}$.

4. $\dfrac{7\frac{2}{3}}{20\frac{3}{5}}$.

5. $\dfrac{3\frac{1}{2} \times 8 \times 7\frac{3}{4}}{12\frac{7}{8} \times 16 \times \frac{1}{2}}$.

6. $\dfrac{12 \times 14\frac{8}{15}}{90 \times 24\frac{3}{4}}$.

7. $\dfrac{80 \times 16\frac{7}{25}}{\frac{3}{25} \times 16}$.

8. $\dfrac{15\frac{1}{5} \times 25\frac{3}{10}}{7\frac{1}{5} \times 12\frac{9}{10}}$.

REVIEW

ORAL EXERCISES

1. What is a mixed number? How is a mixed number related to an improper fraction? Illustrate.

2. How is one fraction divided by another? Illustrate by $\frac{2}{3} \div \frac{4}{5}$.

3. What change should be made in fractions with different denominators before adding or subtracting? Illustrate.

4. Write a mixed number. Explain how to change it to an improper fraction.

5. Illustrate what is meant by reducing a fraction to its lowest terms.

6. How may subtraction of fractions be tested?

7. If two fractions have equal denominators and unequal numerators, which fraction is the larger?

8. If two fractions have equal numerators and unequal denominators, which fraction is the larger?

9. If the numerator of a fraction is multiplied by a certain number, how is the value of the fraction changed?

10. What is the effect on the value of a fraction, if both numerator and denominator are multiplied or divided by the same number?

11. The value of a fraction may be divided by a number by dividing the numerator by that number. Explain why this is true.

12. The value of a fraction may be divided by a number by multiplying the denominator by that number. Explain why this is true.

13. How is the product of an integer and a fraction found?

14. How is the product of two fractions found?

15. How do we divide any number by a fraction?

16. How is the division of fractions tested?

17. What two ways are there of multiplying mixed numbers?

18. Add at sight by giving the sum after the 2d, 3d, 4th addend, and so on to the end: $2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2}$.

19. Add as in Exercise 18 by repeating $3\frac{1}{3}$ until the sum is 30.

Add by repeating each of the following until the sum equals or exceeds 100:

20. $3\frac{1}{3}$. **21.** $12\frac{1}{2}$. **22.** $6\frac{1}{4}$. **23.** $33\frac{1}{3}$. **24.** $16\frac{2}{3}$.

25. In one week a man worked $\frac{1}{2}$ da., $\frac{2}{3}$ da., and $\frac{3}{4}$ da. How many days did he work in all?

WRITTEN EXERCISES

Reduce to lowest terms:

1. $\frac{40}{60}$.
2. $\frac{17}{85}$.
3. $\frac{35}{105}$.
4. $\frac{96}{144}$.
5. $\frac{15}{75}$.
6. $\frac{25}{95}$.
7. $\frac{88}{1331}$.
8. $\frac{75}{250}$.
9. $\frac{39}{65}$.
10. $\frac{16}{84}$.
11. $\frac{69}{126}$.
12. $\frac{95}{210}$.
13. $\frac{72}{144}$.
14. $\frac{36}{96}$.
15. $\frac{12}{20}$.
16. $\frac{34}{51}$.
17. $\frac{121}{220}$.
18. $\frac{22}{121}$.
19. $\frac{30}{225}$.
20. $\frac{12}{108}$.

Add:

21. $\frac{1}{6} + \frac{5}{24} + \frac{3}{8}$.
22. $\frac{1}{12} + \frac{3}{32} + \frac{1}{6}$.
23. $\frac{4}{5} + \frac{7}{15} + \frac{8}{9}$.
24. $\frac{7}{16} + \frac{9}{24} + \frac{11}{12}$.
25. $\frac{8}{17} + \frac{2}{34} + \frac{10}{51}$.
26. $40\frac{8}{9} + 120\frac{3}{27}$.
27. $\frac{2}{3} + \frac{7}{18} + \frac{4}{9}$.
28. $\frac{3}{14} + \frac{5}{21} + \frac{5}{7}$.
29. $\frac{40}{51} + \frac{3}{17} + \frac{1}{2}$.
30. $\frac{13}{36} + \frac{8}{9} + \frac{2}{3}$.
31. $\frac{21}{39} + \frac{4}{65} + \frac{4}{5}$.
32. $32\frac{7}{16} + 68\frac{3}{8}$.
33. $\frac{1}{12} + \frac{3}{16} + \frac{7}{18}$.
34. $\frac{3}{15} + \frac{7}{10} + \frac{1}{12}$.
35. $\frac{9}{16} + \frac{8}{25} + \frac{3}{8}$.
36. $\frac{12}{25} + \frac{7}{15} + \frac{1}{30}$.
37. $\frac{41}{100} + \frac{4}{25} + \frac{5}{6}$.
38. $25\frac{4}{5} + 150\frac{3}{10}$.

Subtract:

39. $\frac{9}{10} - \frac{2}{15}$.
40. $\frac{12}{9} - \frac{2}{21}$.
41. $\frac{25}{72} - \frac{1}{3}$.
42. $\frac{15}{39} - \frac{5}{13}$.
43. $90\frac{4}{5} - 37\frac{13}{20}$.
44. $\frac{211}{20} - 4\frac{2}{15}$.

45. $\frac{7}{8} - \frac{3}{14}$.

46. $\frac{11}{18} - \frac{3}{36}$.

47. $100\frac{7}{12} - 56\frac{2}{3}$.

48. $\frac{11}{8} - \frac{5}{24}$.

49. $\frac{50}{81} - \frac{3}{27}$.

50. $\frac{71}{99} - \frac{5}{11}$.

Multiply:

51. $\frac{7}{16}$ by 8.

52. $\frac{19}{20}$ by 100.

53. $\frac{14}{15}$ by 25.

54. $\frac{75}{95}$ by 35.

55. $\frac{12}{28}$ by 14.

56. $\frac{46}{69}$ by 36.

57. Multiply by $\frac{5}{8}$ the following: $\frac{3}{4}$; $\frac{3}{5}$; $\frac{2}{3}$; $\frac{4}{5}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{3}{8}$.

58. Multiply by $\frac{3}{7}$ the following: $\frac{3}{5}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{3}{8}$; $\frac{7}{8}$; $\frac{4}{5}$; $\frac{7}{8}$.

Divide:

59. $\frac{9}{10} \div 90$.

60. $\frac{17}{25} \div 34$.

61. $100 \div \frac{3}{4}$.

62. $75 \div \frac{2}{3}$.

63. $\frac{7}{8} \div \frac{9}{16}$.

64. $\frac{16}{49} \div 14$.

65. $\frac{12}{85} \div 51$.

66. $900 \div \frac{8}{9}$.

67. $25 \div \frac{3}{5}$.

68. $\frac{5}{16} \div \frac{3}{32}$.

69. $\frac{9}{16} \div 81$.

70. $\frac{39}{72} \div 78$.

71. $144 \div \frac{9}{16}$.

72. $320 \div \frac{11}{20}$.

73. $\frac{210}{256} \div \frac{21}{16}$.

74. 16 is what part of 96? of 144? of 256?

75. 48 is what part of 196? of 144? of 320?

76. 32 is $\frac{7}{8}$ of a certain number. What is the number?

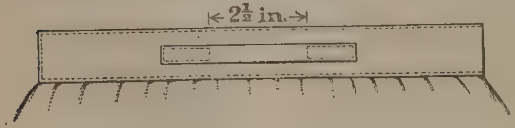
77. 63 is $\frac{5}{9}$ of a certain number. What is the number?

78. Simplify $\dfrac{3\frac{1}{2} \times 4\frac{2}{3}}{5\frac{1}{4} \times 15 \times 120}$.

79. Also $\dfrac{16\frac{2}{3} \times 10\frac{2}{5}}{24\frac{5}{6} \times 20\frac{3}{16}}$.

Solve:

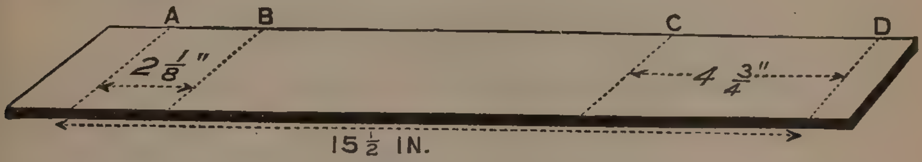
80. A breadth of canvas is $22\frac{3}{4}$ in. wide after a hem of $\frac{5}{8}$ in. has been made along each edge. How wide was the canvas before the hems were turned?



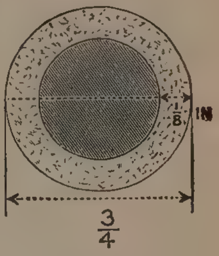
81. A tape loop sewed to a band has a free length of $2\frac{1}{2}$ in. as shown in the picture. If $\frac{3}{4}$ in. is turned under at each end, and the tape is stitched down $1\frac{1}{4}$ in. from each end, how long was the piece of tape?

82. Four half-inch holes are bored at equal intervals in a towel-rack. The distance from each hole to the next one is 4 in., and the outer ones are $1\frac{1}{2}$ in. from the ends of the rack. What is the length of the rack?

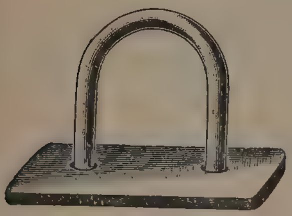
83. Distances in inches are laid off on an iron band as shown in the figure. Find the distance from B to C.



84. The outer diameter of this flat ring and its width are shown. What is the diameter of the circle cut from the center?



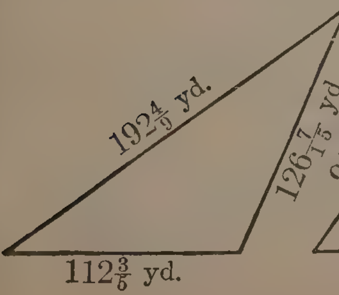
85. The picture shows an iron staple set in a piece of iron $\frac{1}{8}$ in. thick. The length of the staple from one end to the other above the plate is $3\frac{7}{8}$ in. Before it was riveted it projected through the plate $\frac{1}{8}$ in. at each end. How long must the piece of rod be cut to make one staple?



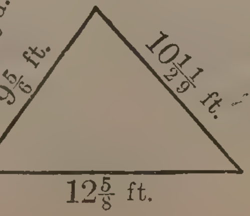
86. A piece of woolen was $78\frac{5}{6}$ yd. long; after washing it was $69\frac{5}{16}$ yd. long. How many yards did it shrink?

Find the length of fence around each of the following fields:

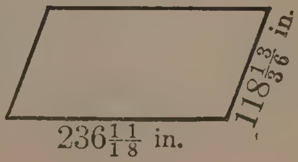
87.



88.



89.



90. A rectangular garden is $21\frac{5}{8}$ ft. wide and $2\frac{1}{3}$ times as long. How long a fence will be required to enclose it?

91. A bicycle rider goes $\frac{1}{2}$ mi. in the first minute of a race, $\frac{3}{8}$ of a mile in the second minute, and $\frac{7}{10}$ of a mile in the third. How far does he ride altogether?

92. Mr. Smith burned $\frac{1}{4}$ of a ton of coal in his furnace in October, $\frac{7}{8}$ of a ton in November, $1\frac{13}{16}$ tons in December, $2\frac{1}{5}$ tons in January, $1\frac{17}{40}$ tons in February, $1\frac{3}{4}$ tons in March, $1\frac{1}{16}$ tons in April, and $\frac{3}{10}$ of a ton in May. How many tons of coal did he use altogether?

93. A farmer planted $63\frac{3}{4}$ acres of his farm in wheat, $16\frac{7}{8}$ acres in oats, $46\frac{5}{8}$ acres in corn, and as much as $\frac{1}{8}$ of the sum of these in rye. How many acres did he plant?

94. How many feet of iron rod are needed to make 2 dozen wire staples, requiring $2\frac{5}{8}$ in. each?

95. A man sold $579\frac{5}{6}$ lb. of butter, of which $85\frac{3}{4}$ lb. were in one tub, $250\frac{2}{3}$ lb. in another, and $143\frac{1}{2}$ lb. in another. The rest was in the fourth tub. How many pounds was this?

96. A grocer had 558 bu. of apples and sold $\frac{4}{9}$ of them. How many bushels had he left?

97. From a pile of potatoes containing 413 bu., $57\frac{5}{9}$ bu. were taken away. What fraction of the whole was taken away?

98. A bicyclist traveled $45\frac{3}{10}$ mi. one day, $56\frac{3}{4}$ mi. the next, $35\frac{10}{32}$ mi. the third, and $55\frac{27}{40}$ mi. the fourth. How far did he travel in the four days?

99. $\frac{2}{7}$ of the value of a house was \$826. What was the whole value of the house?

100. A merchant had 95 yd. of cloth, and sold $\frac{1}{4}$ of it to one customer, $\frac{1}{5}$ of the remainder to another, then $\frac{1}{3}$ of what was left to another. How many yards had he left?

101. A man earns \$215 per month; he spends $\$36\frac{1}{2}$ for rent, and $1\frac{1}{2}$ times as much for his other living expenses. How much has he left?

CHAPTER III

DECIMAL FRACTIONS

NOTATION AND NUMERATION

Tenths and Hundredths

133. PREPARATORY.

1. In the number $3.56 there are how many whole dollars? How many dimes, or *tenths* of a dollar? How many cents, or *hundredths* of a dollar?

2. Name the figure in tenths' place in the above number. Also the figure in hundredths' place.

3. Read the following numbers: 3.5 ft.; 6.25 ft.; .9 gal.; .09 mi.; .18 yd.

4. How many decimal places must be used to express completely a number of hundredths?

134. The place at the right of the decimal point, next to units' place, is called **tenths'** place.

The next place at the right is called **hundredths'** place.

135. Zeros annexed at the right of a decimal do not change its value. For, since $\frac{3}{10} = \frac{30}{100}$, then .3 = .30.

136. Numbers expressed in decimal form are said to be **written decimally.**

ORAL EXERCISES

Read:

1. 10.3 dollars; 10.3 quarts; 10.3 yards.

2. 6.8; 8.8; 5.8; 105.6; 55.5; .9.

3. 2.05 dollars; 2.05 feet; 2.05 ounces.

4. .99 of a dollar; .99 of a quart; .99 of a year.

5. 3.25; 8.90; 7.08; .63; 73.40; 17,289.12.

6. Express as tenths: .50; .20; 4.70; .90.

7. Express as hundredths: .350; .750; .100; .050; 9.6.

WRITTEN EXERCISES

Write decimally:

1. Seven tenths.
2. Nineteen and nine tenths.
3. Eleven hundredths.
4. 35 hundredths.
5. 12 and 39 hundredths.
6. 1 and 1 hundredth.
7. Twenty and two tenths.
8. Twenty-two and two tenths.
9. Five hundredths.
10. Ninety-nine hundredths.
11. 40 hundredths.
12. 5 and 5 hundredths.

13. $\frac{3}{10}$.
14. $\frac{5}{10}$.
15. $\frac{14}{10}$.
16. $\frac{39}{10}$.
17. $\frac{11}{10}$.
18. $\frac{45}{10}$.
19. $\frac{1}{10}$.
20. $\frac{6}{10}$.
21. $\frac{9}{10}$.
22. $\frac{8}{10}$.
23. $\frac{7}{10}$.
24. $\frac{4}{10}$.
25. $\frac{3}{100}$.
26. $\frac{25}{100}$.
27. $\frac{55}{100}$.
28. $\frac{10}{100}$.
29. $\frac{9}{100}$.
30. $\frac{5}{100}$.
31. $\frac{15}{100}$.
32. $\frac{175}{100}$.

Write in words:

33. 21.3.
34. 26.6.
35. .15.
36. .99.
37. 11.1.
38. 41.1.
39. 1.09.
40. 86.06.
41. 55.5.
42. 37.2.
43. .25.
44. .09.
45. 33.3.
46. 99.9.
47. 10.08.
48. 35.35.

Write in the form of a common fraction:

49. .3.
50. 1.5.
51. .15.
52. .09.
53. 1.25.
54. .86.
55. 7.1.
56. .1.
57. 1.09.
58. .99.
59. 1.01.
60. .45.
61. 23.4.
62. 11.1.
63. 10.08.
64. 1.75.
65. .29.
66. 1.17.
67. 3.6.
68. 20.3.
69. .05.
70. 56.25.
71. 5.75.
72. 2.90.

Thousandths

137. PREPARATORY.

1. How many blocks like Figure 1 does Figure 2 contain? How many blocks like Figure 2 does Figure 3 contain?

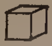


Figure 1.

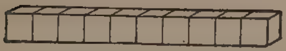


Figure 2.

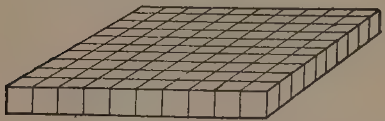


Figure 3.

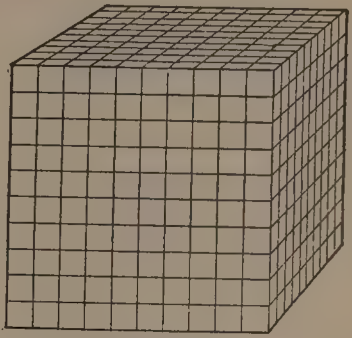


Figure 4.

2. How many blocks like Figure 3 does Figure 4 contain? How many like Figure 2? Like Figure 1?

3. If Figure 4 represents a cubic foot, what part of a cubic foot does Figure 3 represent? Figure 2? Figure 1?

4. What part of a dollar is 1 cent? What part of a dollar is $\frac{1}{10}$ of a cent?

$\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, is called a *mill*. The mill is often used in problems relating to taxes; otherwise it has little use.

5. 5 mills are what part of a dollar? 8 mills are what part of a dollar?

6. 25 mills are what part of a dollar? 125 mills are what part of a dollar?

138. A number of thousandths is expressed by 3 decimal places, the third place at the right of the decimal point being called **thousandths' place.**

Thus, 3 thousandths is expressed decimally by .003. 15 thousandths is expressed decimally by .015; and 625 thousandths by .625.

WRITTEN EXERCISES

Express decimally:

1. Seventeen thousandths.

2. Eight hundred forty-three thousandths.

3. One hundred twenty-five thousandths.

4. Nine thousandths.

5. Eighty-five thousandths.

6. Sixty-six thousandths.

7. Nine hundred ninety-five thousandths.

8. One hundred and five thousandths.

9. Two hundred and twenty-two thousandths.

10. 13 tenths. 40 tenths. 3 tenths.

11. 105 tenths. 40 hundredths.

12. Express as hundredths: .3; .5; .7; .6; 8; 15.

13. Express as thousandths: .05; .01; .3; .08; 10; 25.

14. $\frac{45}{1000}$.
15. $\frac{125}{1000}$.
16. $\frac{222}{1000}$.
17. $\frac{15}{100}$.
18. $\frac{56}{10}$.
19. $\frac{9}{1000}$.
20. $\frac{809}{1000}$.
21. $\frac{999}{1000}$.
22. $\frac{21}{10}$.
23. $\frac{106}{10}$.
24. $\frac{1}{1000}$.
25. $\frac{205}{1000}$.
26. $\frac{400}{1000}$.
27. $\frac{11}{100}$.
28. $\frac{209}{100}$.

Write in words:

29. .125.
30. .675.
31. .101.
32. .025.
33. .806.
34. .444.
35. .007.
36. .001.
37. .005.
38. 30.030.
39. 94.940.
40. 83.066.

Write each number with the decimal as a common fraction:

41. 10.9.
42. .110.
43. .015.
44. .225.
45. 21.001.
46. 121.121.
47. 1.006.
48. 46.046.
49. .005.
50. 1.09.
51. .001.
52. .365.
53. 50.07.
54. .044.
55. .076.
56. .625.

ADDITION AND SUBTRACTION

139. PREPARATORY.

Read the following problems aloud and state the sums:

1. 5 ft.	**2.** $5	**3.** .5	**4.** .05	**5.** .015
3 ft.	$3	.3	.03	.025

Read the following problems aloud and state the differences:

1. 16 yd.	**2.** $16	**3.** .16	**4.** 1.6	**5.** 2.8 ft.
12 yd.	$12	.12	1.2	1.9 ft.

140. EXAMPLES:

1. A farmer sowed 4 pieces of land and used 5.3 bu., 2.5 bu., 10.1 bu., and .8 bu. of seed, respectively. How much did he sow in all?

$$\begin{array}{r} 5.3 \text{ bu.} \\ 2.5\phantom{\text{ bu.}} \\ 10.1\phantom{\text{ bu.}} \\ \underline{.8}\phantom{\text{ bu.}} \\ 18.7 \text{ bu.} \end{array}$$

We place the whole numbers under the whole numbers, and the decimals under the decimals; and we add as with whole numbers, placing the decimal point in the sum under the decimal points of the addends.

2. An acre of land produced 28.45 bu. of wheat. When sown with oats it produced 32.75 bu. How many more bushels of oats than of wheat did it produce?

$$\begin{array}{r} 32.75 \text{ bu.} \\ \underline{28.45}\phantom{\text{ bu.}} \\ 4.30 \text{ bu.} \end{array}$$

We place the whole number to be subtracted under the whole number of the minuend, and the decimal under the decimal of the minuend. We subtract as with whole numbers, placing the decimal point in the difference under the decimal points of the given numbers.

141. *To add or subtract decimals, write like orders in a column and proceed as with integers, placing the decimal point in the result under the decimal points in the given numbers.*

WRITTEN EXERCISES

1. A man bought 1 bu. of oats at $.40, 1 bu. of wheat at $.90, 1 bu. of beans at $1.20, and 1 bu. of grass-seed at $3.30. How much did he pay for all?

2. When apples are bought at $1.86 per bushel and sold at $2.30, what is the gain per bushel?

3. Archie's cyclometer stood at 147.64 mi. when he started on a bicycle ride. On his return it stood at 161.87 mi. How far did he ride?

4. Three tanks for distilling water produce 1.20 gal., .75 gal., and 2.08 gal., respectively, per hour. How many gallons do they distil together in one hour?

5. The length of a rectangular court is 25.75 yd., and its width is 18.9 yd. What is the length of a fence around the court?

6. An automobile traveled 288.4 mi. one day, 240.85 mi. the next, and 301.38 mi. the next. How many miles did it travel in the three days?

7. A man's school tax was 15 mills per dollar of valuation, his land tax 7 mills per dollar, and his water tax 1 mill per dollar. How much did he pay per dollar in taxes?

8. The inside of a freight car was 31.65 ft. long and 6.78 ft. wide. How much longer than wide was the car?

9. A tank contained 525.45 gal., from which 399.75 gal. were drawn. How many gallons were left?

10. One building is 86.88 ft. high, and another 186.66 ft. Find the difference in their heights.

11. One piano wire is 4.125 ft. long, and another is 2.365 ft. long. What is the difference in their lengths?

12. An airship was sailed from towns A to B 10.136 mi., then to town C in the same straight line 15.085 mi.; the ship came back the same way and landed 18.943 mi. from C. How far was this from A?

MULTIPLICATION

Multiplication of a Decimal by an Integer

142. PREPARATORY.

1. How many are 3 times 3 tops? How many are 3 times 3 tenths? Express 9 tenths decimally.

2. How many are 7 times 6 hours? How many are 7 times 6 hundredths? Express the result decimally.

3. How many feet are 3 times 5 feet? How many hundredths are 3 times 5 hundredths? Express decimally 15 hundredths.

4. 3 × 8 hundredths = ( )? **5.** 6 × 21 hundredths = ( )?

143. EXAMPLE: A triangle measured 2.55 rd. on each side. What was the distance around it?

$$\begin{array}{r} 2.55 \\ 3 \\ \hline 7.65 \end{array}$$

The work is the same as in multiplying 255 by 3; but as the multiplicand is a number of hundredths, the result is a number of hundredths; and we indicate it by pointing off two places in the product.

144. *To multiply a decimal by an integer, multiply as with integers, and point off as many decimal places in the product as there are decimal places in the multiplicand.*

WRITTEN EXERCISES

Express the results decimally:

1. 5 times 9 tenths. 5 times .9.

2. 3 times 12 tenths. 3 times 1.2.

3. 4 times 202 tenths. 4 times 20.2.

4. 3 times 108 hundredths. 3 times 1.08.

5. 15 times 375 hundredths. 15 times 3.75.

6. 4 times 16 thousandths. 4 times .016.

7. 25 times 146 thousandths. 25 times .146.

Multiply:

8. .7 9	**13.** 3.9 9	**18.** .60 5	**23.** 9.6 24
9. 2.9 15	**14.** .18 3	**19.** 2.5 4	**24.** 200.1 78
10. .07 19	**15.** .8 6	**20.** 104.9 46	**25.** .75 38
11. .9 7	**16.** 10.6 97	**21.** .003 12	**26.** .105 105
12. 20.06 214	**17.** 1.265 318	**22.** 4.999 135	**27.** 15.088 26

Solve:

28. What does it cost to ride from New York City to Buffalo, a distance of 440 miles, at the rate of $.02 a mile?

29. The side of a square building is 43.98 ft. Find the distance around the building.

30. How long is a walk containing 116 sections each 4.75 ft. long?

31. How high is a card file containing 12 sections each 4.25 in. high?

32. How far can a motor cycle go in 7 hours if it travels 28.87 mi. per hour?

33. At $2.05 a day, how much does a man earn in 26 days?

34. How many miles will a train travel in 24 hr. if its rate is 40.478 mi. per hour?

35. A truck is 10 ft. long, and a teamster wishes to place 3 boxes 39.79 in. long within this length. Can he do it?

36. If 125 bricks 2.255 in. thick are laid one upon the other, how high will the pile be?

37. The wheel on an automobile was 8.665 ft. in circumference. How many feet did the machine travel while the wheel made 510 revolutions?

Multiplication of a Decimal by 10 and by 100

145. Preparatory.

1. How many tenths of a dollar are ten times one tenth of a dollar? How many dollars?

2. How many tenths of a dollar are ten times two tenths of a dollar? How many dollars?

3. 10 times 1 tenth = ( )? 10 times .2 = ( )?

4. 10 times 35 tenths = ( )? 10 times 3.5 = ( )?

5. 10 times 405 tenths = ( )? 10 times 40.5 = ( )?

146. *A decimal is multiplied by 10 by moving the decimal point 1 place to the right.*

147. Preparatory.

1. How many hundredths in 1.20? In 4.50? In 7.25? 10.05? 9.75? 20.09? 16.3? 24? 12.5?

2. 100 × 625 hundredths = ( )?
100 × 6.25 = ( )?
100 × 62.5 = ( )?

148. *A decimal is multiplied by 100 by moving the decimal point two places to the right.*

ORAL EXERCISES

1. What is the cost of 10 Dictionaries at $9.85 each? How is the result found?

2. What is the cost of 10 pairs of skates at $2.75 a pair?

3. At the rate of .45 mi. per minute, how far does a train travel in 100 minutes? How is the result found?

4. How is a decimal multiplied by 10? By 100?

5. The following are prices of various kinds of tea sold in 10-lb. lots:

Common Japan	$.22	Extra Hyson	$.29
Choice Japan	.27	Fancy Oolong	.37
Green Japan	.23	Ceylon	.25

What is the cost of 10 lb. of each kind? Of 100 lb. of each?

6. Give the cost of 100 articles of each kind:

Trunks	$5.50	Dictionaries	$9.75
Tables	18.75	Coats	16.60
Wagons	62.25	Typewriters	60.50
Sleds	3.50	Desks	8.35

WRITTEN EXERCISES

Multiply by 10 by moving the decimal point:

1. .3. **3.** .7. **5.** .9. **7.** 1.1. **9.** 11.93.
2. 20.12. **4.** 29.8. **6.** 196.75. **8.** 400.3. **10.** 287.1.

Multiply by 100 by moving the decimal point:

11. .25. **13.** .50. **15.** 1.25. **17.** 1.13. **19.** 4.05.
12. 13.65. **14.** .01. **16.** 2.225. **18.** .675. **20.** 3.125.

BILLS

149. Hundredths occur in problems relating to dollars and cents. Making and footing bills, or accounts, is one of the chief uses of decimals, requiring addition and multiplication.

ORAL EXERCISES

1. What is the cost of 2 lb. of tea at $.45 a pound?

2. What is the cost of 3 lb. of coffee at $.40 a pound?

3. What is the cost of 5 lb. of lard at $.08 a pound?

State the cost of:

4. 4 yd. lace, @ $.15.

5. 8 sticks of braid, @ $.06.

6. 8 yd. woolen, @ $.75.

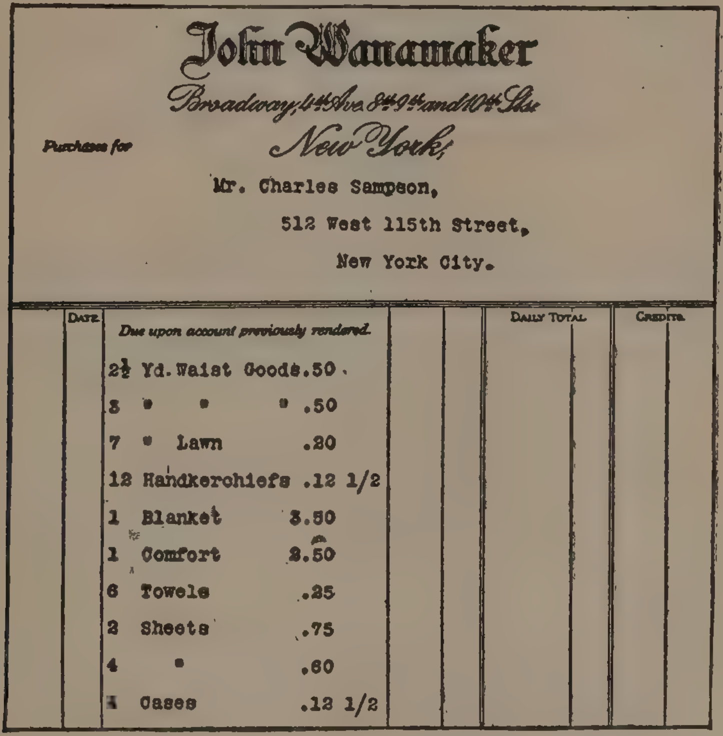
7. 2½ yd. silk, @ $1.00.

8. ½ doz. doilies, @ $.12 each.

9. 10 yd. calico, @ $.06½.

WRITTEN EXERCISES

1. Copy the items of the following bill; find the cost of each article and the total of the bill:



John Wanamaker
Broadway, 4th Ave. 8th 9th and 10th Sts.
New York,

Purchases for

Mr. Charles Sampson,
512 West 115th Street,
New York City.

Date	*Due upon account previously rendered.*			Daily Total	Credits
	2½ Yd. Waist Goods .50				
	3 " " " .50				
	7 " Lawn .20				
	12 Handkerchiefs .12 1/2				
	1 Blanket 3.50				
	1 Comfort 2.50				
	6 Towels .25				
	2 Sheets .75				
	4 " .60				
	[illegible] Cases .12 1/2				

Find the amount of each of the following bills:

2. A customer bought of a baker and confectioner:

Candy	$.50	Cheese	$.18	Bread	$.20
Cookies	.10	Wafers..........	.08	Cake	.30
Pies.............	.50	Nuts	.25	Fruit	.35

3. A family bought from a grocer:

Jan. 1.	Figs	$.18	Jan. 9.	Eggs	$.35	Jan. 20.	Gelatine	$.15
	Dates	.08		Soap	.25		Bananas	.10
	Coffee	.35		Clothes line	.30		Saltines	.15
	Peas	.30		Corn	.30		Salmon	.25
	Oranges	.25	Jan. 15.	Butter	.90	Jan. 26.	Crackers	.10
	Bananas	.10		Coffee	.30		Butter	.90
	Nuts	.15		Celery	.10		Oil	.13
	Butter	.90		Potatoes	.20		Coffee	.35
	Celery	.15		Oil	.13		Tea	.60

4. A lady purchased at a dry-goods store:

3 bolts of lace	at $1.00	4 sheets	at $.70
18 rolls cotton batting	“ .11	2 sheets	“ .58
9½ yd. outing flannel	“ .12	18 yd. muslin	“ .07

5. A restaurant keeper purchased at the market:

2 lb. beefsteak	at $.16	2½ lb. veal	at $.15
3 “ chicken	“ .18	3 “ boiled ham	“ .13
4 “ kettle roast	“ .08	3½ “ beef stew	“ .10
2 qt. oysters	“ .35	2½ “ sausage	“ .12

6. Bought of Baker and Taylor: 25 copies Shakespeare (school ed.), at $1.25 each; 70 copies Scott's "Lady of the Lake," at $.75 each; 1 gross (12 doz.) copy books, at $.75 per doz.; 12 class registers, at $.25 each.

7. Bought of Macy and Co.: Aug. 1, 25 yd. Am. prints at $.07 a yd.; 24 yd. Janesville sheeting at $.08; 12 yd. damask toweling at $.15; Aug. 15, 5 cotton spreads at $3.25 each; 15 yd. pillow tubing at $.25.

8. Bought of Best & Co. May 2, 12 children's suits at $2.15 each; 40 pairs of children's shoes at 58 cts. a pair. June 10, 36 children's coats at $3.75 each; 24 children's bonnets at 39 cts. each.

9. Bought of Roger, Nash, & Co.: Sept. 1, 20 men's suits at $12.40 each; 35 overcoats at $19.60 each; Oct. 5, 15 silk vests at $2.85 each; 4 dozen men's hats at $1.45 each.

10. Using your own name as purchaser, write bills for Exercises 6 and 9.

DIVISION

Division of a Decimal by an Integer

150. Example: Divide $36.12 by 6.

$6.02
6)$36.12

In dividing $36.12 by 6, how do we know when the dollars, or the integral part of the quotient, have been found?

151. *To divide a decimal by an integer, divide as with integers and place the decimal point in the quotient as soon as the integral part of the dividend has been divided.*

Example: Divide $7 by 3.

$2.33⅓
3)$7.00

In this example, if the division were continued there would always be a remainder; but it would be useless to carry the division farther than cents, because we have no coins of lower value. Stopping when two decimal places of the quotient have been found is called carrying the division to *two decimal places.* In dividing numbers other than cents, it may be necessary to carry the quotient to more places.

WRITTEN EXERCISES

Divide, carrying any inexact divisions to two decimal places:

1. 3.00 ÷ 5.
2. 32.8 ÷ 8.
3. 1.47 ÷ 7.
4. .169 ÷ 13.
5. 3.2 ÷ 9.
6. 3.5 ÷ 5.
7. .25 ÷ 7.
8. 2.5 ÷ 5.
9. 9.6 ÷ 12.
10. .49 ÷ 7.
11. 3.05 ÷ 6.
12. 4.65 ÷ 15.
13. 3.00 ÷ 7.
14. 3.16 ÷ 3.
15. 3.12 ÷ 6.
16. 37.2 ÷ 9.
17. 1.625 ÷ 25.
18. 18.42 ÷ 42.
19. 224.3 ÷ 81.
20. 99.74 ÷ 34.
21. 40.35 ÷ 19.
22. 28.16 ÷ 14.
23. 17.09 ÷ 13.
24. 2.05 ÷ 45.
25. 12.15 ÷ 55.
26. 89.65 ÷ 75.
27. 30.63 ÷ 36.
28. 2.005 ÷ 15.
29. 1.008 ÷ 11.

Solve:

30. When 8 pairs of shoes cost $17.20, what is the price per pair?

31. An implement dealer bought 9 farm wagons of the same kind for $436.50. What did he pay for each?

32. A farmer sold 65 bu. of oats for $32.75. How much did he receive a bushel?

33. A fence 43.2 yd. long contains 12 equal lengths. How many yards in each length?

34. A teamster drew away 13.5 cu. yd. of earth in 15 equal loads. How many cubic yards did he draw per load?

35. From a roll of 862.4 yd. of wire, 28 equal lengths are cut. What is the length of each piece?

36. A lot of 45 overcoats of the same kind cost a merchant $735.75. How much did he pay per coat?

37. 38 sets of books were sold at wholesale for $361. How many dollars did they bring per set?

Division of a Decimal by 10 and by 100

152. PREPARATORY.

1. How may the result of multiplying a decimal number by 10 be found easily? The result of multiplying by 100?

2. 10 × .3 = ( )? Then, .1 of 3 = ( )?

3. 10 × 3.2 = ( )? Then, .1 of 32 = ( )?

4. 100 × .05 = ( )? Then, .01 of 5 = ( )?

5. 100 × 3.25 = ( )? Then, .01 of 325 = ( )?

153. *To divide a decimal by 10, move the decimal point one place to the left.*

154. *To divide a decimal by 100, move the decimal point two places to the left.*

ORAL EXERCISES

Divide by 10 and state the answer:

1. 3.5.
2. 9.5.
3. 2,005.
4. 9,675.
5. .5.
6. 27.5.
7. 20.
8. 35.
9. 20.8.
10. 200.5.

Divide by 100 and state the answer:

11. 47.11.
12. 862.5.
13. 17.5.
14. 60.05.
15. 3,500.
16. 37.50.
17. 4,309.
18. 60.95.
19. 35.
20. 15.
21. 350.
22. 27.
23. 625.
24. 9.4.
25. 100.5.
26. 20.40.
27. 6,250.
28. 1,025.
29. 36.81.
30. 30.09.

Solve:

31. The area of a rectangle is 25.5 sq. in., and the height is 10 in. What is the length of the base?

Plan: Since 10 × base = 25.5, the base = $\frac{1}{10}$ of 25.5.

32. The area of a rectangle is 23.5 sq. ft., and the base is 10 ft. What is the height?

Plan: Since 10 × altitude = 23.5, the altitude = $\frac{1}{10}$ of 23.5.

33. The area of a rectangle is 327.5 sq. in., and the base is 100 in. Find the height. Use A to indicate height.

Plan: Since 100 × A = 327.5, A = $\frac{1}{100}$ of 327.5.

34. If it costs $2.50 to ride 100 mi. on the railroad, what is the rate per mile?

35. A man received $18.50 for 10 days' work. How much was this per day?

36. When the freight on 1000 lb. of books is $16.80, what is the rate per pound? Per 100 lb?

37. An automobile was driven 295.5 mi. in 10 hr. How many miles was this per hour?

38. The following shows the cost of 10 articles of each kind. What was the cost of 1?

Chairs, $42.50. Stoves, $200.00. Tables, $82.50.
Desks, $95.50. Bookcases, $150.50. Pictures, $74.20.

REDUCTION

Reduction of a Decimal Fraction to a Common Fraction

155. PREPARATORY.

1. $.5 = \frac{5}{(\ )}$? $\frac{5}{10} = \frac{(\ )}{2}$? **2.** $.04 = \frac{4}{(\ )}$? $\frac{4}{100} = \frac{(\ )}{25}$?

156. *To reduce a decimal fraction to a common fraction, supply the denominator of the decimal fraction and reduce to lowest terms.*

$$\text{Thus, } .025 = \tfrac{25}{1000}.\quad \frac{\overset{1}{\cancel{25}}}{\underset{40}{\cancel{1000}}} = \frac{1}{40}.$$

$$\text{Also, } .16\tfrac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{\overset{1}{\cancel{50}}}{3} \times \frac{1}{\underset{2}{\cancel{100}}} = \frac{1}{6}.$$

$$\text{Also, } 2.05 = 2\tfrac{5}{100} = 2\tfrac{1}{20}.$$

WRITTEN EXERCISES

Reduce to common fractions in lowest terms:

1. .3.
2. .4.
3. .8.
4. .625.
5. 9.75.
6. .45.
7. .12.
8. .01.
9. .40.
10. 17.5.
11. .125.
12. 4.25.
13. .075.
14. 4.05.
15. .375.
16. .16.
17. .25.
18. .008.

Reduction of a Common Fraction to a Decimal Fraction

157. PREPARATORY.

1. How is $\frac{1}{5}$ changed to an equal fraction whose denominator is 10? How is $\frac{2}{10}$ expressed decimally?

2. How is $\frac{3}{25}$ changed to an equal fraction whose denominator is 100? How is $\frac{12}{100}$ expressed decimally?

3. How is $\frac{7}{8}$ changed to an equal fraction whose denominator is 1000? How is $\frac{7}{8}$ expressed decimally?

4. How is $\frac{19}{200}$ expressed as thousandths? Express $\frac{19}{200}$ decimally.

WRITTEN EXERCISES

Express decimally:

1. $\frac{2}{5}$.
2. $\frac{3}{4}$.
3. $\frac{6}{25}$.
4. $\frac{3}{2}$.
5. $\frac{5}{8}$.
6. $\frac{9}{50}$.
7. $\frac{4}{5}$.
8. $\frac{6}{8}$.
9. $\frac{9}{8}$.
10. $\frac{13}{4}$.
11. $\frac{1}{4}$.
12. $\frac{9}{10}$.
13. $\frac{40}{125}$.
14. $\frac{128}{50}$.
15. $\frac{51}{200}$.
16. $\frac{19}{40}$.
17. $\frac{11}{25}$.
18. $\frac{3}{20}$.

158. *To express any fraction decimally, divide the numerator by the denominator.*

Example: Express $\frac{2}{3}$ decimally.

$$\begin{array}{r} .66\frac{2}{3} \\ 3\overline{)2.00} \end{array}$$

The 2 units are regarded as 20 tenths. This gives a quotient .6 with a remainder of .2. These are regarded as 20 hundredths, and the resulting quotient is .06, with a remainder of .02. The further division of this remainder by 3 may be indicated by $\frac{2}{3}$ placed in the quotient. More than two or three decimal places are seldom required.

WRITTEN EXERCISES

Express decimally to two places:

1. $\frac{1}{3}$.
2. $\frac{5}{8}$.
3. $\frac{1}{11}$.
4. $1\frac{2}{3}$.
5. $\frac{1}{8}$.
6. $\frac{6}{7}$.
7. $\frac{7}{11}$.
8. $5\frac{5}{6}$.
9. $\frac{1}{6}$.
10. $\frac{3}{8}$.
11. $\frac{1}{12}$.
12. $2\frac{7}{12}$.
13. $\frac{5}{6}$.
14. $\frac{7}{8}$.
15. $\frac{5}{12}$.
16. $9\frac{5}{6}$.
17. $12\frac{7}{8}$.
18. $7\frac{3}{4}$.
19. $8\frac{5}{8}$.
20. $16\frac{2}{3}$.

159. Some decimals are more convenient to use in multiplication and division when expressed in the common fraction form.

TABLE OF EQUIVALENTS

$.05 = \frac{1}{20}$
$.20 = \frac{1}{5}$
$.25 = \frac{1}{4}$
$.12\frac{1}{2} = \frac{1}{8}$
$.16\frac{2}{3} = \frac{1}{6}$
$.33\frac{1}{3} = \frac{1}{3}$
$.37\frac{1}{2} = \frac{3}{8}$
$.50 = \frac{1}{2}$
$.62\frac{1}{2} = \frac{5}{8}$
$.66\frac{2}{3} = \frac{2}{3}$
$.75 = \frac{3}{4}$
$.87\frac{1}{2} = \frac{7}{8}$

ORAL EXERCISES

1. What part of a dollar is 12½ cts.? What is the cost of a dozen collars at 12½ cts. each?

2. What part of $100 is $12½? What is the cost of 9 harnesses at $12½ each?

3. What part of $100 is $33⅓? What is the cost of 6 suites of furniture at $33⅓ a suite?

4. What part of $100 is $16⅔? What is the cost of 18 tables at $16⅔ each?

5. A real-estate agent bought some property for $2,400 and sold it so as to gain .33⅓ of what he paid for it. For how much did he sell it?

6. .25 is how many fourths? To divide a number by .25 is to divide it by what fraction?

WRITTEN EXERCISES

Divide by .25, using the shortest method:

1. 17.5.
2. $46.50.
3. $3.75.
4. 20.125 lb.
5. 25.50.
6. 3.03 ft.
7. 4.25 mi.
8. 50.775.

Solve:

9. What is the cost of 42 dozen pencils at 33⅓ cts. per dozen?

10. .33⅓ is how many thirds? To divide by .33⅓ is to multiply by what number?

11. When .33⅓ of a yd. of cloth costs $1.70, what is the price per yard?

12. When silk is 66⅔ cts. a yard, what is the cost of 30 yd.?

13. What is the cost of 40 bu. of potatoes at 87½ cts. a bushel?

14. What is the cost of 12 lb. of tea at 66⅔ cts. a pound?

15. What is the cost of 65 lb. of butter at 20 cts. a pound?

16. What is the cost of 36 lb. of cheese at 16⅔ cts. a pound?

17. What is the cost of 24 lb. of coffee at 37½ cts. a pound?

REVIEW

ORAL EXERCISES

1. Express as hundredths: .4; .9; .12.

2. Express as tenths: .40; .80; 2.60.

3. Express as thousandths: .04; .07; .6.

4. Add 2.5, .4, .6, .3.

5. Subtract .9 from 2.6. .3 from 3. 1.4 from 5.6.

6. At 12½ cts. per yd., what is the cost of 3 yd. of ribbon? Of 5 yd.? 7 yd.? 8 yd.? 11 yd.?

7. Multiply .4 by 9. .7 by 4. .8 by 5. .5 by 9.

8. Multiply 3 by .16. 3 by 1.6. 4 by 1.5. 4 by .15.

9. Express in hundredths: 1.40; 6.30; 20.8; 9.6; 4.05.

10. How many times 33⅓ cts. is $1? $1.33⅓? $1.66⅔? $2? $2.66⅔? $2.33⅓? $5?

11. At the rate of .55 mi. in a minute, how far does a train go in 10 min.? In 100 min.?

12. What is the cost of 100 chairs at $5.55 apiece? Of 100 tables at $13.40 apiece? Of 10 books at $1.63 apiece?

13. What is the cost of 3 lb. of sugar at $.15 a lb.? Of 4 lb. of tea at $.45? Of 6 lb. of candy at $.35? Of 3½ yd. of silk at $1.50? Of ½ doz. buttons at $.12 each?

14. At 12½ cts. a pound, how many pounds of sugar can be bought for $1?

15. Divide by 10: 6.5; 3.4; 3.5; 48.6; 912.7.

16. The area of a rectangle is 45.5, and the altitude is 10 in. What is the length of the base?

17. If it costs $3 to ride 100 mi. on the railroad, what is the rate per mile?

18. A train went 365.5 mi. in 10 hr. How many miles did it go in one hour?

19. What part of 25 cts. is $12\frac{1}{2}$ cts.? What part of $1 is 25 cts.? What part of $1 is $12\frac{1}{2}$ cts.?

20. How many times $12\frac{1}{2}$ cts. is $37\frac{1}{2}$ cts.? What part of a dollar is $37\frac{1}{2}$ cts.?

WRITTEN EXERCISES

1. Reduce to common fractions in lowest terms: .6; 4.36; .03; .705; .075; .5; .004.

2. Express decimally: $\frac{3}{5}$; $\frac{2}{4}$; $\frac{7}{8}$; $\frac{35}{125}$; $\frac{125}{25}$.

3. Divide, carrying any inexact division to two decimal places: 4.5 ÷ 5; .36 ÷ 6; 7.2 ÷ 9; .143 ÷ 17; 8.00 ÷ 7; 2.004 ÷ 11.

4. Express decimally to two places: $\frac{2}{3}$; $\frac{6}{8}$; $\frac{7}{8}$; $\frac{1}{12}$; $2\frac{5}{12}$; $\frac{4}{11}$.

5. A field is 25.4 ft. on one side, 14.06 ft. on another, 32.35 ft. on another, and 13.6 ft. on the fourth side. Find the distance around the field.

6. An acre of land produced 36.25 bu. of corn; when sown with wheat it produced 24.65 bu. How many more bushels of corn than of wheat did it produce?

7. The water runs into 3 cisterns at the rate of 12.06 gal., 5.6 gal., and 1.35 gal., respectively, per hour. How many gallons run into all three cisterns in one hour? In 10 hr.?

8. One building is 74.9 ft. high, and another 145.03 ft. Find the difference in their heights.

9. How far can a train go in 6 hr. if it travels 37.92 mi. per hour?

10. A man earns $3.35 a day. How much does he earn in 14 days?

11. If 123 boxes each 6.09 in. in height are piled one on the other, what will be the height of the pile?

12. A square is 2.4 ft. on a side. Find its perimeter.

13. A triangle is 13.7 in. on each side. Find its perimeter.

14. A subscription agency offers the following sets of magazines at the prices quoted:

ALL OFFERS ARE FOR ONE FULL YEAR.

Walker's and **McClure's**	$2.50
Walker's and **Scribner's Magazine**	4.00
Walker's and **Harper's Magazine**	4.50
Walker's and **Century Magazine**	4.50
Walker's and **McClure's** and **Munsey's**	3.35
Walker's and **McClure's** and **Cosmopolitan**	3.35
Walker's and **Munsey's** and **Cosmopolitan**	2.80
Walker's and **McClure's** and **Ainslee's**	3.50
Walker's and **Munsey's** and **Ainslee's**	2.90
Walker's and **Ainslee's** and **Pearson's**	2.80
Walker's and **Cosmopolitan**	2.00
Walker's and **Munsey's**	2.10
Walker's and **Ainslee's**	2.35
Walker's and **Ledger Monthly**	1.75
Walker's and **Youth's Companion**	2.85

If the regular subscription price of Walker's and McClure's magazines is $1.50 each per year, how much is saved in a year by accepting the offer of the agency?

15. If the regular price of Scribner's is $3, how much is saved by accepting the second offer of the agency?

16. Answer the same question for the third, fourth, and fifth offers, taking the regular price of Harper's as $4, of the Century as $4, and of Munsey's as $1.

17. Make and solve 10 other problems, taking the regular price of the Cosmopolitan as $1, of Ainslee's as $1.50, of Pearson's as $1, of the Ledger Monthly as $1, and of the Youth's Companion as $1.75.

18. A man wishes to fence a rectangular lot 60 ft. by 110 ft. with netting at 40 cts. per roll of 100 ft. If broken rolls are not sold, how much must he pay for the netting?

19. Find the cost of 12 yd. of broadcloth at $2.30 per yard and 18 yd. of lining silk at 59 cts. per yard.

20. The table shows the cost per person of various necessities during a recent period of 8 years:

YEAR.	BREADSTUFFS.	MEATS.	CLOTHING.
1	$10.504	$7.058	$13.602
2	10.587	7.529	13.808
3	12.783	7.694	14.663
4	13.483	7.988	15.021
5	14.898	8.906	16.324
6	14.904	9.430	15.098
7	20.534	11.628	15.533
8	17.473	9.269	17.136

Find the increase or decrease in the cost of each item from year to year.

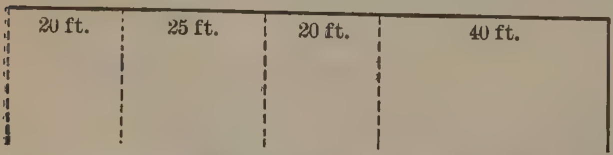
21. Find the cost of a roll, 150 running feet, of wire netting at 38 cts. per 100 ft.

22. Find the cost of 27 fruit-jars at 84 cts. per dozen, 18 tumblers at 72 cts. per dozen, 9 goblets at $1.80 per dozen, and 40 cups at $1.50 per dozen.

23. Find the cost of $\frac{1}{2}$ gross of photographic paper (1 gross = 12 dozen) at 94 cts. per gross, $\frac{5}{6}$ of a gross of pencils at $1.50 per gross, $\frac{3}{4}$ dozen inkstands at $.35 each.

24. At 3 cts. per foot, what is the cost of sufficient picture-molding to go around a room 14 ft. by 17 ft.?

25. A man bought a 25-foot lot at $15.50 per foot. Later he bought two adjacent 20-foot lots at $24.75 per foot. Still later he bought a 40-foot corner lot adjacent to one of these at $41.25 per foot. How much did he pay for all?



26. The lots of Exercise 25 were 125 ft. deep. What is the cost of a fence surrounding the whole plot at 65 cts. per running foot?

CHAPTER IV

DENOMINATE NUMBERS

160. Concrete numbers relating to units of measure are called **denominate numbers.**

Thus, $2; 6 oz.; 3.5 ft.; 4 gal.; 2 qt., are denominate numbers.

161. Denominate numbers expressed in terms of two or more units are called **compound numbers.**

Thus, 2 mi. 60 rd. 10 ft. is a compound number.

162. MEASURES OF CAPACITY

Liquid Measure

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

Barrels (bbl.) are of various sizes, but 31½ gal. is to be taken as the capacity of a barrel unless otherwise stated. 2 bbl. = a hogshead.

Dry Measure

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

ORAL EXERCISES

1. 1 qt. is what part of a gallon?

2. 1 pk. is what part of a bushel?

Express:

3. 5 gal. as quarts.

4. 3 bu. as pecks.

5. 1 pk. as pints.

6. 2 pt. as gills.

7. 2 gal. as pints.

8. 1 gal. as gills.

9. 2½ qt. as pints.

10. 2 pk. as quarts.

11. 2½ bu. as pecks.

WRITTEN EXERCISES

1. How many barrels in a cistern containing 630 gal.?

2. How many bushels in 800 one-quart crates of berries?

3. How many 1-pint bottles can be filled from 90 qt. of milk?

4. How many 5-gallon kegs can be filled from 20 bbl. of vinegar?

5. How many 3-gallon oil tanks can be filled from 6 bbl. of gasoline?

6. How many 2-gallon cans can be filled from 188 qt. of milk?

7. How many half-gallon pails can be filled from 37 gal. 2 qt. of syrup?

163. MEASURES OF WEIGHT AND VALUE

The tables in **bold type** are in everyday use and should be learned thoroughly. The other tables are of limited use.

Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.).
2,000 pounds = 1 ton (T.).

100 lb. is still sometimes used as a unit of measure and abbreviated cwt. 2,000 lb. is often called a *short ton*, and the ton of 2,240 lb., used at the mines in weighing coal and ores, is called the *long ton* or *gross ton.* A pound avoirdupois (avd.) contains 7,000 grains.

Troy Weight

Valuable metals like gold and platinum are still weighed by troy weight.

24 grains (gr.) = 1 pennyweight (pwt. or dwt.).
20 pennyweights = 1 ounce (oz.).
12 oz. = 1 pound (lb.).

The *carat* weight, used in weighing diamonds, varies: but it should be taken as $3\frac{1}{5}$ troy grains unless otherwise stated.

In speaking of gold as "so many carats fine," one means so many twenty-fourths of pure gold. Thus, gold 14 carats fine is $\frac{14}{24}$ pure gold and $\frac{10}{24}$ alloy (cheaper metals).

Apothecaries' Weight

In preparing prescriptions druggists use apothecaries' weight.

20 grains (gr.) = 1 scruple (sc. or ℈).
3 scruples = 1 dram (dr. or ʒ).
8 drams = 1 ounce (oz. or ℥).
12 ounces = 1 pound (lb.).

In the following pages all references to weight pertain to avoirdupois weight unless otherwise stated.

United States Money

Review the table of United States money.

10 cents (¢ or cts.) = 1 dime.
10 dimes = 1 dollar ($1).
100 cents = 1 dollar.

ORAL EXERCISES

1. How many pounds avoirdupois in 32 oz.?

2. How many 100-pound bales of hay make a ton?

3. How many ounces in $2\frac{1}{2}$ lb.? In 5 lb.?

4. How many dimes in $1.40? In $5.50?

5. How many cents in $3.50? In $3.05?

WRITTEN EXERCISES

Express:

1. 96 oz. as pounds (avd.).
2. 2,500 lb. as tons (short).
3. $9\frac{1}{2}$ lb. as ounces (avd.).
4. $24\frac{1}{2}$ lb. as ounces (avd.).
5. 120 oz. as pounds (avd.).
6. 5,600 lb. as tons (long).
7. 256 oz. as pounds.
8. 1872 oz. as pounds (avd.).

9. How many short tons of coal could be sold from a carload weighing 50 long tons?

10. How many pounds in 48.75 long tons?

Questions on Tables Not Generally Used

1. How many pennyweights in a pound troy?

2. What is meant by saying that a ring is 18 carats gold?

3. How many grains troy does a diamond weigh that weighs 3 carats?

4. How many scruples in an ounce apothecaries'?

5. How many drams in a pound apothecaries'?

6. How many grains in a dram?

7. How many 5-grain powders can be made from a dram of quinine?

8. Express 5 oz. as scruples. Also as drams.

9. Express $2\frac{1}{2}$ lb. as grains (troy). $2\frac{1}{2}$ lb. as grains (apoth.).

10. Find the difference between the number of grains in a pound avoirdupois and the number in a pound troy.

11. How many pennyweights of pure gold in a pin 14 carats fine, weighing 12 pwt.?

12. How many grains in an ounce apothecaries'?

13. How many 4-grain powders can be made from $\frac{1}{3}$ of an ounce of salol?

164. MEASURES OF TIME

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
12 months (mo.) = 1 year (yr.).
100 years = 1 century.

April, June, September, and November have 30 days each. The rest, except February, have 31. February has 28 days (29 in leap years).

There are 365 days in a common year and 366 days in a leap year. Every year whose number is divisible by 4 is a leap year, except that century years are leap years only when divisible by 400.

ORAL EXERCISES

1. How many minutes in 2 hr.? $1\frac{1}{2}$ hr.? $1\frac{1}{3}$ hr.? $\frac{5}{6}$ hr.?
2. How many seconds in $\frac{1}{3}$ min.? $\frac{2}{3}$ min.? $1\frac{5}{6}$ min.?
3. How many days in 10 wk.? In 8 wk.?
4. How many weeks and days in a 30-day month?

165. MEASURING BY TWELVES

12 units = 1 dozen (doz.).
12 dozen = 1 gross (gr.).

12 gross = 1 great gross.

ORAL EXERCISES

1. How many pencils in $2\frac{1}{2}$ doz.? In $3\frac{1}{4}$ doz.?
2. How many dozens in $\frac{3}{4}$ gr.? In $2\frac{1}{2}$ gr.? In $\frac{5}{6}$ gr.?
3. How many dozens in 108 eggs? In 144 eggs? In 252?

166. MEASURES OF LENGTH, AREA, AND VOLUME

Linear Measure

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
16.5 feet or 5.5 yards = 1 rod (rd.).
320 rods = 1 mile (mi.).
1,760 yards = 1 mile.
5,280 feet = 1 mile.

A unit of 6 ft., called the *fathom,* is used to measure depths at sea. A *cable length* is 120 fathoms.

Distances at sea are measured by a *nautical mile,* which equals 1.15 common or statute miles.

Draughtsmen often use the symbols ′ and ″ to express feet and inches. Thus, 5 ft. 6 in. is written 5′ 6″.

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
30¼ square yards = 1 square rod (sq. rd.).
160 square rods = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).
36 square miles = a township.

Cubic Measure

1,728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).

Surveyors still use a special table as follows:

100 links (7.92 in.) = 1 chain (4 rd. or 66 ft.).
80 chains = 1 mile.
10 square chains = 1 acre.

In measuring excavations, 1 cu. yd. is regarded as a *load.*

In measuring masonry, $24\frac{3}{4}$ cu. ft. is regarded as a *perch* of stone. This is not uniformly used, however.

In measuring wood, a pile of 4-foot wood 8 ft. long and 4 ft. high is called a *cord.* 128 cu. ft. = 1 cord.

A *cord of stove wood* is a pile of wood cut in stove lengths (usually 16 in.) 8 ft. long and 4 ft. high.

ORAL EXERCISES

1. How many inches are there in a yard?

2. How many feet are there in 4 rd.?

3. How many yards are there in $\frac{1}{3}$ of a mile?

4. How many square rods in $\frac{1}{2}$ of an acre? In $\frac{1}{4}$ of an acre?

5. How many acres in a square $\frac{1}{2}$ mi. on a side?

6. How many cubic feet in $\frac{1}{3}$ of a cubic yard ?

WRITTEN EXERCISES

1. How many cords of wood in 384 cu. ft.?

2. How many acres in a township?

3. How many cubic inches in $5\frac{1}{2}$ cu. ft.?

4. A farm measured 45.8 square chains. How many acres in the farm?

5. Some metal roofing cost at the rate of 1 ct. per square inch. How much was this per square yard?

6. When a balloon was $1\frac{1}{2}$ mi. above the ground, how many feet high was it?

7. How many cubic inches in a block of marble 2 ft. wide, 2 ft. thick, and 4 ft. long?

8. A court 2 rd. wide and $4\frac{1}{2}$ rd. long was paved with cement. How many square yards of cement in the court?

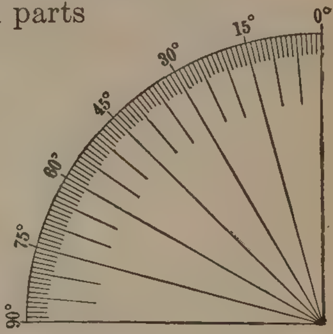
9. How many cubic yards of earth were taken out in excavating a cellar 54 ft. wide, 180 ft. long, and 9 ft. deep?

ANGULAR MEASURE

167. When it is 3 o'clock the hands of a clock form a **right angle.**

When the minute hand moves from XII to III, it turns through a right angle.

In order to describe the size of angles we suppose a right angle to be divided into 90 equal angles, called **degrees,** indicated by the sign °.



Each degree is divided into 60 equal parts called **minutes,** indicated by ′.

Each minute is divided into 60 equal parts called **seconds,** indicated by ″.

When the hands of a clock form a right angle, as at 3 o'clock, what fraction of the clock-face is included between them?

If we think of the lettered circle of the clock-face as measuring the angles, we see that the circle measures 4 right angles.

TABLE

60 seconds (60″) = 1 minute (1′).
60 minutes = 1 degree (1°).
90 degrees = 1 right angle.
360 degrees = 4 right angles.

ORAL EXERCISES

1. How many degrees in each angle of a square?

2. How many degrees in one-half of a right angle?

3. How many degrees in an angle containing 2 right angles?

WRITTEN EXERCISES

1. A pie is cut into 4 equal pieces. How many degrees are there in the angle between the sides of each piece? How many degrees are there in the angle of each piece, if the pie is cut into 6 equal pieces? Into 8 equal pieces?

2. A wheel has 8 spokes. What is the angle between each spoke and the next?

3. A ship is sailing north. Through how many degrees must it turn to change its course to northeast?

4. Through how many degrees does the minute-hand of a clock turn in an hour? In $\frac{1}{2}$ hr.? In $\frac{1}{4}$ hr.? In 10 min.? In 5 min.?

5. What is the angle between the hour-hand and the minute-hand at 2 o'clock? At 5 o'clock?

6. How many seconds in $5\frac{1}{2}'$? In $3\frac{5}{6}'$? In 1°?

7. How many minutes in $1\frac{5}{6}^\circ$? In $5\frac{2}{3}^\circ$?

8. Draw a square and connect the opposite corners by diagonal lines. What is the size of each angle between the cross lines?

9. What is the size of each angle of a rectangle? How many degrees in the sum of the angles of a rectangle?

10. Draw a line from the center of the clock-face to XII, then one to IIII, and one to VIII. How many degrees in each angle made by these lines?

11. Join the center of the clock-face with the point corresponding to III; join the center with the point corresponding to IX. What is the number of degrees in the angle between these lines?

12. A windmill had 4 sails making 4 equal angles. How many minutes in each of these angles?

13. How many minutes in the angle between the hands of the clock at 4 o'clock?

PROCESSES WITH DENOMINATE NUMBERS

Reduction

168. Changing the form of a denominate number without changing its value is called **reduction.**

Thus, 3 gal. 2 qt. may be expressed as 14 qt. or 28 pt. or $3\frac{1}{2}$ gal.

169. Changing a number to another of lower denomination is called **reduction descending.**

For example, 2 lb. may be changed to 32 oz.

170. EXAMPLE: Reduce 36 gal. 1 qt. to pints.

$$\begin{array}{rl}
36 & \\
\underline{4} & \\
144 + 1 = & 145 \\
 & \underline{2} \\
 & 290
\end{array}$$

36 gal. 1 qt. = 290 pt.

(1) Since 1 gal. = 4 qt., 36 gal. = 36 × 4 qt. = 144 qt.
(2) 144 qt. + 1 qt. = 145 qt.
(3) Since 1 qt. = 2 pt., 145 qt. = 145 × 2 pt. = 290 pt.

In computing the result, we multiply 36 by 4 instead of multiplying 4 by 36, because the work is easier, and the result is the same.

WRITTEN EXERCISES

Reduce:

1. 1 rd. 5 ft. to inches.
2. 4 lb. 2 oz. to ounces.
3. 5 rd. 12 ft. to feet.
4. 1 T. 1017 lb. to pounds.
5. 12 gross, 8 doz. to units.
6. $4\frac{5}{8}$ A. to square rods.
7. 3 gal. 2 qt. to pints.
8. 1 bu. 2 pk. 4 qt. to quarts.
9. 5 pk. 7 qt. to pints.
10. 8′ 10″ to inches.
11. 12 yd. 2 ft. to feet.
12. 3 cu. yd. 9 cu. ft. to cu. ft.

171. Changing a number to another of higher denomination is called **reduction ascending.**

For example, 30 in. may be changed to 2 ft. 6 in.

172. Example: Reduce 590 in. to yards.

$$\begin{array}{r} 12\overline{)590} \\ 3\overline{)49} + 2 \\ 16 + 1 \end{array}$$

16 yd. 1 ft. 2 in. =
$16\frac{7}{18}$ yd.

There are as many feet in 590 in. as 12 in. are contained times in 590 in., or 49 with a remainder of 2 in.

There are as many yards in 49 ft. as 3 ft. are contained times in 49 ft., or 16 with a remainder of 1 ft.

Therefore 590 in. = 16 yd. 1 ft. 2 in.

1 ft. 2 in. = 14 in. = $\frac{14}{36}$ yd. = $\frac{7}{18}$ yd. Hence, 16 yd. 1 ft. 2 in. = $16\frac{7}{18}$ yd.

WRITTEN EXERCISES

Reduce:

1. 368 oz. to pounds.
2. 400 pt. to gallons.
3. 340 pk. to bushels.
4. 3,600 sec. to hours.
5. 52 wk. to days.
6. 900 things to gross.
7. 3,456 cu. in. to cubic feet.
8. 1,865 pt. to gallons.
9. 878 hr. to days.
10. 560 in. to yards.
11. 840 sec. to hours.
12. 250 in. to yards.
13. 4,060 yd. to miles.
14. 1,728 cu. in. to cubic feet.
15. 400 sq. in. to square feet.
16. $18\frac{1}{2}$ T. to ounces.
17. 3 yr. 10 da. to hours.
18. 60.5 sq. yd. to square rods.

Addition and Subtraction

173. Examples:

1. Add 12 ft. 8 in. and 25 ft. 7 in.

$$\begin{array}{rr} 12\text{ ft.} & 8\text{ in.} \\ \underline{25\text{ ft.}} & \underline{7\text{ in.}} \\ 37\text{ ft.} & 15\text{ in. or} \\ 38\text{ ft.} & 3\text{ in.} \end{array}$$

Whenever the sum in any column is larger than the number of units in the next higher denomination, the result must be reduced.

Hence 15 in. = 1 ft. 3 in., and the 1 ft. is added to the next column.

2. Subtract 17 lb. 9 oz. from 39 lb. 3 oz.

39 lb. 3 oz.
17 lb. 9 oz.
21 lb. 10 oz.

When the minuend in any column is too small, increase it by the value of a unit from the next left-hand column. Thus, 39 lb. 3 oz. = 38 lb. 19 oz.

174. *To add or subtract compound numbers, add or subtract the numbers of each denomination, and reduce the result, if necessary.*

WRITTEN EXERCISES

Add:

1. 240 lb. 10 oz.
15 lb. 12 oz.

2. 25 gal. 2 qt. 1 pt.
12 gal. 2 qt. 1 pt.

3. 12 yd. 9 in.
18 yd. 6 in.

4. 30 mi. 100 rd.
55 mi. 300 rd.

5. 17 bu. 3 pk.
45 bu. 2 pk.

6. 3 wk. 6 da.
11 wk. 1 da.

7. 40 rd. 10 ft. 6 in.
15 rd. 8 ft. 8 in.
12 rd. 14 ft. 10 in.

8. 12 gal. 3 qt. 1 pt.
40 gal. 2 qt. 1 pt.
67 gal. 1 qt. 0 pt.

9. 7 wk. 6 da. 20 min.
8 wk. 5 da. 35 min.
6 wk. 0 da. 50 min.

10. 83 bu. 2 pk. 6 qt.
17 bu. 0 pk. 7 qt.
25 bu. 3 pk. 0 qt.

Solve:

11. A lawn is 12 yd. 2 ft. 9 in. long and 10 yd. 1 ft. 8 in. wide. What is the distance around it?

12. A grocer bought 3 bu. 1 pk. 5 qt. of cherries from one customer, 4 bu. 3 pk. 6 qt. from another, and 2 bu. 1 pk. 3 qt. from another. Find the total of these three amounts.

Subtract:

13. 25 gal. 1 qt.
12 gal. 2 qt.

14. 55 mi. 100 rd.
35 mi. 300 rd.

15. 10 wk. 5 da.
7 wk. 6 da.

16. 48 lb. 8½ oz.
16 lb. 9¾ oz.

17. 90 bu. 2 pk. 1 qt.
45 bu. 1 pk. 1 qt.

18. 18 rd. 6½ ft.
9 rd. 8¾ ft.

19. 58 gal. 2 qt. 0 pt.
17 gal. 3 qt. 1 pt.

20. 83 bu. 1 pk. 4 qt.
25 bu. 1 pk. 0 qt.

Solve:

21. A dairy had 46 gal. 1 qt. 1 pt. of milk on hand in the morning and sold 41 gal. 3 qt. 1 pt. How much was left?

22. A court is 100 yd. 2 ft. 9 in. long and 80 yd. 1 ft. 10 in. wide. By how much does the length exceed the width?

Multiplication

175. Example: Find 7 times 12 rd. 2½ ft.

12 rd. 2½ ft.
7
84 rd. 17½ ft. or
85 rd. 1 ft.

The product in the first column is 17½ ft. This equals 1 rd. 1 ft. Hence we write 1 ft. and add 1 rd. to the result in the next column.

176. *To multiply compound denominate numbers, multiply each denomination separately aud reduce the result, if necessary.*

WRITTEN EXERCISES

Multiply:

1. 18 lb. 10 oz. by 16.
2. 40 ft. 6 in. by 25.
3. 17 gal. 1 pt. by 32.
4. 20 da. 6 hr. 10 min. by 50.
5. 10 wk. 3 da. 10 hr. by 12.
6. 6 mi. 1250 ft. by 15.
7. 170 yd. 8 in. by 42.
8. 40 gr. 11 doz. by 66.
9. 120 ft. 7 in. by 33.
10. 40 T. 200 lb. by 100.

11. An automobile tank held 5 gal. 3 qt. 1 pt. of gasoline. How much gasoline did it take to fill the tank 6 times?

12. A driving track was 40 rd. 12 ft. 6 in. long. How far did a horse go in traveling over this track 9 times?

Division

177. Example: Ralph divided 47 lb. 6 oz. of sand into 8 equal parts. How much did each part weigh?

Divide 47 lb. 6 oz. by 8.

```
  5 lb.        14¾ oz.
8)47 lb.        6 oz.
  40
   7 × 16 = 112
            118 oz.
              8
             38
             32
              6
```

(1) Dividing 47 lb. by 8 gives 5 lb. and 7 lb. remaining.

(2) 7 lb. is the same as 7 × 16 oz. = 112 oz.

(3) Adding this to 6 oz. makes 118 oz.

(4) Dividing 118 oz. by 8, we have $14\frac{3}{4}$ oz.

Each part weighed 5 lb. $14\frac{3}{4}$ oz.

178. *To divide a compound number by an abstract number, divide each denomination, beginning with the highest, and combine any remainder with the next lower denomination.*

WRITTEN EXERCISES

Divide:

1. 120 ft. 8 in. ÷ 12.

2. 500 lb. 6 oz. ÷ 9.

3. 200 da. 17 hr. ÷ 15.

4. 17 mi. 100 rd. ÷ 40.

5. 50 sq. ft. 80 sq. in. ÷ 24.

6. 262 yd. 2 ft. ÷ 6.

7. 90 rd. $4\frac{1}{2}$ ft. ÷ 8.

8. 290 min. 40 sec. ÷ 7.

9. 18 A. 40 sq. rd. ÷ 20.

10. 1 cu. ft. 500 cu. in. ÷ 27.

Solve:

11. The fence around a square field measured 81 rd. Find the length of one side.

12. There are 5 cans of equal size full of milk. The whole amount is 37 gal. and 2 qt. How much does each can hold?

13. A cement walk 15 rd. 3 yd. 1 ft. 8 in. extends across the front of 3 equal lots. What is the frontage of each lot?

179. Example: How many iron bars 2 ft. 3 in. long can be cut from a rod 7 ft. 5 in. long?

Divide 7 ft. 5 in. by 2 ft. 3 in.

$$7 \text{ ft. } 5 \text{ in.} = 89 \text{ in.}$$
$$2 \text{ ft. } 3 \text{ in.} = 27 \text{ in.}$$
$$89 \text{ in.} \div 27 \text{ in.} = 3\tfrac{8}{27}.$$

3 bars can be cut from the rod and the remainder equals $\frac{8}{27}$ of a bar.

180. *To divide one compound number by another, reduce both to the same denomination and divide.*

WRITTEN EXERCISES

1. 90 ft. 9 in. ÷ 8 ft. 6 in.

2. 20 gal. 3 qt. ÷ 2 gal. 1 qt.

3. 40 bu. 2 pk. ÷ 3 bu. 6 qt.

4. 17 lb. 4 oz. ÷ 2 lb. 3 oz.

5. 45 yd. 2 ft. ÷ 1 ft. 10 in.

6. 48 gal. 1 pt. ÷ 18 gal.

7. 17 yd. 2 ft. ÷ 3 yd. 1 ft.

8. 2 sq. ft. 100 sq. in. ÷ 12 sq. in.

9. 75 rd. 12 ft. ÷ 3 rd. 10 ft.

10. 12 hr. 28 min. ÷ 5 hr. 20 min.

11. 24 A. 30 sq. rd. ÷ 4 A. 10 sq. rd.

12. 10 mi. 10 rd. ÷ 10 ft. 10 in.

13. 20 gal. 2 qt. ÷ 2 gal. 1 pt.

14. 7 wk. 2 da. 10 hr. ÷ 1 da. 20 hr.

Solve:

15. A grocer bought 4 bu. 3 pk. of strawberries and sold them 1 quart at a time. How many sales did he make?

16. How many times can a 1-quart tank be filled from 14 gal. 3 qt. of gasoline?

17. A square rug 110 in. on a side is how many feet around?

18. A car contained 4,950 lb. of coal. How many short tons on the car?

19. How many packages containing 2 lb. 12 oz. can be made from 30 lb. 4 oz. of starch?

20. A milkman has 54 gal. 1 qt. 1 pt. of milk and uses it all to fill an equal number of quart-bottles and pint-bottles. How many of each can he fill?

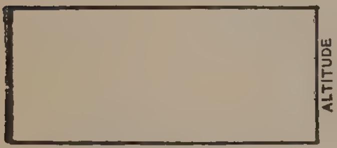
APPLICATIONS OF DENOMINATE NUMBERS

Measuring Surfaces

AREAS OF PARALLELOGRAMS

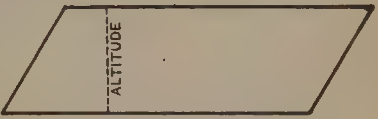
181. A **parallelogram** is a four-sided figure whose opposite sides are parallel, that is, everywhere an equal distance apart.

A **rectangle** is a four-sided figure whose angles are right angles.



RECTANGLE

SQUARE



PARALLELOGRAM

A **square** is a rectangle whose sides are equal.

Squares and other rectangles are also parallelograms.

The side on which a figure seems to stand is called its **base.**

The height of a figure above its base is called its **altitude.**

In finding area, the base and the altitude must be of the same denomination, as feet and feet, or yards and yards.

182. *The number of units in the area of a rectangle or a parallelogram is the product of the numbers measuring its base and altitude.*

Example: What is the area of a rectangle whose base is 1 ft. 6 in. and whose altitude is 8 in.?

Solution: (1) The base 1 ft. 6 in. = 18 in.
(2) 8 × 18 × 1 sq. in. = 144 sq. in.

The base and the altitude expressed in inches are 18 in. and 8 in., respectively. Hence the product of 8 and 18 is the number of square inches in the area of the rectangle.

WRITTEN EXERCISES

1. How many rectangular pieces of cardboard 9 in. by 12 in. can be cut from a piece a yard square?

Find the areas of parallelograms with the following bases and altitudes:

	2.	3.	4.	5.
Bases:	45 yd.	2.75 ft.	46 in.	315 ft.
Altitudes:	1.75 yd.	15 ft.	11.5 in.	.75 ft.

6. A village lot is 50 ft. by 120 ft. How many square feet in its area? How many square yards?

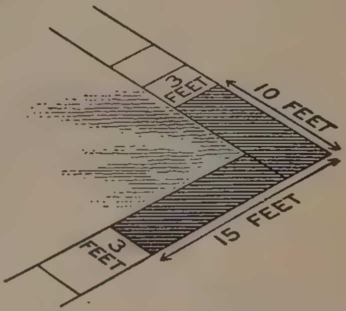
7. The *Place de la Concorde* in Paris is a rectangle 217 yd. by 357 yd. Find its area in square yards.

8. What is the altitude of a parallelogram that is 67.5 in. long and contains 135 sq. in.?

9. Find the area of the shaded portion of the walk shown in the picture.

10. How many acres in a farm in the shape of a rectangle that is 250 rd. long and 125 rd. wide?

11. What is the cost of laying a cement walk 75½ ft. long and 6 ft. wide at $.16½ a square foot?



12. How many feet long is a rectangular park that contains 836,000 sq. yd. and is 1,425 ft. wide?

13. What is the cost of making a playground 75 ft. long, 40 ft. wide, at $1.25 a square yard?

14. A skating pond in the form of a rectangle contained ¼ of an acre and was 4 rd. wide. How long was the pond?

15. How many square yards in a square court 100 ft. on a side?

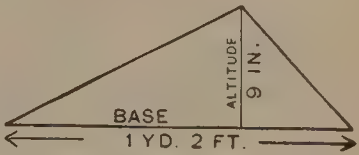
16. A garage had a floor space 42 ft. wide and 84 ft. long. How many square feet in the floor?

AREA OF TRIANGLES

183. A **triangle** is a plane figure having three sides.

184. *The number of square units in the area of a triangle is one-half of the product of the number of units measuring its base and altitude.*

Example: Find the area of a triangle whose base is 1 yd. 2 ft. and whose altitude is 9 inches.



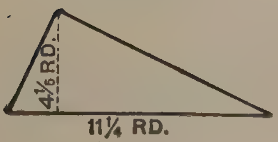
Solution: (1) 1 yd. 2 ft. = 5 ft.
(2) 9 in. = $\frac{3}{4}$ ft.
(3) $\frac{1}{2} \times \frac{3}{4} \times 5 \times 1$ sq. ft. = $1\frac{7}{8}$ sq. ft.

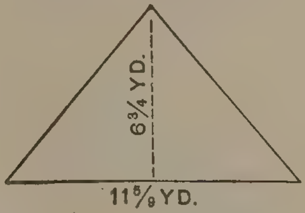
WRITTEN EXERCISES

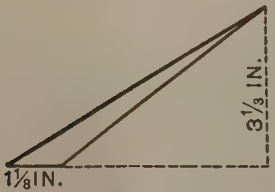
Find the area of each triangle:

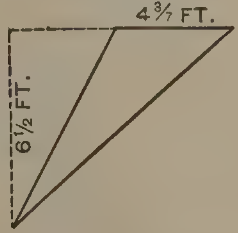
	1.	2.	3.	4.
Base........	3 in.	9 ft.	$2\frac{1}{2}$ yd.	6 ft.
Altitude.....	4 in.	3 ft.	1 yd.	$3\frac{1}{2}$ ft.

5. Find the area of each of the following figures:





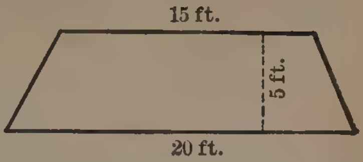




6. A triangular plot of ground had a base of 20 rd. and an altitude of 16 rd. How many acres in its area?

AREA OF TRAPEZOIDS

185. A **trapezoid** is a plane figure of four sides, two sides of which, and only two, are parallel.



186. *The number of square units of area in a trapezoid is one-half the product of the number of units in the altitude and in the sum of the lengths of the bases.*

Example: Find the area of a trapezoid whose parallel bases are 15 ft. and 20 ft., and whose altitude is 5 ft.

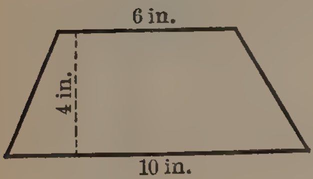
Solution: (1) 15 ft. + 20 ft. = 35 ft.
(2) $\frac{1}{2} \times 5 \times 35 \times 1$ sq. ft. = 87.5 sq. ft., the area of the trapezoid.

WRITTEN EXERCISES

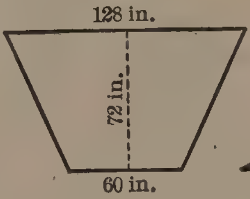
1. Let s represent the sum of the two parallel sides of a trapezoid, and a its altitude. What stands for its area?

Find the areas of the following trapezoids:

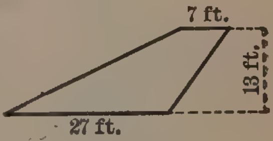
2.



3.

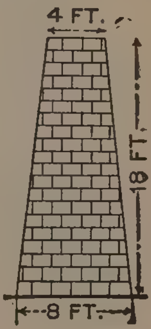


4.



5. A field is in the form of a trapezoid. The parallel sides are 116 yd. and 84 yd., and the distance between them is 50 yd. How much will it cost to plow it and sow it with wheat at 6 cts. per 10 sq. yd?

6. What will the stone for the wall in the figure cost at 97 cts. per square foot of the face?



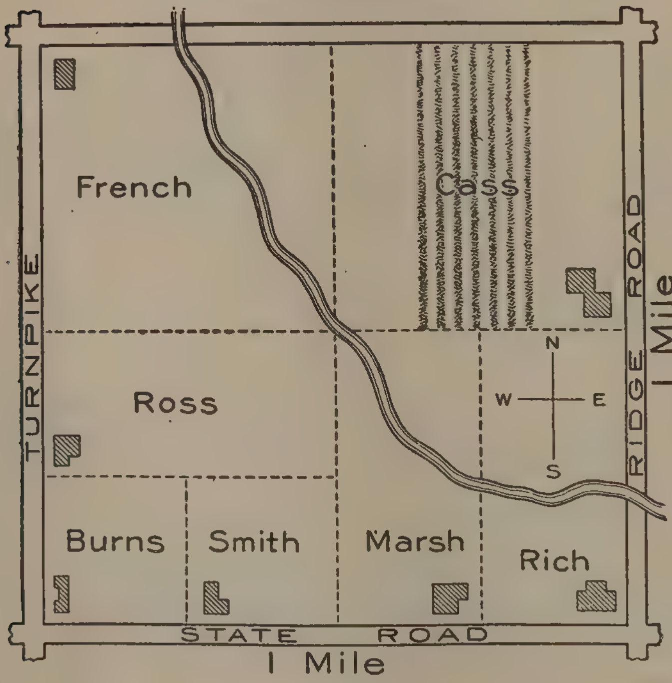
Find the area of each of the following trapezoids:

	LOWER BASE.	UPPER BASE.	ALTITUDE.	AREA.
7.	23 ft.	17 ft.	12 ft.	( )
8.	72 in.	28 in.	20 in.	( )
9.	30 yd.	20 yd.	12 yd.	( )
10.	32 rd.	48 rd.	9 rd.	( )

11. What is the area of a trapezoid of bases 8 in. and 6 in. and of altitude 4 in.?

MEASURING LAND

ORAL EXERCISES



1. How many miles are there in the distance around the tract of land as shown in the picture? How many rods are there in this distance (1 mi. = 320 rd.)?

2. The land is divided into fourths, eighths, and sixteenths. What part of the whole is each farm? How many rods are there in the length of the Cass farm? In its breadth?

3. How many rods of line-fence are there between the Cass and French farms? Between the Cass and Rich farms?

4. What part of the square mile does the Cass farm occupy? The Rich farm? The Ross farm? The Smith farm?

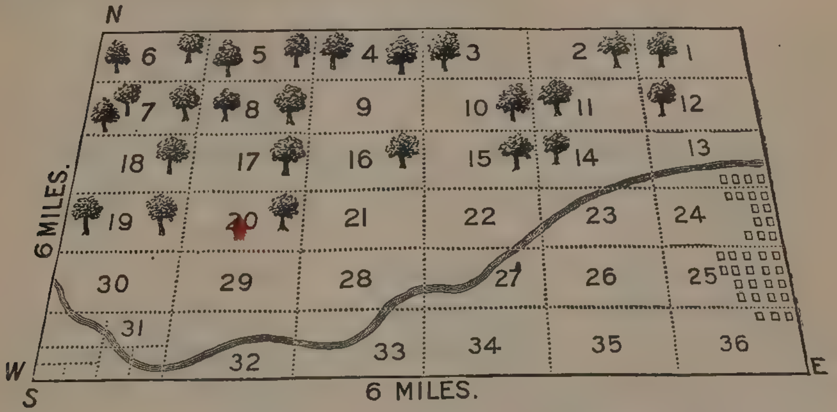
187. A square mile of land contains 640 acres and is called by surveyors a **section.**

ORAL EXERCISES

1. How many acres are there in the French farm? In the Smith farm? In the Burns farm?

2. Which of the above farms occupy a quarter-section?

3. From the dimensions in rods of the Smith farm find its area in square rods. How many acres does it contain? How many square rods are there in an acre?



188. In surveying the newer states, the government surveyors divided the land into **townships** as nearly as possible 6 miles square. Each township is divided into 36 parts called sections, and these are recorded by number as shown in the figure. The township is regarded as square for all computations.

EXERCISES

1. How many square miles are there in a township?

2. In every township the Government sets apart section number 16 for school purposes. How many acres is this? In a locality where such land sells for $25 an acre, what is the gift worth?

3. Section 31 is the one shown on p. 140. What part of the area of a township is this section? How many acres are there in a township?

4. How many acres are there in a row of sections in a township?

5. What will be the cost of a line-fence between section 9 and section 10 at $1.75 a rod?

6. If half of a township is timber land worth $10 an acre, and the rest is farming land worth $25 an acre, what is the value of the township?

7. Make and solve 3 other problems about land.

CARPETING, PAPERING, AND SHINGLING

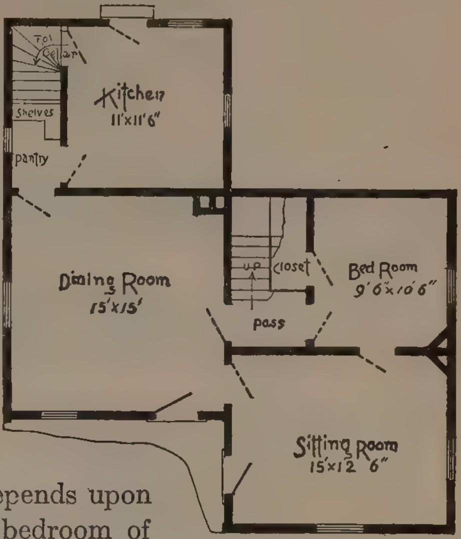
WRITTEN EXERCISES

1. A certain room is 5 yd. long and 4 yd. wide. How many strips of carpet 1 yd. wide, and running the long way, will be required to cover the room? How many yards of carpet will there be?

2. In carpeting the sitting-room shown in the floor plan on p. 143, the strips are to run lengthwise, and 8 in. to the strip is to be allowed for matching the pattern. It is not customary in selling carpet to cut strips lengthwise, but a fraction of a yard in length is sold. Make a drawing of the sitting-room floor showing the strips of carpet.

3. How many yards of carpet $\frac{3}{4}$ yd. wide will be required to carpet the sitting-room, if the strips run the long way of the room?

4. How many yards of carpet 1 yd. wide, that can be matched without waste, are needed to carpet the bedroom if the strips run the long way?



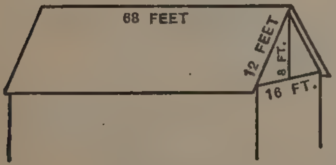
5. A rug 9 ft. by 12 ft. is placed centrally in the dining-room, and yard-wide filling at $33\frac{1}{3}$ cts. per yard is used to cover the rest of the floor. What is the cost of the filling?

Wall-paper is usually sold in rolls of 16 yd. each, the paper being 18 in. wide. Fractions of a roll are not sold. The number of strips that can be cut from a roll depends upon matching the pattern. In the bedroom of the above house the ceiling paper is to run the short way, 5 strips being cut from a roll. In the other rooms the ceiling paper is cut so as to use 3 sixteen-foot strips to the roll.

6. Find the cost of the paper for the ceilings of the bedroom, dining-room, and sitting-room at 40 cts. a roll.

7. Taking the height of the rooms to be 9 ft., 5 strips can be cut from a roll. Allowing $\frac{1}{10}$ of the measured surface for doors, windows, baseboards, and frieze, find the cost of the paper for the three rooms, at 45 cts. a roll.

8. How many shingles will be needed to cover a roof 42 ft. along the ridge and 18 ft. down the slope, allowing 800 shingles per 100 sq. ft.?

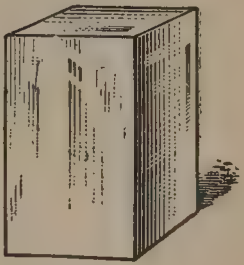


9. Shingles are sold in bundles, 4 bundles to the thousand, and a fraction of a bundle is not sold. Find the cost of the shingles actually used for the roof in the figure at $2.45 per thousand shingles and for the two gables at $3.80 per thousand.

Measuring Volume

VOLUMES OF RECTANGULAR SOLIDS

189. *The number of cubic units in the volume of a rectangular solid, or prism, is the product of the numbers of units in its base and altitude.*



The area of the base must be expressed in square units corresponding to the linear units of the altitude.

Example: Find the volume of a rectangular box whose base is 50 sq. in. and whose altitude is $1\frac{1}{4}$ ft.

Solution: (1) $1\frac{1}{4}$ ft. = 15 in.
(2) $15 \times 50 \times 1$ cu. in. = 750 cu. in.

WRITTEN EXERCISES

1. An ice-wagon has a bed 10 ft. 6 in. by 3 ft. 9 in., inside measure, and the ice is packed 4 ft. 2 in. high. How many cubic feet of ice are there?

2. A street 40 ft. wide and $\frac{3}{4}$ mi. long is to be covered with crushed stone to a depth of 3 in. Find the cost of the stone at $1.45 per cubic yard.

3. How many wagon-loads will the quantity of stone mentioned in Exercise 2 make, if the inside measurements of the wagon are $4\frac{1}{2}$ ft. by $9\frac{1}{3}$ ft. by $2\frac{1}{4}$ ft.?

4. How many cubic feet of asphalt are required to cover a surface 6 ft. wide and 500 ft. long with a layer 3 in. thick?

5. How many cubic feet of material are there in a cement walk 450 ft. long, $4\frac{1}{2}$ ft. wide, and 6 in. thick?

6. How many cubic yards of masonry in a concrete dam 350 ft. long, 12 ft. thick, and 36 ft. high?

7. A tunnel 1000 ft. long, 30 ft. wide, 15 ft. high, was cut through rock. How many cubic feet of rock were removed?

EQUIVALENTS

190. Cubic contents may be expressed in units of capacity.

231 cu. in. = 1 gallon.
2,150.42 cu. in. = 1 bushel.

191. Example: An automobile has a gasoline tank measuring 3 ft. by 14 in. by 8 in. How many gallons will it hold?

Solution: (1) 3 ft. = 36 in.
(2) $36 \times 14 \times 8$ cu. in. = 4,032 cu. in.
(3) 4,032 cu. in. ÷ 231 cu. in. = $17\frac{5}{11}$, the number of gallons.

WRITTEN EXERCISES

1. How many cubic inches in a liquid quart?

2. How many cubic inches in a dry quart?

3. Which is the larger, the dry or the liquid quart? By how many cubic inches?

4. How many cubic feet in a bushel?

5. How many bushels does a bin hold that is 8 ft. by 4 ft. by 6 ft.? (Use 1.244 cu. ft. = 1 bu.)

6. A bin measures 4 ft. by $6\frac{1}{2}$ ft. by $3\frac{1}{2}$ ft. and is filled with oats. How many times can a horse be fed 4 qt. of oats from the contents of the bin? (Neglect remainder.)

7. The tank on a gasoline stove was 16 in. long, 6 in. wide, and 4 in. deep. How many times could it be filled from a 10-gallon tank of oil? (Neglect remainder.)

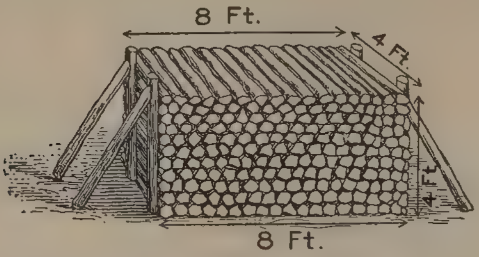
8. A water tank held 500 gal. of water when full. What was its volume in cubic feet?

9. A tank is $28\frac{4}{5}$ in. long, $6\frac{5}{16}$ in. wide, and $8\frac{4}{7}$ in. deep. How many cubic inches does it contain? How many gallons?

10. How many bushels of grain does a bin hold that is $12\frac{1}{2}$ ft. long, $8\frac{3}{4}$ ft. wide, and 4 ft. deep? (Use 1.244 cu. ft. = 1 bu.)

MEASURING WOOD AND LUMBER

192. The picture shows a cord of wood (see page 127).


8 Ft.
4 Ft.
4 Ft.
8 Ft.

ORAL EXERCISES

1. How many cubic feet are there in a cord of wood?

2. How many cords of wood are there in a pile 32 ft. long, 4 ft. wide, and 4 ft. high? In a pile 32 ft. by 4 ft. by 2 ft.?

3. How many cords of wood are there in 12 rows of 4-foot sticks, each row being 32 ft. long and 4 ft. high?

4. How many cords of wood are there in a pile 64 ft. long and 10 ft. high?

5. How many stove lengths (see page 127) can be cut from one 4-foot length?

6. What are the measurements of a cord of stove wood?

Wood is usually sold at the wood-yard in this form. Thus, if a dealer says maple wood is $2.50 a cord, he means a cord of stove wood.

WRITTEN EXERCISES

1. How many cords of 16-inch stove wood can be cut from a cord of 4-foot wood?

2. How many cords of 4-foot wood are required to cut 75 cords of 16-inch wood? 96 cords? 69 cords? 243 cords?

3. A dealer buys 4-foot wood at $4 a cord and, after cutting it into 16-inch wood, sells it for $2.50 a cord. How much does he gain on a cord of the 4-foot wood?

193. A square foot of lumber 1 in. or less in thickness is called a **board foot.** In lumber more than 1 in. thick, the number of board feet depends upon the thickness.

Thus, a board 8 in. wide, 3 ft. long, and $\frac{1}{2}$ in. thick contains 2 board feet.

8 in. = $\frac{2}{3}$ ft. Lumber $\frac{1}{2}$ in. thick is reckoned as if it were 1 in. thick.

$\frac{2}{3} \times 3 \times 1$ board ft. = 2 board ft.

A board 8 in. wide, 3 ft. long, and $1\frac{1}{2}$ in. thick contains 3 board feet.

$\frac{2}{3} \times 3 \times 1\frac{1}{2}$ board ft. = 3 board ft.

WRITTEN EXERCISES

How many feet of lumber are there in:

1. 3 boards each 16 ft. long, 10 in. wide, and 1 in. thick?
2. 12 planks each 12 ft. long, 14 in. wide, and 2 in. thick?
3. 150 boards each 14 ft. long, 16 in. wide, and $\frac{3}{4}$ in. thick?
4. 25 joists each 16 ft. long, 4 in. wide, and 4 in. thick?

Find the cost of the following at $22 per thousand board feet:

	FEET LONG.	INCHES WIDE.	INCHES THICK.
5. 28 boards......	16	$10\frac{1}{2}$	$\frac{1}{2}$
6. 52 boards......	14	14	1
7. 65 scantlings...	16	4	2
8. 48 joists.......	14	10	$2\frac{1}{2}$
9. 12 timbers.....	$18\frac{1}{2}$	12	10
10. 125 rafters.....	20	6	2
11. 36 plates......	$16\frac{1}{2}$	8	$4\frac{1}{2}$

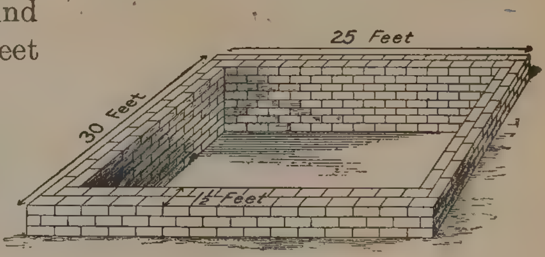
Solve:

12. How many feet of lumber in a box, excluding the top, made of boards $\frac{3}{4}$ in. thick, and having a bottom 6 ft. by $3\frac{1}{2}$ ft. and an inside depth of $4\frac{1}{3}$ ft.?

13. How many feet of lumber are required to lay a floor 16 ft. wide and 80 ft. long, of planks $3\frac{1}{2}$ in. thick?

COMPUTING THE CONTENTS OF MASONRY

194. Example: Find the number of cubic feet of stone in a cellar wall 25 ft. by 30 ft. outside measurement and $1\frac{1}{2}$ ft. thick and 8 ft. deep.



Solution: (1) Take the end walls to be 25 ft. long, $1\frac{1}{2}$ ft. thick, and 8 ft. deep.

Then $25 \times 8 \times 1\frac{1}{2} \times 1$ cu. ft. = 300 cu. ft.

The two end walls = 2×300 cu. ft. = 600 cu. ft.

(2) Take the side walls to be 27 ft. long, $1\frac{1}{2}$ ft. thick, and 8 ft. deep.

Then $27 \times 8 \times 1\frac{1}{2} \times 1$ cu. ft. = 324 cu. ft.

The two side walls = 2×324 cu. ft. = 648 cu. ft.

(3) Therefore, the number of cubic feet in the cellar wall is 600 cu. ft. + 648 cu. ft. = 1,248 cu. ft.

WRITTEN EXERCISES

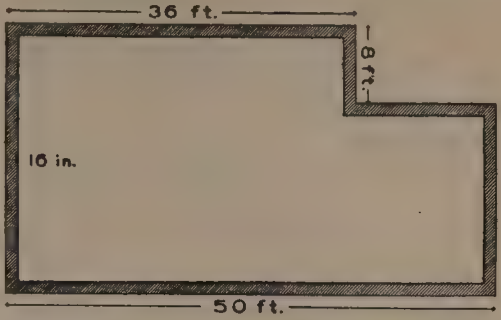
1. At $1.56 per cubic yard, what is the cost of the stone for a wall $1\frac{1}{2}$ ft. thick, built within a cellar 30 ft. by 40 ft. and 7 ft. deep?

2. Find how many cubic yards of masonry there are in a foundation wall in the form of a rectangle whose inside dimensions are 30 ft. by 50 ft. and 9 ft. high, and whose thickness is 18 in. Draw a ground-plan of the wall.

3. The inside dimensions of a hollow brick pier are 15 ft. long, 12 ft. wide, and 7 ft. high. How many cubic feet in the wall which is 8 in. thick?

4. How many cubic yards of stone in the walls of a basement, the inside dimensions of which are 100 ft. by 75 ft. by 12 ft., the wall being 20 in. thick?

5. Find the number of cubic yards of masonry in a wall 9 ft. high built on the foundation shown in the picture.



6. A cistern in the form of a hollow prism has for inside dimensions, length 16 ft., width 5 ft., depth 7 ft. Taking the top to be flat and the thickness of the walls of the cistern including the top and bottom to be 8 in., find the number of cubic feet of cement used in making the cistern.

7. A vat built of bricks and mortar had for inside dimensions, length 10 ft., width 8 ft., depth 12 ft. Taking the thickness of the wall to be 6 in., compute the number of cubic feet in the masonry, omitting the top and the bottom.

8. A glass cube 12 in. on an edge has a hollow cubical space within, 6 in. on an edge. How many cubic inches of glass are there?

Finding the Difference Between Dates

ORAL EXERCISES

1. How many days are there from January 1 to February 25, including Jan. 1, but excluding Feb. 25?

2. How many days are there from April 1 to May 20, including April 1, but excluding May 20?

3. Longfellow was born June 18, 1819, and Lowell was born Feb. 22, 1819. What was the difference in their ages?

4. How long after the beginning of the American Revolution, April 19, 1775, did the French Revolution, May 5, 1789, begin?

5. The Spanish-American War began April 21, 1898, and ended April 11, 1899. How long did it last?

195. When the difference in time is large, the following written method of solution should be used:

Example: How long after the Battle of Hastings, Oct. 14, 1066, did the Battle of Waterloo, June 18, 1815, take place?

Solution:

	YR.	MO.	DA.
June 18, 1815 =	1815	6	18
Oct. 14, 1066 =	1066	10	14
	748 yr.	8 mo.	4 da.

196. *To find the difference between dates of different years, write the year, the number of the month, and the days in each date and subtract.*

WRITTEN EXERCISES

1. Victoria was born May 24, 1819, and was crowned queen, June 28, 1836. How old was she at this time?

2. Napoleon was born Aug. 15, 1769, and died May 5, 1821. What was his age at the time of his death?

3. George Washington was born Feb. 22, 1732, and died Dec. 14, 1799. What was his age at the time of his death?

4. Queen Elizabeth was born Sept. 7, 1533, and died March 24, 1603. How old was she at the time of her death?

5. The British Museum celebrated its 150th anniversary in 1909. In what year was it founded?

6. Abraham Lincoln was born Feb. 12, 1809, and died April 14, 1865. How old was he when he died?

7. How many years, months, and days is it from the discovery of America to the present day?

8. How many years, months, and days is it since the battle of Lexington, April 19, 1775?

9. The Declaration of Independence was adopted July 4, 1776. How long ago was that?

10. How long is it since the Boston massacre, which occurred March 5, 1770?

REVIEW

WRITTEN EXERCISES

Add:

1. 40 gal. 2 qt. 1 pt.
86 gal. 3 qt. 1 pt.

2. 12 bu. 3 pk. 2 qt. 1 pt.
15 bu. 2 pk. 7 qt. 1 pt.

3. 2 mi. 16 rd. 1 ft. 6 in.
5 mi. 43 rd. 14 ft. 3 in.

4. 17 T. 1000 lb. 8 oz.
25 T. 1500 lb. 11 oz.

5. 48 sq. yd. 8 sq. ft. 100 sq. in.
36 sq. yd. 3 sq. ft. 106 sq. in.
17 sq. yd. 7 sq. ft. 54 sq. in.

6. 100 cu. yd. 20 cu. ft. 12 cu. in.
276 cu. yd. 12 cu. ft. 1720 cu. in.
106 cu. yd. 15 cu. ft. 200 cu. in.

Subtract:

7. 16 mi. 60 rd. 10 ft. 11 in.
7 mi. 58 rd. 11 ft. 10 in.

8. 700 T. 409 lb. 3 oz.
80 T. 1500 lb. 9 oz.

9. 12 bu. 3 pk. 2 qt. 1 pt.
3 bu. 3 pk. 7 qt. 1 pt.

10. 40 gal. 2 qt. 0 pt.
17 gal. 3 qt. 1 pt.

Multiply:

11. 16 sq. yd. 3 sq. ft. 100 sq. in.
7

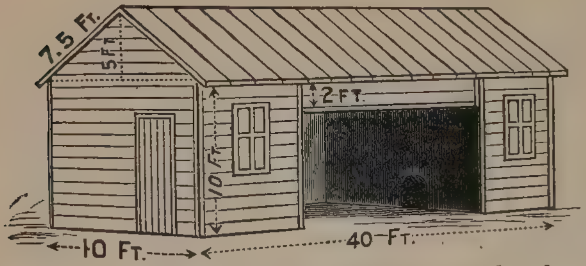
12. 10 cu. yd. 20 cu. ft. 1000 cu. in.
25

Divide:

13. 15)25 sq. yd. 8 sq. ft. 100 sq. in.

14. 22)17 mi. 181 rd. 15 ft. 6 in.

15. From the dimensions given in the figure, find the area of the space covered by the poultry-house.


7.5 Ft.
5 Ft.
2 Ft.
10 Ft.
10 Ft.
40 Ft.

16. Find the number of square feet of lumber required for the walls, the front opening being 20 ft. wide. Also for the roof, taking the length of the roof to be 40 ft.

17. How many yards of wire netting 1 yd. wide are required to cover the opening to the shed?

18. How many flagstones 2 ft. 8 in. by 5 ft. are required to lay a walk 5 ft. wide and 56 ft. long?

19. How many acres are there in a farm $1\frac{1}{2}$ mi. long and $\frac{3}{4}$ of a mile wide?

20. What is the cost of renovating a rug $8\frac{1}{4}$ ft. × $10\frac{1}{2}$ ft. at 10 cts. per square yard?

21. How many feet of inch boards will be necessary to build a sign-board 40 ft. long and 12 ft. high? What will the lumber cost at $6 per thousand feet?

22. What is the volume of a rectangular solid having a square base 8 in. on a side and an altitude of 10 in.?

23. A rectangular stone pier has a base of 25 sq. yd. and a height of 35 ft. What is the number of cubic feet in its volume? What was the cost of the pier at $8.50 per cu. yd.?

24. Air is composed of oxygen and nitrogen; in any quantity of pure air about .21 is oxygen and .79 is nitrogen. How many cubic feet of each are there in 2,700 cu. ft. of air?

25. If the average amount of air inhaled by a pupil at one breath is 24 cu. in., how many times must he breathe to inhale 1 cu. ft.? If he breathes 20 times a minute, how long will it take him to inhale a cubic foot?

26. How long will it take 30 pupils to inhale as much air as there is in a schoolroom 25 ft. by 30 ft. by 12 ft.?

27. A vat is 20 ft. long, $3\frac{1}{2}$ ft. deep, and 6 ft. wide. How many gallons of paper pulp will it hold when full?

28. An elevator has a capacity of 250,000 bu. of grain. How many cubic feet in the cubical capacity of its bins?

29. A freight car is 32 ft. long, $7\frac{1}{2}$ ft. high, and 8 ft. wide inside measure. How many bushels of wheat would it contain?

30. How many cords of wood in a pile 256 ft. long, 4 ft. wide, and 16 ft. high?

31. How many yards of carpet $\frac{3}{4}$ yd. wide are required to cover a room 15 ft. by 18 ft., the strips running the long way and $\frac{7}{9}$ yd. in all being allowed for matching?

32. How many seconds in a day of 24 hours?

33. How many minutes in an angle of 35°?

34. Find the volume of an oblong solid whose base is 15 sq. ft. and whose altitude is 50 ft.

35. The edge of a cube is $4\frac{1}{2}$ in. How many cubic inches does it contain?

36. How many cubic feet are there in a rectangular solid whose edges are $1\frac{1}{5}$ ft., $2\frac{3}{4}$ ft., and $3\frac{1}{3}$ ft.?

37. The dimensions of a box are $14\frac{2}{3}$ in., $12\frac{3}{4}$ in., and $3\frac{7}{11}$ in. How many cubic inches does it contain?

38. Some freight-cars have their measurements in feet and inches stamped on them. How many cubic feet in the contents of each car, as given?

	(1)	(2)	(3)	(4)	(5)
Length.........	33 — 9	50 — 0	44 — 0	33 — 10	34 — 0
Width.........	6 — 2	9 — 4	8 — 6	8 — 1	8 — 6
Height.........	7 — 0	8 — 10	8 — 7	7 — 1	8 — 0

Plan the work, cancel when possible, and find the results within 1 cu. ft., but not exceeding the actual capacity of the cars.

CHAPTER V

THE SOLUTION OF PROBLEMS

197. The following shows the *four steps* used in solving a problem:

Example: Find the altitude of a rectangle whose base is 8 in. and whose area is 152 sq. in.

(1) *Read* carefully, noting that the base and the area of a rectangle are given, and that the altitude is required.

(2) *Plan:* Recall that the area of a rectangle is the product of the numbers expressing its length and breadth. Hence 152 is 8 times the required number and must be divided by 8.

(3) *Computation:* $8\overline{)152}$ with quotient 19

(4) *Test:* $8 \times 19 = 152$.

THE FOUR ESSENTIALS

1. **Read the problem;** note carefully what is given and what is required.

2. **Plan the work;** determine how to find what is required from what is given.

3. **Make the computations** as planned.

4. **Test the result.**

Make the planning of a problem a distinct and important feature of its solution. In very simple problems, the plan and the work need not be separated, but any uncertainty as to *how* a problem is to be worked shows clearly that a separate, correct plan is necessary. Never work at random.

WRITTEN EXERCISES

Plan the solution; do not make the computation:

1. How many feet per minute is a train moving when traveling 42 mi. per hour?

2. A train left Cincinnati at 8:15 A.M. and arrived at St. Paul, 702 mi. distant, at 2:57 A.M. the next day. Find its average speed in miles per hour.

3. The base of a rectangle is 16 in., its area is 3 sq. ft. Find the altitude in inches.

4. A man earns $75 a month. He spends $10 per month for room rent, $3.50 per week for board, $125 per year for clothing and other expenses. In how many years will he save $1000 at this rate?

5. 40 ft. of wire weighed 1 lb. What was the weight of 31 mi. of this wire?

6. A, B, and C own a store; A owns $\frac{4}{7}$ of it, and B owns $\frac{1}{2}$ as much as A. What part does C own?

7. The base of a rectangle is 17 in.; its perimeter is 48 in. What is its altitude?

8. A 100-acre farm contains 4 lots. Three of the lots contain $55\frac{1}{2}$ acres, $20\frac{3}{4}$ acres, and $7\frac{5}{8}$ acres respectively. How many acres does the fourth lot contain?

Plan and solve:

9. $\frac{2}{3}$ of the distance from Detroit to Chicago is 190 mi. What is the whole distance?

Plan: (1) $\frac{1}{3}$ of the distance = $\frac{1}{2}$ of 190 mi., or (   ) mi.
(2) $\frac{3}{3}$, or the whole distance, = 3×95 mi. = (   ) mi.

10. A dressmaker paid $1.40 for $\frac{7}{8}$ of a yard of velvet. What was the price per yard?

Plan: (1) $\frac{1}{8}$ yd. cost $\frac{1}{7}$ of $1.40, or $(   ).
(2) $\frac{8}{8}$ yd., or 1 yd., cost $8 \times \$.20$ = (   ).

11. When 13 ounces of cinnamon cost $.65, what is the cost of cinnamon per pound?

12. In 1900, $\frac{5}{39}$ of the population of the United States was 10 millions. Find the population.

13. $24 is $\frac{3}{5}$ of the cost of a wagon. What is its cost? What is $\frac{8}{15}$ of its cost?

14. $\frac{7}{8}$ of a ton of coal costs $7.35. What is the cost of a ton? Of $\frac{3}{4}$ of a ton?

15. $\frac{3}{7}$ of the number of passengers on a street-car is 12. How many are there in the car?

16. $\frac{5}{17}$ of the population of Chicago in a certain year was 500,000. What was the population?

17. $\frac{3}{20}$ of the population of Pittsburg in a certain year was 48,000. What was the population? What was $\frac{5}{8}$ of it?

18. $\frac{2}{9}$ of the total newspaper product in the United States in a recent year was 60,000 tons. What was the total product?

19. At a concert $\frac{7}{11}$ of the audience sat on the main floor, and 360 in the gallery. How many people were there in all?

20. 300 men march 4 abreast in ranks 6 ft. apart. How long is the procession that they form?

UNITARY ANALYSIS

198. Problems are often solved by a method called **unitary analysis.**

EXAMPLES:

1. When 16 books cost $5.12, what is the cost of 9 at the same rate?

Plan: (1) Since 16 books cost $5.12, 1 book will cost $\frac{1}{16}$ of $5.12.
(2) 9 books will cost 9 times as much as 1 book, or $9 \times \frac{1}{16}$ of $5.12.

Computation: (1) $\$5.12 \div 16 = \$.32$.
(2) $9 \times \$.32 = \$2.88$.

In this solution we first found the cost of 1 book, from which we found the cost of 9. The plan of solving a problem by making a unit the basis of calculation is the essential feature of unitary analysis.

2. Find the cost of 150 ft. of wire netting at 38 cts. per 100 ft.

Plan: (1) Since the price is quoted per 100 ft., this is taken as the unit of computation.

(2) There are as many 100-ft. lengths in 150 ft. as 100 is contained times in 150.

(3) Since a 100-foot length costs \$.38, the whole will cost \$.38 multiplied by the number of 100-foot lengths.

Computation: (1) 150 ft. ÷ 100 ft. = $1\frac{1}{2}$.

(2) \$.38 × $1\frac{1}{2}$ = \$.57.

WRITTEN EXERCISES

In the following problems write out the plan, or analysis, and make the computation in steps, as in the examples above.

1. When 12 yd. of broadcloth cost \$27.60, what is the cost of 17 yd. at the same rate?

2. When 45 hats cost \$135, what is the cost of 2 dozen at the same rate?

3. $\frac{3}{4}$ of the production of book paper in a recent year was 100,500 tons. What was the total product?

Suggestion: *1 fourth* of the production is the *unit quantity* to be found first.

4. $\frac{19}{27}$ of the total product of writing-paper in a recent year was 100,000 tons. What was the total product?

5. Wire netting comes in rolls of 150 ft. each. When only unbroken rolls are sold, how many rolls must be bought to make a fence 575 ft. long? What is the cost at 50 cts. per roll?

6. A man wishes to fence a rectangular lot 50 ft. by 115 ft. with netting at 40 cts. a 100-ft. roll. If broken rolls are not sold, how much must he pay for the netting?

7. A dealer received a special price of 90 cts. a copy on books bought in lots of 500 each. During the season he purchased 4,500 copies in this way. How much did the books cost him?

8. A real estate company sold $\frac{5}{8}$ of a tract of farm land for $35,000. At this rate how much was the whole block worth?

9. $\frac{4}{7}$ of a man's yearly income was $1,800. What was the whole of his yearly income?

APPROXIMATING RESULTS

199. Before solving a problem *make a rough estimate of the result.* The habit of doing this will prevent the errors that would result from working at random.

ORAL EXERCISES

1. A rectangle is 12 in. long and 6.5 in. wide. Is its area greater or less than 72 sq. in.? Why? Is its area greater or less than 84 sq. in.? Why?

2. A rectangle is 5.5 ft. wide and 11.3 ft. long. Is its area greater or less than 55 sq. ft.? Why? Is its area greater or less than 72 sq. ft.? Why?

3. What is the cost of 15 lb. of sugar at 5½ cts. a pound? Is the cost more or less than 75 cts.? Why? Is it more or less than 90 cts.? Why?

4. A train runs from New York to Montreal, 383 miles, in 12 hours. Estimate the rate of speed per hour.

5. What is the cost of 11 yd. of cloth at 29 cts. a yard? How would you estimate the result?

6. What is the cost of a gross of paste-bottles at 9 cts. each? How would you estimate the result?

7. What is the area of a triangle 22 in. long and 19 in. wide? How would you estimate the result?

WRITTEN EXERCISES

Estimate the results and then solve:

1. A rectangular box is 8 ft. by 4 ft. by 5½ ft. Find its volume.

2. What is the area of a triangle of base 6.5 in. and altitude 14.5 in.?

3. What is the area of a rectangle of base 5½ yd. and altitude 3½ yd.?

4. When bottles of ink cost $15.12 per gross, what is the cost per bottle?

5. When bottles of library paste cost $5.04 per gross, what is the cost per bottle?

6. A crate of eggs contains 6 layers of 6 dozen each. How many eggs are there in the crate?

7. What is the cost of two 16-quart crates of blueberries at 8 cts. a quart?

8. How many hours does it take a train to travel 440 mi. at the rate of 35 mi. an hour?

200. A close approximation of the result is often all that is required in solving a business problem.

ORAL EXERCISES

1. What is the cost of 3 yd. of muslin at 12½ cts. a yard? How much does a customer who makes this purchase pay for the cloth?

2. What is the cost of 8⅔ yd. of ribbon at 10 cts. a yard? How much does a customer who makes this purchase pay for the ribbon?

3. How much does a customer pay for 8¼ lb. of rice at 9 cts. a pound?

4. If the computed cost of an article is $.37½, what is the amount usually paid for it?

5. One cent is what part of a dollar? To how many decimal places need the value of anything be expressed to show its value correct "to the nearest cent."

In determining results to the nearest cent, all decimal places beyond the third are useless and should not be calculated.

WRITTEN EXERCISES

1. On a certain railroad, the distance from New York to Chicago is 1,024 mi. and the fare is $17. How much is this per mile correct to the nearest tenth of a cent?

2. The cost of operating a railway has been classified thus:

Cost of maintenance of way, .2733 of the operating expenses.
Cost of maintenance of equipment, .1551 of the operating expenses.
Conducting transportation, .5284 of the operating expenses.
General expenses, .0432 of the operating expenses.

In one year the operating expenses of the Missouri Pacific railway were $10,500,000. How would the sum be distributed under the above heads? Find the results correct to the nearest $1000.

Suggestion: To find the cost of maintenance of way, we find:

$$\begin{aligned} &.2733 \times \$10{,}500{,}000 \\ &= 2733 \times \$1{,}050 \\ &= \$2{,}869{,}650 \\ &= \$2{,}870{,}000 \text{ to the nearest } \$1000. \end{aligned}$$

RATIO

201. PREPARATORY.

1. What part of 9 qt. are 2 qt.? 5 qt.? 3 qt.? 7 qt.?

2. What part of 10 bu. are 2 bu.? 4 bu.? 5 bu.? 8 bu.?

3. What fraction of 7 days are 2 days? 3 days? 5 days? 6 days?

4. What fraction of 1 square foot is 1 sq. in.? What fraction are 12 sq. in.? 36 sq. in.? 72 sq. in.?

5. What part of $\frac{12}{17}$ is $\frac{4}{17}$? What part of $\frac{12}{25}$ is $\frac{4}{25}$?

6. What fraction of $1 is $16\frac{2}{3}$ cts.? 25 cts.? $33\frac{1}{3}$ cts.? $37\frac{1}{2}$ cts.? $62\frac{1}{2}$ cts.?

7. What part of 12 is 4? What is the quotient of 4 divided by 12?

202. The quotient of two like numbers is often called their **ratio.**

The questions

What *part* of 12 is 4?
What *fraction* of 12 is 4?
What is the *quotient* of $4 \div 12$?
What is the *ratio* of 4 to 12?

are all answered by the fraction $\frac{1}{3}$.

ORAL EXERCISES

1. State the ratio of 16 oz. to 20 oz. Of 10 min. to 15 min.

State the ratio of the gain to the cost:

	2.	**3.**	**4.**	**5.**	**6.**
Cost.......	$8	$$\frac{3}{4}$	$.50	$1.75	$26.00
Gain.......	2	$\frac{1}{4}$	$.12\frac{1}{2}$	.25	5.20

203. Many problems involve finding the average of several numbers.

EXAMPLE: On three consecutive days a traveler spent $7.85, $6.15, and $6. How much did he spend in all? Had he spent an equal amount each day, how much must it have been to make the same total expenditure? This is called his *average* daily expenditure for the 3 days.

204. The **average** of several numbers is the quotient of their sum divided by their number.

Thus, the average of the three numbers, 6, 8, and 13, is their sum divided by 3, or

$$\frac{6 + 8 + 13}{3} = 9$$

WRITTEN EXERCISES

1. Write the ages to the nearest year of 4 pupils in your grade. Find the average age of these pupils.

2. The following table shows the attendance of a class. Find the average number of days that Mary attended per week. Find the average weekly attendance for each other pupil.

NAME.	1ST WEEK.	2D.	3D.	4TH.	5TH.	6TH.	7TH.
Mary	5 days.	4 da.	5 da.	5 da.	5 da.	3 da.	4 da.
Helen ...	5 "	5 "	4 "	3 "	5 "	5 "	4 "
Leslie	3 "	5 "	4 "	4 "	5 "	5 "	5 "
Charles .	5 "	5 "	5 "	5 "	5 "	5 "	5 "
James ..	5 "	4 "	5 "	3 "	5 "	4 "	5 "

3. Find the average daily attendance of the class for the first week. For each of the following weeks.

4. The following table shows a number of birds and the number of insects destroyed by them in a period of 5 days:

	BLACK-BILLED CUCKOO.	YELLOW-BILLED CUCKOO.
Number of birds..........	46	109
Destroyed		*Destroyed*
Caterpillars..............	906	1,865
Beetles..................	44	92
Grasshoppers.............	96	242
Spiders..................	15	86
Others...................	130	112

According to the above table, what was the average number of insects destroyed per each of the black-billed cuckoos? Per each of the yellow-billed cuckoos?

5. There were 155 birds. What was the average number of insects destroyed per bird?

6. If it took 5 days to devour these insects, what was the average number destroyed per day?

7. A Swanson hawk destroys on the average 100 grasshoppers per day. How many grasshoppers do 300 of these birds destroy in a month of 30 days?

THE USE OF LETTERS

205. The use of letters is of value in solving certain kinds of problems.

PREPARATORY.

1. If there are 8 books on one shelf and 12 books on another, how many books are there on both shelves? If there are a books on one shelf and b books on another, how many books are there on both shelves?

This sum is written $a + b$.

2. Goods costing $6 were sold by a merchant for $8. How many dollars did he gain? If goods costing b dollars were sold at a profit for a dollars, how many dollars were gained?

The expression for the gain is $a - b$.

3. If a barrel of apples costs $3, what will 3 bbl. cost? 4 bbl.? 12 bbl.? n bbl.?

The last result is written $3\ n$.

4. If there are 4 rows of apple-trees in an orchard and 8 trees in each row, how many trees are there in the orchard? How many trees are there in an orchard with b rows of 8 trees each? How many trees are there in an orchard with b rows of a trees each?

The product of a and b, or $a \times b$, is written ab.

5. How many tons of coal at $5 a ton can be bought for $25? How many tons at b dollars a ton can be bought for a dollars?

The operation a divided by b is written $a \div b$ or $\frac{a}{b}$.

206. Examples:

1. The metal types used in printing are made of 4 parts by weight of lead to 1 part of antimony. How much lead is there in 85 lb. of type? How much antimony?

Solution: (1) Let a stand for the number of pounds of antimony.
(2) The number of pounds of lead is $4a$.
(3) $a + 4a = 5a$. Why?
(4) Therefore, $5a = 85$, and $a = \frac{85}{5} = 17$.
(5) There are 17 lb. of antimony and 68 lb. of lead in 85 lb. of type.

2. A rectangular park is twice as long as it is wide and has a perimeter of 18 mi. Find its length and breadth in miles.

Solution: (1) Let x stand for the number of miles in the width of the park.
(2) The number of miles in the length is $2x$.
(3) The perimeter is $2x + 4x = 6x = 18$ mi.
(4) Therefore, the width = 3 mi., and the length 6 mi.

ORAL EXERCISES

1. What is the value of $a + b$ when $a = 6$ and $b = 14$? When $a = 8$ and $b = 20$? When $a = 16$ and $b = 14$?

2. What is the value of $a - b$ when $a = 25$ and $b = 10$? When $a = 15$ and $b = 7$? When $a = 30$ and $b = 14$?

3. What is the value of ab when $a = 3$ and $b = 4$? When $a = 50$ and $b = 14$?

4. What is the value of $\frac{a}{b}$ when $a = 30$ and $b = 6$? When $a = 42$ and $b = 7$? When $a = 56$ and $b = 7$?

WRITTEN EXERCISES

1. The perimeter of a rectangular farm is $1\frac{1}{2}$ mi. The length is $\frac{1}{4}$ of a mile more than the breadth. Find its dimensions.

2. A man left $\frac{3}{4}$ of his estate to his children and divided the remainder equally among 4 charitable institutions, each receiving $6,000. What was the value of his estate?

3. A newsboy delivers 75 papers per day. He delivers twice as many in the morning as in the evening. How many does he deliver in the morning?

4. A man owned $\frac{9}{14}$ of a store; he sold $\frac{2}{3}$ of his share for $3,000. Find the value of the store.

5. A contractor, wishing to dig a cistern, finds it most convenient to make it 5 ft. square. How deep must it be dug to contain 750 gal.? (Use 1 cu. ft. = $7\frac{1}{2}$ gal.)

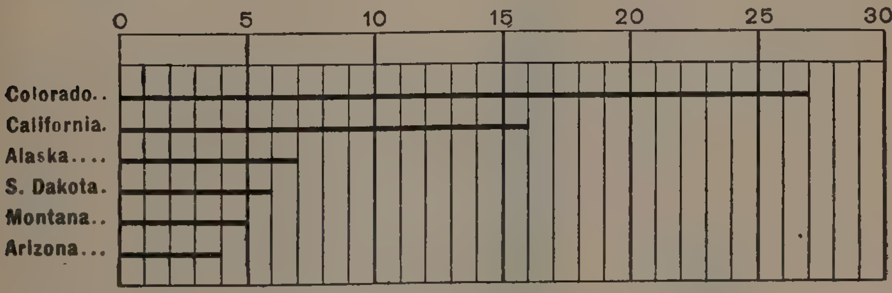
NUMBER PICTURES, OR GRAPHS

207. Numbers may be represented by pictures, called **graphs,** and the results of problems can be read from these pictures directly without computation.

The following list gives the value of the gold produced by the states mentioned in a recent year:

Colorado	$27,000,000	Alaska	$7,000,000	Montana	$5,000,000
California	16,000,000	S. Dakota	6,000,000	Arizona	4,000,000

These values are more easily compared if they are represented by parallel lines. Thus each *division* on the scale represents *1 million.*



WRITTEN EXERCISES

1. Draw on the blackboard the same set of lines for the exercise above, using 1 inch for each division of the scale.

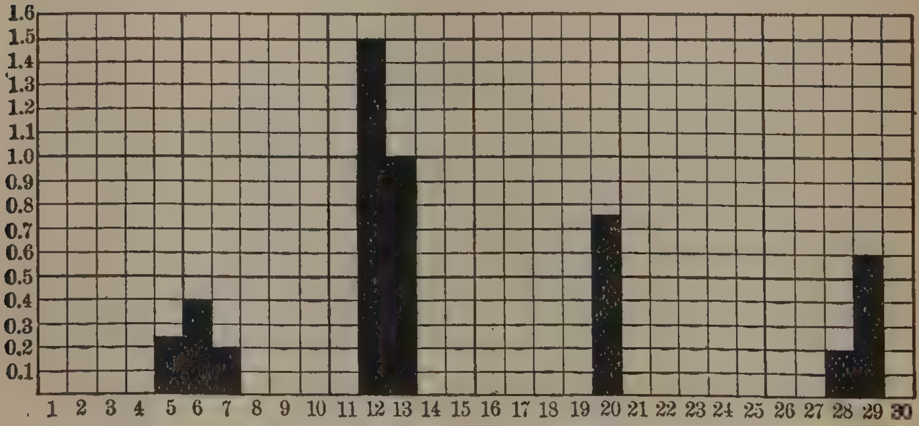
2. The following tables show the approximate production of certain articles of trade. Draw lines to represent the numbers in each table:

(1) SILVER.	*Million dollars.*	(2) COPPER.	*100,000 tons.*	(3) COTTON.	*Million bales.*
Nevada......	1.5	Utah.........	0.2	Arkansas	$\frac{7}{8}$
California....	4	California	0.3	Alabama.....	1
Utah........	12	Arizona......	0.5	Mississippi....	$1\frac{1}{5}$
Montana	18	Michigan.....	0.7	Georgia......	$1\frac{1}{4}$
Colorado	26	Montana.....	2.5	Texas........	3

3. This table gives a daily record of rainfall in September. What was the rainfall for the month?

DAY.	5TH.	6TH.	7TH.	12TH.	13TH.	20TH.	28TH.
Inches....	.25	.4	.2	1.5	1	.75	.2

4. The adjacent lines in the figure are $\frac{1}{10}$ in. apart. How high is each of the dark columns? How do the heights of the columns compare with the records of rainfall in the table of Exercise 3?



Find, from the diagram above, the rainfall on the 29th of September.

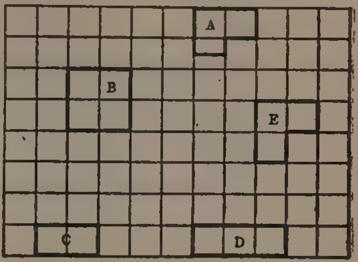
5. Robert measured the rainfall for April. The records were:

DAY.	5TH.	6TH.	10TH.	11TH.	12TH.	24TH.	30TH.
Inches....	.8	.3	.2	1	.5	.6	.2

Represent the rainfall graphically as in Oral Exercise 4.

6. What is the difference between the greatest and the least rainfall for the month?

7. Insurance agents use squared paper to show distances between buildings. If the side of each small square represents 10 ft., how far apart, according to the figure, are the buildings, E and B?



8. How far apart are C and D? C and B? E and D?

9. What is the greatest length and greatest breadth of building A? Of B? Of C? Of D? Of E?

10. The table shows the average annual rainfall of various places:

Chicago........	34.8	Sacramento....	20.9	Pensacola......	57.1
Boston.........	45.0	Denver........	14.5	Mobile.........	62.2
St. Louis.......	41.1	New York City.	44.8	Yuma..........	3.0

Draw on the blackboard horizontal lines to represent the numbers in this table.

11. Represent graphically these readings of a thermometer (above zero):

SUN.	MON.	TUES.	WED.	THURS.	FRI.	SAT.
27½°	28°	30°	29°	31½°	29½°	31°

REVIEW

Plan the solution, but omit the computation:

1. The cyclometer of a bicycle read as follows at the close of each of these days: Monday, 841.62 mi., Tuesday, 873.84 mi., Wednesday, 910.05 mi., Thursday, 919.42 mi. Find the average daily ride.

2. On Monday morning a cistern contained 627.5 gal. of water; Tuesday it rained, and 127.46 gal. ran into the cistern; Thursday it rained, and 472.01 gal. ran into the cistern; the following Monday morning there were 710.91 gal. of water in the cistern. Find the average daily consumption of water.

3. In a recent year 2,895,661 dozen pairs of leather gloves and mittens were made in the United States. How many pairs was this per inhabitant, the population being 76,303,387?

4. The distance from Cincinnati to St. Louis is 341 mi. How long will it take a train to make the run, if it goes at the rate of 28.3 mi. per hour the first three hours, 26 mi. the next 2.7 hr., and 30.7 mi. the remainder of the time?

Solve:

5. By density of population is meant the average number of inhabitants per square mile. For Massachusetts the density is 348.9. The land area is 8,040 sq. mi. Find the population of Massachusetts.

6. The population of Mississippi is 1,551,270; its density of population is 33.5. Find its land area.

7. The population of Wisconsin is 2,069,042; its land area is 54,450 sq. mi. Find its density of population.

8. Two steamers left New York for Liverpool at 10 A.M., Saturday. The average speed of the first was 386.4 mi. per day; of the second, 495.35 mi. How far apart were they at 10 P.M. the following Wednesday?

9. If 8 hr. of labor are understood to constitute a day's work, and a workman receives $1.50 a day on this basis, how much does he receive for 36 hours' work? How much does he receive per hour? For 3 hours' work?

10. On the basis of Exercise 9, how many dollars does a man earn in a week who works 8½ hr. on Monday, 6 hr. on Tuesday, 8 hr. on Wednesday, 7 hr. on Thursday, 9 hr. on Friday, and 4 hr. on Saturday?

11. The following table shows the number of hours worked by A, B, C, D in one week at a factory; also their wages per day on the basis of an 8-hour day. On this basis how many days did each man work during the week?

	MON.	TUES.	WED.	THUR.	FRI.	SAT.	WAGES.	TOTAL.
A	8	7¼	7½	6	9	8	$1.25	
B	7½	8½	8	8¾	6	8	1.50	
C	8	6½	0	8	0	6	2.00	
D	6	5½	6¾	9	8½	8	1.75	

12. What is the average number of hours per day for each man in Exercise 11?

13. According to Exercise 11, what are the total wages of each for the week?

14. What is the average amount earned daily by each man for the 6 days?

15. What is the total pay-roll for the four men?

16. How many hours did C's week lack of being six 8-hour days?

17. A man earned $840 a year and spent $780. What part of his earnings did he spend? What is the ratio of his expenditures to his income?

18. A copper ore mined in Alaska contains 48 lb. of copper, 27 oz. of silver, and .035 oz. of gold per ton. How many pounds of each metal in 1000 tons of ore?

CHAPTER VI

DECIMAL FRACTIONS—PERCENTAGE

DECIMAL FRACTIONS

Notation and Numeration

208. The following table shows the names of the places, or orders, of decimals from tenths to millionths.

Tens	**Units**	**Decimal Point**	**Tenths**	**Hundredths**	**Thousandths**	**Ten-thousandths**	**Hundred-thousandths**	**Millionths**
3	7	.	4	0	6	3	6	9

The number in the table is read *thirty-seven and four hundred six thousand, three hundred sixty-nine millionths.*

209. A unit of any order is 10 times a unit of the next order at the right, and $\frac{1}{10}$ of a unit of the next order at the left.

ORAL EXERCISES

1. Counting from the decimal point, what decimal place is occupied by: Hundredths? Ten-thousandths? Millionths? Hundred-thousandths?

2. How many decimal places are needed to express: Ten-thousandths? Thousandths? Millionths? Hundred-Thousandths?

Read:

3. .303.
4. .065.
5. 1.863.
6. .0078.
7. .00005.
8. .10001.
9. .00015.
10. .01708.
11. .88888.
12. .70536.
13. .136732.
14. .250875.

WRITTEN EXERCISES

Express in figures:

1. Thirty and 5 thousandths.

2. Eighty-seven and 25 thousandths.

3. 100 and 263 ten-thousandths.

4. 408 and 1879 ten-thousandths.

5. 96 and 497 millionths.

6. Four hundred seventy-five hundred-thousandths.

7. Thirty-four thousand, three hundred forty-six hundred-thousandths.

8. Nine hundred twenty-four thousand, five hundred sixty-three millionths.

Reduction of a Decimal Fraction to a Common Fraction

210. Example: Reduce .0825 to the form of a common fraction.

$$.0825 = \frac{825}{10000} = \frac{33}{400}.$$

211. *Write the decimal fraction as a common fraction, and reduce it to its lowest terms.*

WRITTEN EXERCISES

Reduce to common fractions in their lowest terms:

1. .625.
2. .3125.
3. .0565.
4. .0075.
5. .07525.
6. .1275.
7. .0625.
8. .15625.
9. .04565.
10. .00375.
11. .875.
12. .0025
13. .00475.
14. .12575.
15. .400375.
16. .01275.

Reduction of a Common Fraction to a Decimal Fraction

212. *To reduce a common fraction to a decimal, divide the numerator by the denominator, annexing as many zeros to the numerator as are necessary to obtain in the quotient the desired number of decimal places.*

The plus sign is used to indicate that the quotient is not complete and that therefore the decimal is approximate.

Example: Reduce $\frac{3}{11}$ to a decimal of 5 places.

$$\begin{array}{r} 11\overline{)3.00000} \\ .27272+ \end{array}$$

If the denominator is greater than 12 it is better to use long division. If the divisor has small factors, as 105 = 3×5×7, it is better to divide successively by the factors as 3, 5, 7.

WRITTEN EXERCISES

Reduce to decimals containing not more than 5 places:

1. $\frac{3}{16}$.
2. $\frac{7}{15}$.
3. $\frac{7}{10}$.
4. $\frac{8}{33}$.
5. $\frac{8}{9}$.
6. $\frac{11}{12}$.
7. $\frac{4}{11}$.
8. $\frac{8}{15}$.
9. $\frac{2}{3}$.
10. $\frac{5}{12}$.
11. $\frac{9}{13}$.
12. $\frac{7}{9}$.
13. $\frac{6}{7}$.
14. $\frac{9}{20}$.
15. $\frac{5}{24}$.
16. $\frac{15}{16}$.

Addition and Subtraction

WRITTEN EXERCISES

Add as rapidly as possible:

From 1 to 1½ minutes per problem is sufficient.

1.
46.072
19.678
7.125
8.005
10.86
6.905
.697

2.
12.4032
7.96
6.8704
.0009
36.6333
9.3008
3.1225

3.
9.6789
4.33
.0085
6.2376
3.1405
9.999
12.3777

4.
15.3009
4.5
7.311
.453
1.008
.721
22.1145

5. 88.1263
44.9977
30.4436
6.7384
3.0097
.0009
18.5555
9.2121

6. .12245
.67809
.00043
.13067
.40856
.20458
.00777
.33333

7. .845631
.234652
.198273
.203674
.098765
.123456
.210987
.446655

8. .05213
2.1072
.452
3.11127
.5207
.0095
.525
.0255

Subtract as rapidly as possible:

9. 4.6633
1.8459

10. .86354
.55634

11. 306.
31.72

12. 7.0638
6.0987

13. .56337
.48732

14. 517.3
25.666

15. 12.0066
5.4399

16. .178962
.054673

17. 9.
.4561

Solve:

18. A business house found that its rent was .0125 of its gross business, that its salaries were .3375 of its gross business. What part of its gross business did rent and salaries equal?

19. A filing cabinet consists of three parts whose heights are respectively 1.25 ft., 1.75 ft., 1.5 ft. The cabinet stands on a base 1.75 ft. high. Find the total height of the piece.

20. The cabinet mentioned in Exercise 19 stands in a room 9 ft. 9 in. high. Express decimally the height of the ceiling above the top of the cabinet.

21. A farmer's state tax was $.005 per dollar, his school tax $.016, and his county tax $.0065. What was his total tax per dollar?

22. The tax rate in a certain city is $.028 per dollar, and in a village the rate is $.0185. By how much does the city tax exceed the village tax?

Multiplication

213. Preparatory.

1. Observe the relation between these two multiplications:

$$\begin{array}{rcr} 4.5 & \qquad & 4.5 \\ \underline{23} & & \underline{2.3} \\ 135 & & 135 \\ \underline{90} & & \underline{90} \\ 103.5 & & 10.35 \end{array}$$

The result of multiplying 4.5 by 23 is how many times as great as that of multiplying it by 2.3? Then what part of 103.5 must be taken to get the product of 4.5 and 2.3?

$\frac{1}{10}$ of 103.5 = ( )?

214. The numbers multiplied to form a product are called the **factors** of the product.

215. Preparatory.

1. Name the factors in each of the above multiplications.

2. How does the number of decimal places in the product compare with the total number in the factors in each case?

3. Observe the relation between these two multiplications:

$$\begin{array}{rcr} 2.136 & \qquad & 2.136 \\ \underline{38} & & \underline{.038} \\ 17088 & & 17088 \\ \underline{6408} & & \underline{6408} \\ 81.168 & & .081168 \end{array}$$

The result of multiplying 2.136 by 38 is how many times as great as that of multiplying it by .038?

$\frac{1}{1000}$ of 81.168 = ( )?

216. *To multiply decimals, proceed as with integers, and set off from the right as many decimal places in the product as there are decimal places in the factors. If there are not figures enough in the product to fill all of the decimal places, supply zeros.*

WRITTEN EXERCISES

Multiply:

1. $\begin{array}{r} 4.96 \\ \underline{2.5} \end{array}$

2. $\begin{array}{r} 53.9 \\ \underline{1.3} \end{array}$

3. $\begin{array}{r} 100.1 \\ \underline{.25} \end{array}$

4. $\begin{array}{r} 5.87 \\ \underline{86.5} \end{array}$

5. $\begin{array}{r} 65.85 \\ \underline{1.44} \end{array}$

6. $\begin{array}{r} 46.75 \\ \underline{1.25} \end{array}$

7. $\begin{array}{r} 6.25 \\ \underline{6.25} \end{array}$

8. $\begin{array}{r} 1111 \\ \underline{3.33} \end{array}$

9. $\begin{array}{r} 13.25 \\ \underline{4.2} \end{array}$

10. $\begin{array}{r} 1965 \\ \underline{4.6} \end{array}$

11. $\begin{array}{r} 40.05 \\ \underline{1.8} \end{array}$

12. $\begin{array}{r} 1.24 \\ \underline{6.25} \end{array}$

13. $\begin{array}{r} 17.28 \\ \underline{1.2} \end{array}$

14. $\begin{array}{r} 9015 \\ \underline{18.5} \end{array}$

15. $\begin{array}{r} 50.05 \\ \underline{202} \end{array}$

16. $\begin{array}{r} 112.2 \\ \underline{6.64} \end{array}$

17. $\begin{array}{r} 96.15 \\ \underline{8} \end{array}$

18. $\begin{array}{r} 200.7 \\ \underline{.14} \end{array}$

19. $\begin{array}{r} 16.3 \\ \underline{29.8} \end{array}$

20. $\begin{array}{r} 9.02 \\ \underline{4.05} \end{array}$

21. $\begin{array}{r} 77.7 \\ \underline{9.9} \end{array}$

22. $\begin{array}{r} 69.9 \\ \underline{38} \end{array}$

23. $\begin{array}{r} 425 \\ \underline{.88} \end{array}$

24. $\begin{array}{r} 7.07 \\ \underline{5.05} \end{array}$

25. Find the area of a rectangle of sides 45.5 ft. and 5.5 ft.

26. Find the area of a square 3.25 yd. on a side.

27. What is the cost of 26.5 ft. of moulding at 8.5 cts. a foot?

28. What is the cost of 46.5 yd. of cloth at 6.5 cts. a yard?

29. What is the cost of 136.8 sq. yd. of cement walk at $3.25 per square yard?

30. How many miles does an automobile travel in 7.8 hr., if it goes at the rate of 23.75 mi. per hour?

31. What is the cost of 5½ yd. of cloth at $.15 a yard?

32. For what does 26¾ lb. of butter sell, at $.25 a pound?

33. The base of a tower had 8 sides each 17.75 ft. long. How many feet was it around the base of the tower?

34. A train travels at the rate of 38.95 mi. per hour. How far will it travel in 18⅞ hr.?

Division

217. PREPARATORY.

1. Multiply both divisor and dividend by 10 in this expression: 3.6 ÷ .6. Then 3.6 ÷ .6 is the same as 36 ÷ 6.

2. How can you show that 1.25 ÷ .25 is the same as 125 ÷ 25?

3. How can you show that .625 ÷ .5 is the same as 6.25 ÷ 5?

218. In dividing by a decimal it is easier first to multiply both dividend and divisor by a power of 10 such as to make the divisor an integer. This is done by moving the decimal point in each the same number of places.

EXAMPLE: The area of a rectangle is 4.472 sq. ft., and its altitude is .26 ft. How may the length of the base be found?

Divide 4.472 by .26.

Solution:

$$\begin{array}{r} 17.2 \\ 26\overline{)447.2} \\ \underline{26} \\ 187 \\ \underline{182} \\ 5.2 \\ \underline{5.2} \end{array}$$

In the division at the right, 26 is taken instead of .26, and 447.2 instead of 4.472. Why does 447.2 ÷ 26 = 4.472 ÷ .26?

How is the position of the decimal point in the quotient related to that of the decimal point in the dividend?

219. *To divide decimals, make the divisor a whole number by multiplying both divisor and dividend by the proper power of 10. Place the first figure of the quotient over the last figure of the part of the dividend first used; place the decimal point of the quotient over the decimal point of the dividend. If the number of decimal places in the divisor exceeds that in the dividend, zeros may be added to the dividend.*

220. The work of division is tested by comparing the dividend with the product of the quotient and the divisor plus the remainder.

WRITTEN EXERCISES

Divide and test:

1. 3.75 ÷ 5.
2. .625 ÷ .25.
3. 42.63 ÷ .213.
4. 69.76 ÷ .032.
5. 121.67 ÷ 2.3.
6. 3.06 ÷ 6.
7. 4.494 ÷ .70.
8. 423.4 ÷ 7.3.
9. 885.8 ÷ 1.11.
10. 80.48 ÷ 1.64.
11. 3.06 ÷ .06.
12. 81.54 ÷ 27.
13. 6.636 ÷ .79.
14. 97.336 ÷ .47.
15. 160.05 ÷ .05.

Divide to not more than 3 decimal places:

16. 5.5 × 11.25 by 6.75.
17. 20.9 × 35.75 by 14.8 × 7.35.
18. 15.5 × 18.3 by 40.5.
19. 36.75 × 13.6 by 17.8 × 40.35.
20. 19.05 × 7.63 by 3.125 × .17.
21. 40.15 × 3.09 by 6.005 × 3.6.
22. 18¾ × 6.25 by 11.86 × 5⅝.

Solve:

23. At 2.4 cts. per mile, the fare from New York to Chicago would be $21.60. Find the distance.

24. At an average speed of 25.6 mi. per hour, how long would it take a train to go from Chicago to Denver (1,028 mi.)?

25. A rod contains 16.5 ft. How many rods are there in a mile (5,280 ft.)? In 2,541 ft.?

26. The pendulum of a clock made 120 swings in a minute. How many seconds did it take the pendulum to make 1 swing?

27. A cubic inch of mercury weighs about .5 lb. How many inches high must a tube be whose base is 1 sq. in. in order to hold 100 lb. of mercury?

28. There are 30.25 sq. yd. in a square rod. How many square rods are there in 226.875 sq. yd.?

29. A train traveled 1080.3 mi. in 19.5 hr. How many miles was this per hour?

PERCENTAGE

Meaning of Rate Per Cent

221. Preparatory.

1. Of 5 hundred strawberry plants, 3 per hundred died. How many died?

2. Of 600 pupils in a school 2 per hundred were absent. How many were absent?

3. A contractor bought 12,000 bricks; 14 per hundred were defective. How many defective bricks were there in the lot?

In expressions like the above, **per cent** is commonly used instead of *per hundred.*

Per cent is derived from the Latin *per centum* meaning *per hundred.*

Thus, 6 per cent (6%) means 6 per hundred. The symbol % is read *per cent.*

222. The phrase **per cent** is used when a *rate per hundred* is meant. The term **percentage** is used to express the actual number resulting from the application of the rate.

Thus, in the expression "10 per hundred of the total population of 10,000 were Americans", *10 per hundred* is the *rate* and may be read *10 per cent.* The *actual number* of Americans, or *1000* out of *10,000*, is the *percentage* of Americans.

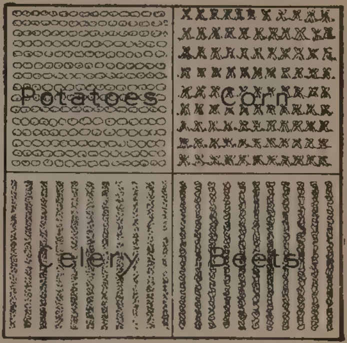
ORAL EXERCISES

1–6. The rate per cent of salt in the bodies of water named is approximately:

Caspian Sea	1	Atlantic Ocean	4
Black Sea	1	Mediterranean Sea	4
Great Salt Lake	20	Dead Sea	$6\frac{1}{2}$

In each case, how many pounds of salt are there in every 100 pounds of water?

7. A vegetable garden contained four kinds of plants as shown in the picture. What part of the whole garden was used for potatoes?



8. What part was occupied by potatoes and corn together?

9. How many hundredths of the garden were used for potatoes? For corn? For both of these?

10. One-fourth of the garden is how many hundredths of it? How many per cent of it?

$\frac{1}{4}$ of the garden = .25 of it = 25% of it.

11. One-half of the garden is how many hundredths of it? How many per cent of it?

$\frac{1}{2}$ of the garden = .50 (or .5) of it = 50% of it.

12. $\frac{3}{4}$ of the garden = ( ) hundredths of it = ( )% of it?

13. The whole garden = ( ) hundredths of it = ( )% of it?

14. $\frac{9}{10}$ of a farm = ( ) hundredths of it = ( )% of it?

15. $\frac{1}{5}$ of a farm = ( ) hundredths of it = ( )% of it?

16. $\frac{2}{5}$ of a dollar = ( ) hundredths of it = ( )% of it?

17. $\frac{4}{5}$ of a dollar = ( ) hundredths of it = ( )% of it?

18. $\frac{3}{5}$ of a dollar = ( ) hundredths of it = ( )% of it?

WRITTEN EXERCISES

1. Draw a square and divide it into 10 equal rectangles. Divide each rectangle into 10 equal squares. How many small squares are there in the figure?

2. Shade 1% of the squares in the figure mentioned in Exercise 1. Shade 5% of them; 10%; 25%; 50%; 75%; 80%.

3. Draw a circle and shade 25% of it; 50% of it; 75%.

4. Draw a rectangle whose breadth is 20% of its length. One whose breadth is 75% of its length.

5. On paper ruled in squares draw a square 10 in. on a side. Shade 1% of its area. Shade a square whose area is 25% of that of the given square.

6. Draw on the board a square 10 in. on a side. What is its area? How many square inches are there in 25% of it? In 75% of it? In 40%? In 53%?

7. Draw a rectangle 10 in. long whose area shall equal 10% of that of the square of Exercise 6. Draw other rectangles each 10 in. long to equal 20%; 30%; 50%; 70% of the given square.

8. Draw a rectangle 1 in. wide whose area is 5% of that of the square of Exercise 6. Draw others of the same width with areas 7%; 3%; 15%; 29% of the area of the square. How long would such a rectangle be in order to equal 100% of the square?

Finding a Percentage

223. PREPARATORY.

1. .45 of a number is how many per cent of it?

2. 25% of a number is how many hundredths of it? How many fourths of it?

3. Taking $\frac{1}{4}$ of a number is taking how many per cent of it?

4. Taking $\frac{3}{4}$ of a number is taking how many per cent of it?

5. Taking $\frac{1}{3}$ of a number is taking how many per cent of it?

224. EXAMPLES:

1. Find 47% of 216.

Solution: (1) 47% of 216 = .47 × 216.
(2) .47 × 216 = 101.52.

2. Find $33\frac{1}{3}$% of 216.

Solution: (1) $33\frac{1}{3}$% of 216 = $\frac{1}{3}$ of 216.
(2) $\frac{1}{3}$ of 216 = 72.

225. *To find the percentage of a number at any rate per cent, multiply it by the corresponding number of hundredths or the corresponding common fraction.*

TABLE OF CORRESPONDING NUMBERS

PER CENT.	DECIMAL.	FRACTION.	PER CENT.	DECIMAL.	FRACTION.
5%	.05	$\frac{1}{20}$	50%	.50 or .5	$\frac{1}{2}$
10%	.10 or .1	$\frac{1}{10}$	62.5%	.625	$\frac{5}{8}$
12.5%	.125	$\frac{1}{8}$	$66\frac{2}{3}\%$	$.66\frac{2}{3}$	$\frac{2}{3}$
$16\frac{2}{3}\%$	$.16\frac{2}{3}$	$\frac{1}{6}$	75%	.75	$\frac{3}{4}$
20%	.20 or .2	$\frac{1}{5}$	$83\frac{1}{3}\%$	$.83\frac{1}{3}$	$\frac{5}{6}$
25%	.25	$\frac{1}{4}$	87.5%	.875	$\frac{7}{8}$
$33\frac{1}{3}\%$	$.33\frac{1}{3}$	$\frac{1}{3}$	90%	.90 or .9	$\frac{9}{10}$

226. The number to which the rate is applied is called the **base.**

227. The relation of the three numbers—Base, Rate, and Percentage—may be expressed as follows:

Base multiplied by Rate equals Percentage.

WRITTEN EXERCISES

Find:

1. 20% of 200?

2. 25% of 500?

3. $33\frac{1}{3}\%$ of 2,760?

4. $66\frac{2}{3}\%$ of 3,615?

5. 43% of 186?

6. 10% of 1,000?

7. 50% of 2,000?

8. 87.5% of 900?

9. 37.5% of 1,200?

10. 48% of 160?

11. 75% of 1,600?

12. 60% of 1,200?

13. 74% of 286?

14. 16% of 1,400?

15. 9% of 963?

Solve:

16. In a certain territory 576,660 bu. of corn were grown in 1 yr., of which $66\frac{2}{3}\%$ was exported. How many bushels was this?

17. In a recent year the imports of Mexico from the United States were 44% of the exports to the United States. How many dollars' worth of imports did Mexico receive per $100-worth of exports to the United States? For every $500? For every $50? For every $25?

18. If 75% of a certain ore is copper, how many pounds of copper are there in 100 lb. of ore? In 4,480 lb. of ore?

19. If on an average 75% of the eggs placed in an incubator hatch, and 90% of the chickens transferred to the brooder live to become marketable fowls, how many fowls are grown from 80 doz. eggs?

20. Diminish 100 by 10% of itself; the result by $33\frac{1}{3}\%$ of itself; this result by $33\frac{1}{3}\%$ of itself. What is the final result? How many per cent of 100 is it?

21. A merchant began with $5,000 capital, and at the end of each year added his profit to his capital. His profit each year is shown by the following percentages of the capital invested: first year, 20%; second year, 25%; third year, 10%. What was his capital for the fourth year?

22. A man bought 320 acres of land and sold 90% of it to one neighbor, $33\frac{1}{3}\%$ of the remainder to another, and 12.5% of the remainder to another. How many acres had he left?

23. A boat carried 3,125 tons of iron ore. This ore yielded 43.8% of iron. How many tons of iron in the cargo?

Finding How Many Per Cent One Number is of Another

228. PREPARATORY.

1. 5 is what part of 20? $\frac{5}{20}$, or $\frac{1}{4}$, is how many hundredths? 5 is how many per cent of 20?

2. 4 is what part of 20? $\frac{4}{20}$ or $\frac{1}{5}$ is how many hundredths? 4 is how many per cent of 20?

3. Divide 735 by 3,500 and express the result in hundredths. 735 is how many hundredths of 3,500? How many per cent?

229. EXAMPLES:

1. 34 is how many per cent of 170?

Solution: (1) $34 \div 170 = .2 = .20$.
(2) Therefore, 34 is 20% of 170.

2. $266\frac{2}{3}$ is how many per cent of 400?

Solution: (1) $266\frac{2}{3} \div 400 = \frac{800}{3} \div \frac{400}{1} = \frac{2}{3} = .66\frac{2}{3}$.
(2) Therefore, $266\frac{2}{3}$ is $66\frac{2}{3}$% of 400.

230. *To find how many per cent one number is of another, divide the first number by the second. The number of hundredths in the quotient is the rate per cent.*

231. If we employ the terms Base, Rate, and Percentage, the above process may be stated:

Percentage divided by Base equals Rate.

The phrase "What per cent" is often used as an abbreviation for "How many per cent" or for "What rate per cent."

WRITTEN EXERCISES

1. 48 is how many per cent of:
96? 144? 200? 500? 480? 1000?

2. 28 is how many per cent of:
84? 140? 168? 200? 500? 700? 1,400?

3. 35 is how many per cent of:
70? 105? 210? 700? 770? 250? 1000?

4. 125 is how many per cent of:
125? 250? 1000? 500? 650? 900? 1,250?

5. 80 is how many per cent of:
240? 100? 80? 40? 20? 10? 8? 5?

6. A bank has a capital of $100,000 and keeps $25,000 on hand. How many per cent is this reserve of the whole capital?

7. A trust company had a capital of \$3,000,000, and kept in reserve \$450,000. How many per cent was this reserve of the whole capital?

8. A man purchased a block of land for \$250,000 and paid at once \$50,000. How many per cent of the whole amount was this cash payment?

9. A house and lot cost \$6,500; the lot alone cost \$1,500. How many per cent was this of the whole cost?

10. To test flower-seeds, 200 seeds were planted; 40 plants sprouted. How many per cent of the seeds sprouted?

11. Fifty pounds of corn contain 35 lb. of starch. How many per cent of corn is starch?

Finding a Number from a Given Percentage

232. PREPARATORY.

1. If 7 is $\frac{1}{2}$ of a number, what is the number?

2. If 8 is $33\frac{1}{3}\%$ of a number, that is, $\frac{1}{3}$ of it, what is the number?

3. If 12 is 6% of some number, what is 1% of it? What is 100% of it?

233. EXAMPLES:

1. 20 is 4% of what number?

Solution: (1) $20 \div 4 = 5$, or 1% of the number.
(2) $100 \times 5 = 500$, or the whole of the required number.
Steps 1 and 2 are the same as $20 \div .04 = 500$.

2. 16 is 6% of what number?

Solution: $16 \div .06 = 266\frac{2}{3}$, the required number.

3. If 45 is $62\frac{1}{2}\%$ of a number, or $\frac{5}{8}$ of it, what is the number?

Solution: (1) $\frac{1}{8}$ of the number is $45 \div 5$, or 9.
(2) $\frac{8}{8}$ of the number is 8×9, or 72.
(3) Steps (1) and (2) are the same as $45 \div \frac{5}{8} = 72$.

234. *To find a number of which the percentage at a certain rate is given, divide the percentage by the number of hundredths corresponding to the rate per cent. Or, divide the percentage by the common fraction corresponding to the rate.*

235. The above process may be stated in terms of Base, Rate, and Percentage as follows:

Percentage divided by Rate equals Base.

WRITTEN EXERCISES

1. 24 is 8% of what number?

2. 25 is 5% of what number?

3. 10% of ( ) = 15?

4. 25% of $N = 7$. Find N.

5. 20% of $B = 12$. Find B.

6. 8% of $N = 24$. Find N.

7. A bag of grass seed contained 86 lb. of good seed, which was $61\frac{3}{7}$% of its weight. What was the weight of all the grass seed?

8. A standard bushel of clover seed contains 48 lb. of good seed; this is 80% of the weight of a standard bushel. Find that weight.

9. A bushel of clover seed costing $5.50 was found to contain 75% of good seed. At this rate what was the cost of a bushel of good seed?

10. A piece of woolen cloth lost 18% of its length in shrinking. The present length is 41 yd. If the cloth cost $60 before shrinking, what was the price per yard?

11. When wheat is ground, $77\frac{1}{2}$% by weight of the product is fine flour. How many bushels of wheat (60 lb.) are required to produce 1000 bbl. of fine flour (196 lb.)?

12. In drying green sycamore leaves 45% of their weight is lost by the evaporation of the water they contain. A quantity of leaves after drying weighed $4\frac{1}{2}$ oz. What did they weigh before drying?

REVIEW

ORAL EXERCISES

1. Counting from the decimal point, what place is occupied by hundredths? By thousandths? By millionths?

2. How many decimal places are needed to express any number of:

Hundredths? Thousandths? Millionths?
Ten-thousandths? Tenths? Hundred-thousandths?

Read:

3. .033. **5.** .8763. **7.** .63399. **9.** .333333.
4. .005. **6.** 5.00789. **8.** .600853. **10.** .900009.

11. 25% of a number is what fractional part of it? Answer the same question for 50% of it; 75%; 10%; 60%; 40%; 80%; $33\frac{1}{3}\%$; $66\frac{2}{3}\%$; 35%; 85%; 15%; $12\frac{1}{2}\%$; $37\frac{1}{2}\%$; $62\frac{1}{2}\%$; $87\frac{1}{2}\%$; $16\frac{2}{3}\%$; $83\frac{1}{3}\%$; 84%; 6%; 5%; 4%.

12. $\frac{1}{3}$ of a number is how many per cent of it? Answer the same question for $\frac{1}{2}$ of it; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{1}{20}$; $\frac{1}{25}$; $\frac{1}{50}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{2}{5}$; $\frac{5}{6}$; $\frac{3}{8}$; $\frac{4}{9}$; $\frac{7}{10}$; $\frac{5}{12}$; $\frac{4}{25}$; $\frac{11}{30}$.

13. Which of the following percentages would be found more easily, if the rates were expressed as common fractions: 25% of 1000? 40% of 1000? $46\frac{1}{2}\%$ of 1000? $12\frac{1}{2}\%$ of 888? 81% of 400?

14. A tract of land contained 640 acres, of which 80 acres were wooded. How many per cent of the tract was wooded? How many per cent was cleared?

WRITTEN EXERCISES

Add:

1.	.1275	**2.**	16.875	**3.**	2.8645	**4.**	.00084
	.1363		19.008		7.0089		.00176
	.9908		67.479		.6374		.08932
	.4532		99.001		10.9009		.46754

Subtract:

5. 46.93	**6.** 30.0008	**7.** .900005	**8.** 400.005
19.884	16.9909	.063548	33.399

Reduce to common fractions:

9. .875. **11.** .0075. **13.** .9995. **15.** .8888.

10. .6275. **12.** .000125. **14.** .6335. **16.** .00085.

Reduce to decimal fractions of not more than 4 places:

17. $\frac{5}{8}$. **19.** $\frac{5}{6}$. **21.** $\frac{9}{16}$. **23.** $\frac{15}{32}$.

18. $\frac{7}{15}$. **20.** $\frac{11}{20}$. **22.** $\frac{8}{11}$. **24.** $\frac{9}{64}$.

Multiply:

25. 17.19 × 14.6. **27.** 99.25 × 8451. **29.** .087 × 1.009.

26. 30.96 × 21.8. **28.** 106.3 × 2.009. **30.** .045 × .099.

Divide to not more than 4 decimal places:

31. 6.25 ÷ 3.369. **33.** 4.006 ÷ 3.008. **35.** 185.6 ÷ 206.001.

32. 84.51 ÷ 7.635. **34.** 72.36 ÷ .0025. **36.** .01263 ÷ .00018.

Solve:

37. A collector reported the following collections for one week. How much did he collect in all?

Mon.	$286.17	Tues.	$905.36	Wed.	$1000.33
Thurs.	$363.25	Fri.	$200.75	Sat.	$2059.48

38. The three sides of a triangular field measure 102.75 rd., 89.45 rd., 70.9 rd. What is the distance around the field?

39. From 5.068 oz. of platinum, 2.909 oz. were used. How many ounces were left?

40. A man earns $1.87½ a day. How much is this for a week?

41. When the price of calico is 7½ cts. per yard, what is the cost of 81 yd.? Express the result in decimal form.

42. How much does a man earn in 11.75 da. at $2.25 a day?

43. A tank holds 4.35 gal. How many times can it be filled from 34.8 gal. of oil?

44. Fill the blanks in the table:

STATE.	POPULATION.	PERSONS OF SCHOOL AGE: 5–20 YEARS.	PER CENT OF TOTAL.
Illinois...........	4,800,000	1,600,000	( )
Indiana...........	( )	800,000	32
Kansas...........	1,500,000	( )	33⅓
Tennessee.........	2,000,000	( )	40
Texas.............	( )	1,200,000	40

45. Various commercial fertilizers are sold. They are intended to supply nitrogen, phosphoric acid, and potash to the soil. If nitrogen is valued at 16 cts. a pound, phosphoric acid at 4½ cts. a pound, and potash at 5 cts. a pound, what is the value of these substances in 100 lb. of a fertilizer of which 2½% is nitrogen, 10% available phosphoric acid, and 1½% potash?

46. Beef loses about 15% of its weight by boiling and 20% by roasting. How much will a piece of beef, weighing 10 lb. when raw, weigh after having been roasted? Boiled?

47. The water surface of Kentucky is 1% of the land surface; the total area is 40,000 sq. mi. Find the area of the land surface of Kentucky. Of the water surface.

48. The land surface of New Hampshire is about 9,000 sq. mi.; the water surface is 3⅓% of the land surface. Find the total area.

49. 10 lb. of apples were placed in an oven to dry. After drying they weighed 1 lb. 12 oz. How many pounds of water had evaporated from the apples? How many per cent of the weight had been lost in drying?

50. Potatoes contain 78% of water. How many pounds of water are there in a bushel of potatoes? (1 bu. of potatoes weighs 60 lb.)

51. Flour contains 12½% of water. How much water is there in a barrel of flour? (1 bbl. of flour weighs 196 lb.)

52. Walter took a 3-lb. sample of soil from a forest and dried it; afterward it weighed 2 lb. 4 oz. What part of the soil was water? How many per cent? 1 cu. ft. of the soil weighed 72 lb. How many pounds of water did it contain?

53. Horace sifted soil with a coarse sieve, thus retaining gravel in the sieve; he sifted the residue through a finer sieve, retaining sand in the sieve. He weighed soil, gravel, and sand in 6 experiments and found the weights as follows:

SAMPLE.	1ST.	2D.	3D.	4TH.	5TH.	6TH.
Soil taken.	24 oz.	36 oz.	72 oz.	84 oz.	90 oz.	120 oz.
Gravel	6 "	10 "	27 "	20 "	15 "	36 "
Sand.	10 "	16 "	32 "	36 "	25 "	53 "

How many per cent of each lot of soil was gravel? Sand? Gravel and sand together?

54. Henry sifted 2 lb. of soil through a coarse sieve. The gravel remaining in the sieve weighed 5.4 oz. How many per cent of the soil was gravel?

55. John weighed out 10 oz. of green oak leaves; he dried them thoroughly and weighed them again; this time they weighed 5.1 oz. The loss of weight is due to the evaporation of water from the leaves while drying. How many per cent of the green leaves was water?

56. A cement walk consists, by weight, of 15% of gravel, $33\frac{1}{3}$% of cement, and $51\frac{2}{3}$% of sand. How many tons of each kind of material in a cement walk 100 ft. long, $4\frac{1}{2}$ ft. wide, and 6 in. deep, if each cubic foot weighs 75 lb.?

57. Fresh beef furnishes on the average 15.5% of protein, the chief tissue-building element of all foods. How many ounces of protein are there in 300 oz. of fresh beef?

58. Green corn furnishes on the average 3.1% by weight of protein. How many ounces of protein are there in 200 oz. of green corn?

59. 13.1% of eggs are protein. How many ounces of protein are there in 200 oz. of eggs?

60. The following table shows the average relative amounts of waste, water, protein, fat, and heat units derived from various staple foods:

Animal Food.	Waste Per Cent.	Water Per Cent.	Protein Per Cent.	Fat Per Cent.	Heat Units Per Pound.
Beef........	16.9	52.6	15.5	15.0	910
Veal........	21.6	52.0	15.4	11.0	745
Mutton.....	10.3	39.0	13.8	36.9	1,770
Pork, fresh..	12.6	48.0	13.5	25.9	1,320
Chickens....	42.1	43.7	12.8	1.4	305
Fish (cod)...	30.2	58.5	11.1	0.2	220
Eggs........	12.1	65.5	13.1	9.3	635

Vegetable Food.	Carbohydrates Per Cent.	Water Per Cent.	Protein Per Cent.	Fat Per Cent.	Heat Units Per Pound.
Flour.......	75.6	12.0	11.4	1.0	1,635
Beans, dried	63.1	12.6	22.5	1.8	1,520
White bread	54.2	35.3	9.2	1.3	1,200
Corn, green .	20.4	75.4	3.1	1.1	440
Rice........	79.4	12.3	8.0	0.3	1,620

According to the table above, how many units of heat are supplied by the fuel in 1 lb. of fresh beef? How many units are there in 5 lb.?

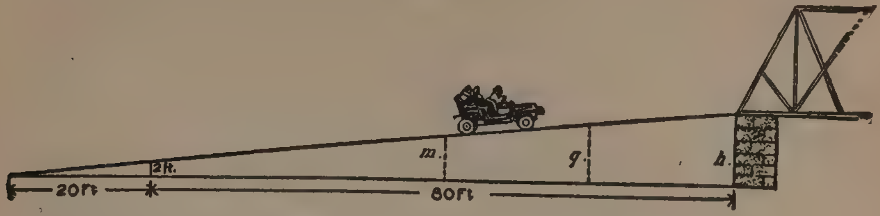
61. Similarly, how much water in 1 lb. of beef? In 36 lb.?

62. How many pounds of protein in 25 lb. of mutton? Of veal? Of rice? Of eggs? Of flour?

63. How many pounds of fat in 32 lb. of pork? Of fish? Of eggs? Of rice? Of beans? Of flour?

64. How much waste in 10 lb. of beef? In 10 lb. of mutton? Of eggs? Of flour? Of rice? Of corn?

65. The approach to this bridge rises 2 ft. for every 20 ft. horizontally. What is the height *h*?



m and *q* in the picture are 20 ft. apart; also *q* and *h*.

66. What is the height *m*? What is the height *q*?

67. If the height *h* were 5 ft. only, and if the approach were 100 ft. long horizontally, what would be the rise of the approach for each 10 ft.?

The rise in the road is called its grade, and it is often indicated by stating the rise for some horizontal distance; e.g., the *grade* of the bridge approach is *2 ft. in 20 ft.*

The grade per cent means the number of feet the grade rises vertically per 100 ft., on the level. Thus, the grade per cent shown in the picture is 10%. The grades of roads are seldom more than 3% or 4%.

68. What is the grade in Exercise 67?

69. What is the grade per cent in Exercise 67?

70. What is meant by a 12% grade? By a 5% grade? By a .5% grade? By a ⅔% grade?

71. How many feet does a 1% grade rise in a mile? A 10% grade? A 12% grade?

72. A horse that can draw 1000 lb. on a level can draw:

900 lb. on a	1% grade,	810 lb. on a	2% grade
750 " "	2.3% "	720 " "	2.5% "
640 " "	3⅓% "	600 " "	4% "

Find the number of feet rise per hundred of horizontal length in each case.

73. According to Exercise 72, on what grade can a horse draw 75% as much as on a level?

CHAPTER VII

BUSINESS APPLICATIONS

GAIN AND LOSS

236. The gain or loss is always reckoned on the cost.

EXAMPLES:

1. A merchant sold goods costing \$540 at a gain of 15%. What was the gain? What was the amount received?

Solution: (1) .15 of \$540 = \$81, the gain.
(2) \$540 + \$81 = \$621, the amount received.

2. A real-estate agent bought a house for \$2,500 and sold it for \$3,000. How many per cent did he gain?

Solution: (1) \$3,000 – \$2,500 = \$500.
(2) \$500 ÷ \$2,500 = $\frac{1}{5}$.
(3) \$500 is $\frac{1}{5}$, or 20%, of \$2,500.
(4) The agent gained 20%.

3. A suit of clothes was sold for \$22 at a gain of 10%. Find the cost.

Solution: (1) $c + .10c$, or $1.10c = \$22$.
(2) Therefore, $c = \dfrac{\$22}{1.10} = \$20$.

Here the rate per cent of gain is expressed by a decimal.

4. A house, sold at a gain of 20%, brought \$2,100. Find its cost.

Solution: (1) Let c = cost. Then $\frac{1}{5}c$ = gain.
(2) $c + \frac{1}{5}c = \frac{6}{5}c$. $\frac{6}{5}c = \$2,100$.
(3) $c = \frac{5}{6}$ of \$2,100 = \$1,750.

Here the per cent of gain is expressed as a fraction.

WRITTEN EXERCISES

1. A man bought wheat in October at $.80 and sold it in December at $.85. How many per cent did he gain?

2. Mr. Williams sold a house for $200 more than it cost; the gain was 5% of the cost. Find the cost.

3. A bicycle, sold at a loss of 10%, brought $45. What was the cost?

4. In 1900 the population of New York City was 3,400,000; in 1890 it was 2,500,000. What was the gain per cent in 10 years?

Find the rate per cent of gain or loss in each case:

	5.	**6.**	**7.**	**8.**	**9.**
Cost..............	$16.	$25.	$40.	$1.20.	$2.00.
Selling Price.......	$20.	$15.	$48.	$1.60.	$1.80.

Find the cost:

	10.	**11.**	**12.**	**13.**
Gain or Loss.........	25%, gain.	20%, loss.	30%, gain.	20%, loss.
Selling Price.........	$2.	$6.	$130.	$1,500.

14. A tailor made 6½ dozen coats with cloth at $1.95 per yard; 3 yd. were required for each coat, and the making cost $19.80 per dozen; he sold the coats at $8.25 each. What was his rate per cent of gain?

15. A dealer bought a carload of coal, 38,000 lb; he sold it at $7.50 per ton, and thus gained $28.50 on the carload. Find the rate per cent of gain.

16. In 1900, there were in the United States about 200 million apple-trees, a gain of 66⅔% since 1890. Find the number in 1890.

17. In a recent year there were produced in the United States about 9½ million bales of cotton (a bale contains about 500 lb.); this was a gain of 26⅔% over the crop of 10 years previous. Find the production in that year.

18. A dealer sold two typewriters, costing the same fixed price, for \$165; on one he gained twice as much as on the other; his total gain was \$45. What was the rate of gain on each?

19. A dealer sold two wagons of equal cost so as to gain five times as many dollars on one as he lost on the other; his total gain was \$200. What was the gain, or loss, on each?

20. A dealer sold a piano at a gain of 30%; the purchaser, after having used it a year, sold it for \$195 at a loss of 40% of its cost to him. Find the cost to the dealer.

COMMISSION

237. The percentage received by an agent for handling goods is called his **commission.**

EXAMPLES:

1. A commercial traveler, who receives a commission of 5% on his sales, earned \$28 in ten days. Find the amount of the sales.

Plan: (1) Let s = the amount of his sales.
(2) $.05 \times s = \$28$.
(3) Therefore $s = \$(\quad)$.

2. A man sold his house and lot through an agent, who charged a commission of 2% and remitted \$9,310. What was the selling price?

Plan: (1) Let s = selling price.
(2) $s - .02 \times s = .98s = \$9{,}310$.
(3) Therefore $s = \dfrac{\$9310}{.98} = \$(\quad)$.

3. Mr. Baker paid \$1,100 for sheep to stock his ranch. This includes the broker's commission of 10%. What did the broker pay?

Plan: (1) Let c = the cost.
(2) $c + 10\%$ of $c = \$1{,}100$.
(3) $1.10\,c = \$(\quad)$.
(4) Therefore $c = \$(\quad)$.

WRITTEN EXERCISES

1. A real-estate agent sold a 200-acre farm at $75 an acre. What was his commission at 2½%?

2. A commercial traveler receives 7% commission on the orders he takes; in 6 months he sells goods to the amount of $9,800. How much does he earn in commissions?

3. An auctioneer received $15 for selling $500 worth of goods. What was his rate of commission?

4. A broker sold a farm for $28,500, his commission being 2%. What was the amount of his commission?

5. A real-estate agent sold a house and lot for $3,200; his commission was 2½%. How much did the owner receive?

6. An agent remitted $142.50 to the owner of goods that he sold, and retained $7.50 commission. For how much did he sell the goods? What was his rate of commission?

7. An agent sold 6 cottages at $1000 each for 2½% commission. What was his commission per cottage?

8. A real-estate agent collects rents amounting to $400 and remits $384 to the owner. What is his rate of commission?

9. A merchant gave bills to the amount of $300 to an agency for collection; the agency charged 2% for collecting ½ of the amount, but 5% for collecting the other half, on account of the trouble in collecting. How many dollars did the merchant receive?

10. A merchant gave bills aggregating $300 to a collector who succeeded in collecting $240, but reported the rest uncollectable. If the collector charged 8% of the amount collected, how much of the $300 did the merchant lose? What was the percentage of loss?

11. Mr. Jones is an agent for an automobile company that pays him a commission of 10% on his sales. In one month he sold one automobile at $760, and another at $925. How much did he earn?

TRADE DISCOUNT

238. When for any reason a reduction is made from a fixed price, such a reduction is called a **discount.**

EXAMPLE: At the close of a season damaged goods were offered as follows:

(1)		(2)	
MARKED PRICE	DISCOUNT	MARKED PRICE	DISCOUNT
\$1.15	20%	\$39	$33\frac{1}{3}\%$

Find the reduced price in each case.

Solution:

(1) $100\% - 20\% = 80\%$. Then, $80\% \text{ of } \$1.15 = .8 \times \$1.15 = \$.92$.
(2) $100\% - 33\frac{1}{3}\% = 66\frac{2}{3}\%$. “ $66\frac{2}{3}\% \text{ of } \$39 = \frac{2}{3} \text{ of } \$39 = \$26$.

239. Many articles of merchandise have fixed prices, which are published in catalogues and price-lists. This price is known as the **list price.**

240. The price after the discount has been taken off is called the **net price.**

WRITTEN EXERCISES

1. Find the reduced price resulting from these discounts:

(1)		(2)	
MARKED PRICE	DISCOUNT	MARKED PRICE	DISCOUNT
\$15	10%	\$42	$16\frac{2}{3}\%$

Determine the numbers to fill the blanks:

	LIST PRICE.	DISCOUNT.	NET PRICE.
2.	\$4.00	6%	( )
3.	\$1.75	20%	( )
4.	\$12.00	( )	\$11.40
5.	\$225.00	( )	\$202.50
6.	( )	10%	\$8.10

	LIST PRICE.	DISCOUNT.	NET PRICE.
7.	( )	15 %	$17.00
8.	$8.40	8⅓%	( )
9.	$4.80	12½%	( )
10.	$855.00	4½%	( )
11.	( )	2 %	$10.29
12.	$1,025.00	( )	$1,000.00
13.	( )	12½%	$87.50
14.	$56.20	5 %	( )

	LIST PRICE.	DISCOUNT.	NET PRICE.
15. Shakespeare (Temple Edn.)	$26.00	20%	( )
16. Roosevelt's Rough Riders..	$2.00	15%	( )
17. Hawthorne's Works, 8vo...	$10.00	25%	( )
18. Poe's Works, 6 vol., cloth..	$7.50	20%	( )
19. Barker's School Stories....	( )	25%	$3.75
20. David Harum............	( )	30%	$1.05
21. Dickens's Works, 12 vol....	$6.00	( )	$4.50
22. Kipling's Works, 15 vol....	$15.00	( )	$10.00

INTEREST

241. PREPARATORY.

A borrowed $500 of B for one year; at the end of the year he returned the $500 and paid B $6 per $100 for the use of the money. How much did he pay for the use of the $500?

242. A payment for the use of money is called **interest.** The rate of payment may be stated by naming the sum to be paid for the use of $100 for one year.

243. Instead of saying "interest at the rate of $6 per $100 per year, or $.06 per dollar," it is customary to say, "interest at the rate of 6% per annum," or, more briefly, "interest at 6%."

EXAMPLE: At the rate of 5% per year, what is the interest for 3 yr. on $300?

Solution: (1) The interest on $300 for 1 yr. is .05 × $300 = $15.
(2) For 3 yr. the interest is 3 × $15 = $45.

ORAL EXERCISES

1. At the rate of 6% per year, what is the interest for 1 yr. on $200? On $500? On $800? On $1000? On $2,000? On $3,200?

2. What is the interest on $100 for 1 yr. at 4%? On $600? On $3,200? On $1?

3. What is the interest on $1 for 1 yr. at 5%? At $3\frac{1}{2}$%? At 6%?

4. At 5% per annum, what is the interest for 2 yr. on $600? On $800? On $1000? On $4,000? On $6,800?

5. At 6% what is the interest on $1 for 1 yr.? For 2 yr.? For 4 yr.? For 7 yr.? For 18 yr.?

6. At 3% what is the interest on $100 for 1 yr.? For 5 yr.? For $\frac{1}{2}$ yr.? For $2\frac{1}{2}$ yr.? For $\frac{3}{4}$ yr.?

244. The sum of money bearing interest is usually called the **principal.**

WRITTEN EXERCISES

Find the interest on the following:

	1.	2.	3.	4.
Principal.............	$200	$250	$483	$1,500
Rate..................	4%	5%	6%	3%
Time..................	1 yr.	2 yr.	1 yr.	5 yr.

5. Which is the greater, the interest on $100 for 3 yr. at 6%, or that on $100 for 4 yr. at 5%?

6. A man borrowed money at 4% and lent it at 6%. How much did he make on $10,000 in 5 yr. in this way?

7. What is the interest at 4% on $500 for 5 yr.?

8. What is the difference in the annual income from $8,600 when invested at 5% and at 4½% interest?

9. How much does $100 earn at 6% in one year? In 2 yr.? In 3 yr.? In 4 yr.? 5 yr.? 10 yr.? 15½ yr.?

10. If $100 has been on interest at 6% per annum long enough to earn $27, how many years has it been lent?

11. How long must $200 be on interest at 5% to earn $20? $50? $35? $15?

12. Mr. Todd bought a house for $1,800. He had $1,100 cash on hand and borrowed the rest from a friend to whom he agreed to pay interest at 5% per annum. How much interest did he pay for each year?

13. At the end of the first year Mr. Todd had saved $185; he paid the interest due and paid the rest of his money to diminish the indebtedness. How much did he still owe? How much interest did he owe at the end of the second year?

245. The principal plus the interest for any given time is called the **amount** for that time.

EXAMPLE: What is the amount of $750 at 6% for 5½ yr.?

Solution: (1) 6% of $750 = $45.
(2) 5½ × $45 = $247.50.
(3) $750 + $247.50 = $997.50, the amount.

WRITTEN EXERCISES

1. What is the amount of $1,075 at 5% for 3 yr.? For 8 yr.? For 15 yr.?

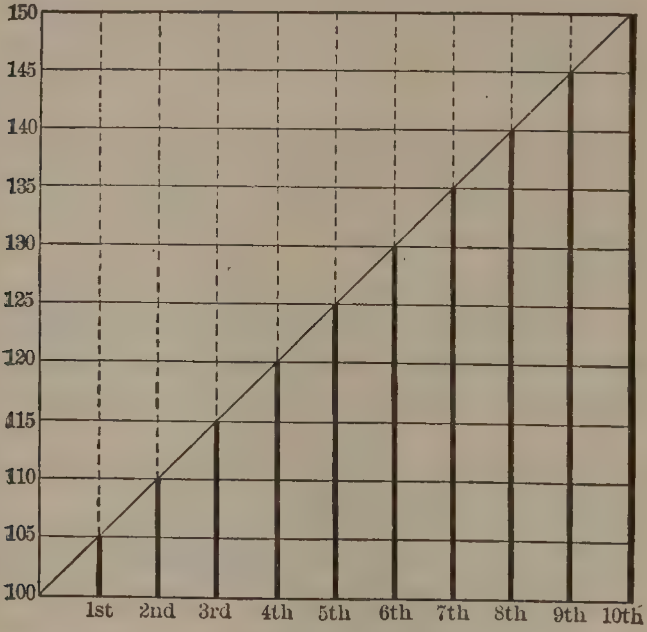
2. A man lent $565 on a note for 3 yr. at 5½%. What was the amount of the note at the end of the 3 yr.?

3. The amount of a given principal for a certain time and rate was $458, and the interest was $79. What was the principal?

4. What is the amount of $2,500 at 3½% for 10 yr.? At 4% for 25 yr.?

ORAL EXERCISES

1. In the figure, each space between the horizontal lines represents $5, and each space on these lines represents 1 year. The heavy lines represent the interest on $100 at 5% for the time indicated. How many dollars interest does the first heavy line represent? The second?



2. What interest does the fourth line represent? The tenth line?

3. The figures at the left express the *amounts* of $100 for various years at 5%. What is the amount of $100 for 2 yr. at 5%? What is the amount for 5 yr.? What is the amount for 8 yr.?

Read from the figure the interest on $100 at 5% for:

	4.	**5.**	**6.**	**7.**	**8.**
Years..........	1	2	3	4	5

Read from the figure the number of years required to earn:

	9.	10.	11.	12.	13.
Interest	$5	$10	$20	$15	$40

WRITTEN EXERCISES

1. Make a similar table for the rate 4%.

2. Find the amount of $500 with interest at $5\frac{1}{2}\%$ for 7 yr.

3. Find the amount of $375 with interest at $3\frac{1}{2}\%$ for 5 yr.

4. Find the amount of $1,050 with interest at 4% for 16 yr.

Interest for a Fraction of a Year

246. PREPARATORY.

1. 6 mo. are what part of a year? 4 mo.? 3 mo.? 8 mo.? 9 mo.? 1 mo. is what part?

2. Express 2 mo. as a fractional part of a year. Also 4 mo. 6 mo. 8 mo. 10 mo.

3. In how many months would the interest on a sum of money equal that for $\frac{1}{4}$ yr. + $\frac{1}{6}$ yr.? That for $\frac{1}{3}$ yr. + $\frac{1}{4}$ yr.?

247. In computing interest, time expressed in months should be expressed as a fraction of a year.

WRITTEN EXERCISES

1. What is the interest on $300 for 1 yr. at 4%? What is the interest for $\frac{1}{3}$ yr., or 4 mo.? For $\frac{1}{4}$ yr. + $\frac{1}{6}$ yr., or 5 mo.?

2. As in Exercise 1, find the interest on $300 at 4% for 7 mo. For 11 mo. For 2 yr. 7 mo. For 3 yr. 11 mo.

What is the interest on $800 at 3% for:

3. 1 yr.?
4. 5 mo.?
5. 9 mo.?
6. 2 mo.?
7. 6 mo.?
8. 10 mo.?
9. 3 mo.?
10. 7 mo.?
11. 11 mo.?
12. 4 mo.?
13. 8 mo.?
14. 12 mo.?

Find the interest on the following at 6%:

	15.	**16.**	**17.**	**18.**
Principal.......	$200	$250	$600	$880
Time..........	1 yr. 4 mo.	2 yr. 3 mo.	4 yr. 8 mo.	1 yr. 10 mo.

Find the interest on:

19. $675 for 11 mo. at 4%.

20. $500 for 3 yr. 9 mo. at 3%.

21. $200 for 2 yr. 3 mo. at 3½%.

22. $385 for 5 yr. 6 mo. at 4½%.

23. $1,500 for 3 yr. 3 mo. at 4%.

24. $325 from January 1, 1901, to Apr. 1, 1904, at 3%.

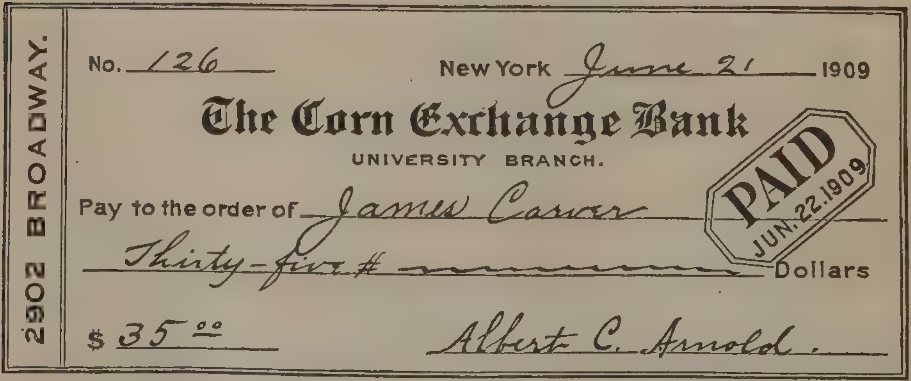
25. $450 from May 1, 1909, to Sept. 1, 1909, at 4%.

26. $218 from Apr. 1, 1909, to Aug. 1, 1911, at 3%.

27. $550 from March 1, 1909, to Sept. 1, 1913, at 3½%.

28. $600 from June 1, 1910, to Oct. 1, 1914, at 4½%.

BANKING



2902 BROADWAY.

No. 126 New York June 21 1909

The Corn Exchange Bank

UNIVERSITY BRANCH.

Pay to the order of James Carver

Thirty-five # Dollars

$ 35 00

Albert C. Arnold.

PAID JUN. 22. 1909

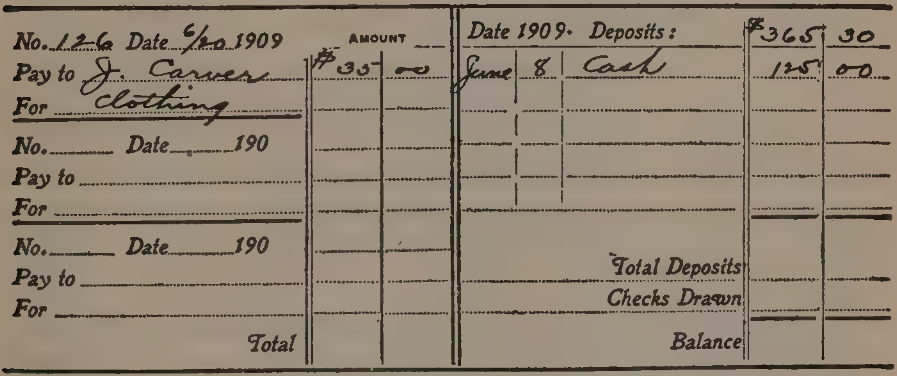
248. When money is deposited in a bank, the bank will pay the money, or any part of it, to any one who presents a written order from the depositor. Such an order is called a **check.**

Read the check. Who made it? On what bank was this order drawn? To whom did the bank pay the money? To whom did the money paid belong?

Checks may be transferred from one person to another, each person writing his name on the back of the check when he transfers it. Each person who thus signs his name is said to endorse the check.

The check is said to be *deposited* when it is handed into the bank for credit.

249. Blank checks are furnished by the banks in book form. When a check is filled out, it is detached from the book and sent to the bank or person to whose order it is drawn. The one issuing the check, called the **maker** of the check, should keep a record of the essential facts about it in the portion of the book made for his purpose, as shown in the illustration. The deposits also may be entered as shown.



	AMOUNT	
No. 126 Date 6/20 1909 Pay to J. Carver For clothing	$35	00
No. ____ Date ____ 190 Pay to ____ For ____		
No. ____ Date ____ 190 Pay to ____ For ____		
Total		

Date 1909.		Deposits:	$365	30
June	8	Cash	125	00
		Total Deposits		
		Checks Drawn		
		Balance		

WRITTEN EXERCISES

1. In the account above find the total deposits shown on the right; also the balance after deducting the check.

2. A depositor had $454.50 to his credit and drew the following checks: $125, $60.75, $34.78, $101.10. How much did he have left in the bank after the bank had cashed the checks?

3. A man opened an account with a bank and deposited $500; later he drew by checks $104, $65.15, $33.25, $75.60. How much remained on deposit?

4. A merchant's deposits for one day were: cash $235.15, checks $360.84; his withdrawals were: cash $110.50, checks $290.35. What was the balance for the day?

Savings Accounts

250. Many banks receive savings deposits in sums from $1 upward and pay interest thereon. The usual rules are:

a. Interest is computed semi-annually; January 1st and July 1st. Usual rates of interest are 3%, 3½%, and 4%.

b. Interest is not computed for fractions of a month, or on fractional parts of a dollar. Many banks do not compute interest for fractions of a quarter (3 months).

c. The interest when computed draws interest thereafter.

WRITTEN EXERCISES

1. Mr. Brown deposited $50 on Jan. 5, and $40 on Feb. 3. Find the amount of interest at 4% credited to him July 1 for the three months ending then, the bank not allowing interest on a part of a quarter year. How much had he on deposit July 1?

2. Mr. Hay deposited in the bank mentioned in Exercise 1: July 6, $80; Sept. 23, $125; Oct. 8, $200; Nov. 1, $150. How much interest was credited Jan. 1?

3. A depositor put $235 in the same bank on Jan. 15 and withdrew $105 on Feb. 10. How much interest, reckoned from April 1, was credited him on July 1 of the same year? How much had he on deposit then?

4. A man deposited $375 on April 10 in a bank which pays interest at 3½% from the 1st of the month following the date of deposit; on June 5 he deposited $118.75. How much money had he to his credit Jan. 1 of the following year?

5. A man deposited $15 in a savings bank regularly the first of each month, beginning July 1, 1909. The bank computed interest at 3% from the date of deposit. How much interest was credited Jan. 1, 1910? If he continued his deposits, how much interest was credited July 1, 1910? How much had he then on deposit altogether?

TAXES AND DUTIES

251. Cities, towns, counties, and states require their citizens to pay a sum of money each year for the support of schools, charitable institutions, public improvements, and many other common interests. Such a charge is called a **tax.**

252. Property is seldom taxed on its full value. The value fixed for the purpose of taxation is called the **assessed value.**

253. One tenth of a cent is sometimes called a **mill,** and the tax per dollar is usually expressed in mills.

Thus, a tax of $\frac{1}{4}$ of 1% is the same as $2\frac{1}{2}$ mills per dollar.

WRITTEN EXERCISES

1. A man's property is valued at $4,000. If his school tax is $\frac{1}{4}$ of 1% of this amount, how many dollars is he taxed?

2. If a man's property valued at $10,000 is assessed at $\frac{3}{4}$ of its full value, on how many dollars does he pay taxes?

3. If the tax in Exercise 2 is $2\frac{1}{2}$ mills on each dollar assessed, what are the man's taxes? What percentage of the assessed value are his taxes?

4. The board of a school district wishes to build a new schoolhouse costing $1,500. If the assessed valuation of the property of the district is $125,000, how much on every dollar must be collected in taxes to pay for the building?

5. According to Exercise 4, what tax will a man pay who owns property assessed at $10,000?

6. What is the real value of the man's property mentioned in Exercise 5, if the assessment is $\frac{2}{3}$ of its value?

7. If the man who collects the tax receives 1% of the tax for his work, how much does the collector in Exercise 5 receive?

8. What is the collector's fee at 1% for collecting a tax of $.002 per $1 on an assessment of $250,000?

254. The National Government does not tax land or personal property, but it taxes certain manufactured products; for example, alcoholic liquors and tobacco. Such charges are called **internal-revenue taxes.** A charge is made also on many articles brought into the country from foreign countries. Government charges levied on imported articles are called **customs,** or **duties.**

WRITTEN EXERCISES

1. If the tax on a gallon of wine of a certain brand is 30 cts., what is the tax on 1,700 gal.?

2. When the tax on a box of cigars of a certain brand is 20 cts., what is the tax on 25,000 boxes?

The duty on :

Carpets is	40%	Skein silk is	35%
Clothing is	50%	Silk fabric is	60%
Porcelain is	55%	Cotton garments is	25%
Flannels is	35%	Fur articles is	35%
Leather goods is	35%	Linen goods is	45%
Velvet is	15%	Oriental rugs is	40%

3. According to the table, what is the duty on 150 yd. of carpet worth 75 cts. a yard?

4. What is the duty on 35 yd. of silk at 95 cts. a yard?

5. What is the duty on an Oriental rug that cost $250? On one that cost $475?

6. Make and solve five other problems from the table.

7. In 1 yr. the United States Government received from:

LIQUORS	TOBACCO	STAMPS	MISCELLANIES
$193,126,913	$51,937,425	$13,442,792	$13,360,130

How much did it receive from all these sources?

8. From what source did it receive the most? The least? The amount received from liquors was how much more than that from tobacco? Than that from stamps?

9. What percentage of the total was from miscellaneous goods? What percentage was received from tobacco?

INSURANCE

255. Companies have been formed to afford protection (**insurance**) against financial loss by fire, storms, shipwreck, theft, injury by accidents, death, and the like.

256. The written or printed agreement that a company makes with a customer to reimburse him in case of loss is called a **policy.** The amount paid to the company for taking the risk is called the **premium.** This is usually stated at a rate per $100 for a period of 1 or 3 yr.

WRITTEN EXERCISES

1. A man had his house insured for $2,700 at $1.10 per $100. What is the premium?

2. What is the annual premium on a brick house valued at $6,500, the rate being $.45 per $100?

Find the premiums for the following amounts of insurance at the specified rates:

3. House, $1,550, contents for $500, at $1.05 per $100.
4. House, $3,400, contents for $1000, at $.95 per $100.
5. House, $2,500, contents for $800, at $.90 per $100.
6. House, $4,500, contents for $1,500, at $1.10 per $100.
7. House, $7,000, contents for $3,000, at $1.20 per $100.

8. What is the annual premium for insuring a business block against loss by fire for $9,000 at $1.50 per $100, its first-floor contents for $5,000 at $1.15 per $100, and the other contents for $3,500 at $1.50 per $100?

9. Find the premium for insuring a building valued at $55,500 against damage by cyclones at $1 per $1000.

10. Find out about the rates of insurance in your locality, also the approximate value of some property. Find the premium that would be paid on the insurance of this property.

Rates of Insurance

257. Insurance rates are fixed by an association representing the various companies and vary with time and place.

TABLE OF INSURANCE RATES PER YEAR

SINGLE DWELLINGS AND CONTENTS	RATE PER $100 INSURED
Brick	$0.30
Brick veneered	0.40
Frame, detached, not less than 50 ft. on all sides	0.50
“ “ “ “ “ 25 “ “ “ “	0.60
“ “ “ “ “ 20 “ “ “ “	0.70
“ all others	0.75
Churches and contents, brick or stone	0.75
“ “ “ frame	1.00
SCHOOLHOUSES	
Brick or stone, with metal or slate roof	0.60
Brick or stone, with shingle roof	0.75
Frame	1.00

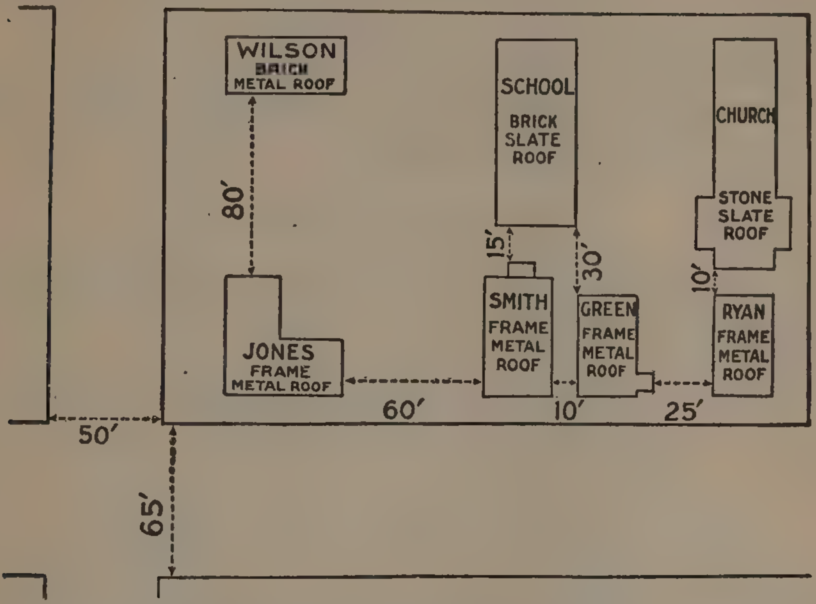
258. EXAMPLE: What is the premium for insuring for 3 yr. a frame dwelling, detached, not less than 25 ft. and valued at $3,500?

Solution: (1) The rate for such a risk, as given in the table, is 60 cts. per hundred per year.

(2) 3 × 35 × $.60 = $63, the premium.

WRITTEN EXERCISES

1. Using the table on page 208, find the premium for insuring the schoolhouse shown in the plan for $10,000 for 5 yr.



2. Find the premium for insuring Mr. Ryan's house and its contents for $3,300 for 2 yr.

3. Using the table, make and solve six other problems about the houses shown on the plan.

RENTS

259. Money paid for the use of houses, land, furniture, etc., is called **rent.** The property used is said to be rented.

260. A contract is usually signed, stating the time for which the property is rented, the amount of rental and the mode of payment, the responsibility of the renter for proper care of the property, etc. Such a contract is called a **lease.**

ORAL EXERCISES

1. According to the lease shown on page 211, to whom did Lewis Grant rent his property?

2. What was the total rental, and how paid, as stated at the end of the lease?

3. How much rent did Mr. Whitcomb pay annually?

4. B leased a house at $750 per annum, payable monthly. How much was his rent per month?

WRITTEN EXERCISES

1. A lady rented a piano at $4.50 per month; she paid $4 for cartage and had the piano 6 mo. What was the total cost?

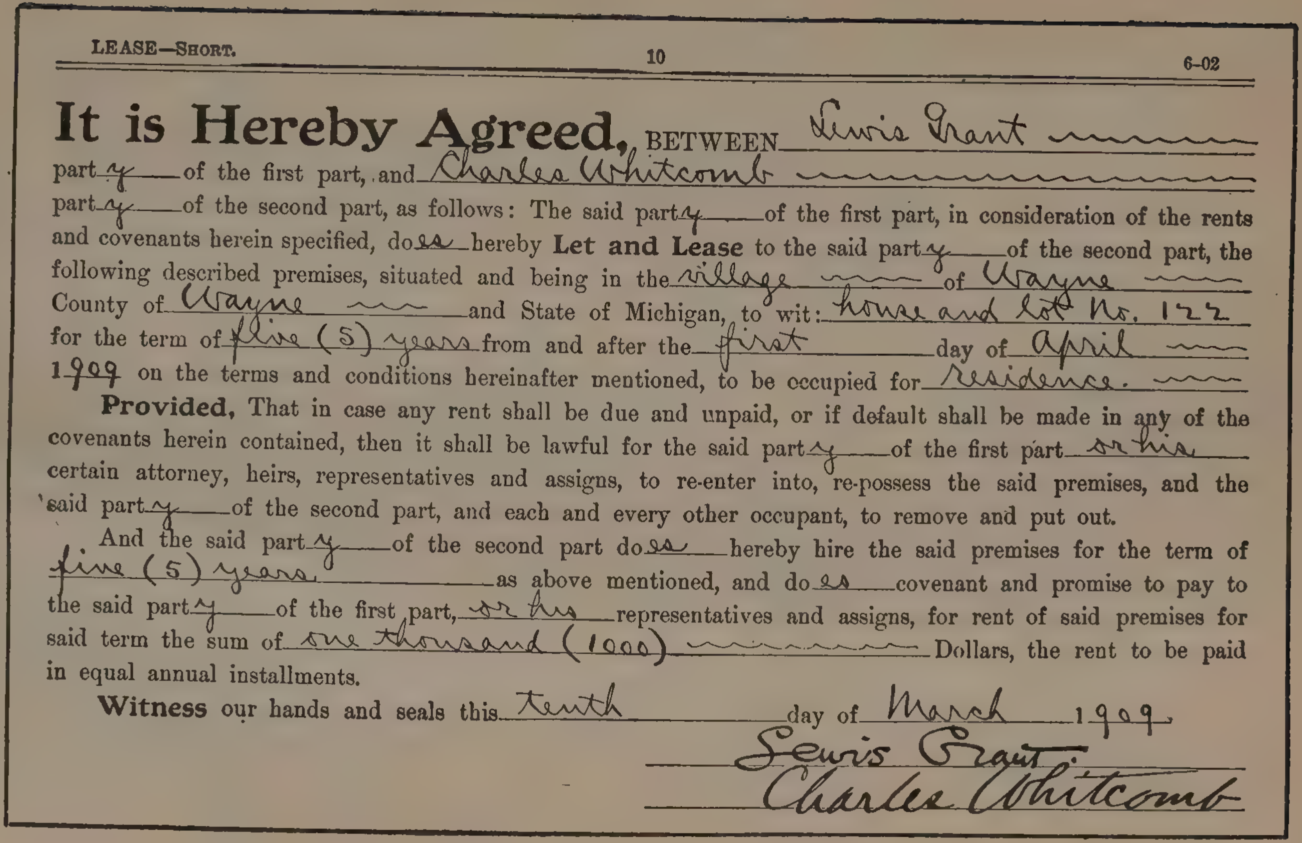
2. If the rent of a house is $250 per year, how much is the rent per month? How much is the rent for 7 mo.? For 19 mo.?

3. A man arranged to rent a bicycle by the hour and to pay his bill monthly; the first month he used the wheel 162 hr. and paid $16.20. What was the rate per hour? The second month he rode 117 hr., and the third month 78 hr. What was the amount of his bill for each month?

4. A merchant rented a block containing 2 stores and 4 suites of rooms for $750 per year. He subrented one store for $200 a year and each suite of rooms for $8 a month. How much rent did the other store cost him per month?

5. An apartment-house contains 6 suites of rooms that rent for $25 a week each, 10 suites that rent for $75 a month each, and 5 that rent for $60 a month. What is the total annual rental for the building?

6. Make up the items necessary to fill out a blank lease for a certain piece of property. Compute the rent named in the lease for 5 yr.



LEASE—SHORT. 10 6-02

It is Hereby Agreed, BETWEEN Lewis Grant part y of the first part, and Charles Whitcomb part y of the second part, as follows: The said part y of the first part, in consideration of the rents and covenants herein specified, do es hereby **Let and Lease** to the said part y of the second part, the following described premises, situated and being in the village of Wayne County of Wayne and State of Michigan, to wit: house and lot No. 122 for the term of five (5) years from and after the first day of April 1909 on the terms and conditions hereinafter mentioned, to be occupied for residence.

Provided, That in case any rent shall be due and unpaid, or if default shall be made in any of the covenants herein contained, then it shall be lawful for the said part y of the first part or his certain attorney, heirs, representatives and assigns, to re-enter into, re-possess the said premises, and the said part y of the second part, and each and every other occupant, to remove and put out.

And the said part y of the second part do es hereby hire the said premises for the term of five (5) years as above mentioned, and do es covenant and promise to pay to the said part y of the first part, or his representatives and assigns, for rent of said premises for said term the sum of one thousand (1000) Dollars, the rent to be paid in equal annual installments.

Witness our hands and seals this tenth day of March 1909

Lewis Grant.
Charles Whitcomb

REVIEW

ORAL EXERCISES

1. When goods costing \$50 are sold at a gain of 10%, how many dollars are gained by the transaction?

2. When \$25 is gained on the sale of goods which cost \$100, what is the rate per cent of gain?

3. A set of books was sold for \$120, at a gain of 20%. What did the books cost?

4. An agent received 5% commission for selling \$500-worth of butter. How many dollars did he receive?

5. An agent received \$35 for his services in selling a building lot; his rate of commission was 7%. For how much did he sell the lot?

6. The list price of an article was \$12.50, and the discount 20%. What was the net price?

7. What is interest? What does "interest at 6%" mean? What is the principal? The amount?

8. What is a bank check? Who is the maker of a check? Who may make checks to be paid by a bank?

9. What are taxes? By whom are taxes levied?

10. How is a small tax-rate usually expressed? What is meant by a tax-rate of 2½ mills?

11. How does the United States Government levy taxes? Name some property that it does not tax. What are government taxes called?

12. What is insurance? What is the premium? What is meant by rate of insurance?

13. What is the insurance on property insured for \$20,000 for 3 yr., at \$.65 per \$100 per year?

14. How is a tax collector paid for his work?

WRITTEN EXERCISES

Find the gain or loss in each of the following:

	COST	RATE OF GAIN	GAIN
1.	$500	$16\frac{2}{3}\%$	( )
2.	$350	28%	( )
		RATE OF LOSS	LOSS
3.	$86.50	35%	( )
4.	$195.75	$15\frac{1}{2}\%$	( )

Find the selling price:

	COST	GAIN		COST	LOSS
5.	$40	16%	**7.**	$25.	$16\frac{2}{3}\%$
6.	$120.50	18%	**8.**	$175.75	6%

Find the cost in each of the following:

	SELLING PRICE	RATE OF GAIN	COST
9.	$165	10%	( )
10.	$185	$16\frac{2}{3}\%$	( )
		RATE OF LOSS	
11.	$583.33	$7\frac{1}{2}\%$	( )
12.	$750.75	25%	( )

Find the rate of gain or loss:

	COST	SELLING PRICE		COST	SELLING PRICE
13.	$150	$175	**15.**	$399.99	$300
14.	$18.75	$25	**16.**	$360	$422.40

17. An agent received a commission of $6\frac{2}{3}\%$ for selling 1000 bu. of potatoes at 50 cts. a bushel. How many dollars in his commission?

18. A collector received $275 for collecting $5,000. What was his rate of commission?

19. The list price of some chairs was $6.30, subject to a 20% discount. What did a customer pay for 4 doz. of these chairs?

20. An agent remitted $396 after deducting a commission of 10%. What was the amount of the collections on which he reckoned his commission?

21. A bill of $875 was subject to a discount of 20%. How many dollars did it take to pay the bill?

22. The discount on a bill was $162.50, resulting from an allowance of 25%. What was the amount of the bill?

23. What is the interest on $675 for 11 mo. at 4%?

24. What is the interest on $500 for 3 yr. 9 mo. at 3%?

25. What is the interest on $200 for 2 yr. 3 mo. at 3½%?

26. A depositor had $595.80 in the bank and drew, in checks, $105.33, $89.60, $204.98. When these checks were paid, what was his balance?

27. What is the amount of a farmer's tax at 2 mills per dollar, whose farm, worth $20,000, is assessed at $\frac{3}{4}$ of its value?

28. What is the duty on 17 cases of leather goods valued at $5 per case when the duty is 35%?

29. If the school tax is $\frac{1}{3}$ of 1%, what is the school tax on property worth $25,000?

30. What is the duty at 60% on 425 yd. of silk worth $2.15 per yard?

31. What is the annual rent on a house whose rental is $83.33 a month?

32. What is the premium for insuring property assessed for $5,500, at 3½%?

33. When the rate of taxation is 2½ mills per dollar and 1% for collecting, what does an owner pay on property assessed at $35,000?

34. A school imported some laboratory instruments, billed at $235.50, on which there was a duty of 1%. What was the total cost?

35. What is the interest on $825 for 1 yr. 9 mo. at 4%?

GENERAL REVIEW

I

1. Name the first three periods in the numeration of integers.

2. Write in figures: Five million, three hundred eight thousand, two hundred thirty-six.

3. Write in words: 8,008,008.

4. Read: 905,006.0025.

5. Express in hundredths and thousandths: 84 thousandths (thus: 8 hundredths, 4 thousandths); 27 thousandths; 43 thousandths; 91 thousandths.

Write decimally :

6. 32 and 28 thousandths.

7. 19 thousandths.

8. 5 and 139 thousandths.

9. 1 and 1 thousandth.

10. Four hundred eight thousandths.

11. Four hundred and eight thousandths.

12. 9,352 thousandths.

13. 82 and 321 thousandths.

14. $\frac{1}{100}$; $\frac{5}{1000}$; $\frac{200}{1000}$; $\frac{25}{1000}$; $\frac{15}{1000}$; $\frac{986}{1000}$; $\frac{100}{1000}$.

Add and test :

15. $37565	**16.** $47826	**17.** $268.42	**18.** $827.99
16342	23247	127.80	777.85
98763	12835	988.75	676.90
47325	45626	878.29	788.75
66232	63412	144.35	425.50
83488	47856	800.20	100.83
24792	32883	723.00	676.97
72687	73632	110.75	900.08

Find the products of:

19. 2848 × 27.
20. 4997 × 36.
21. 9832 × 99.6.
22. .083 × .046.
23. 4978 × .04.
24. 76.7 × .23.
25. 23.4 × 7.09.
26. 1.86 × .0053.
27. 88.3 × 9.06.
28. 17.8 × .87.
29. 40.91 × 4.08.
30. .0096 × .0009.

Multiply:

31. $8056.23
7.79

32. $46.832
.008

33. $1729.85
29

34. $72658
.125

35. $2799.87
79.8

36. $7267.43
8.7

37. $1728.97
34.6

38. $2897.8
27.5

Find the result to the nearest hundredth:

39. 29.5 ÷ 7.5.
40. .029 ÷ 3.6.
41. .286 ÷ 86.
42. 856.432 ÷ 1.6.
43. 72.80 ÷ 3,465.
44. 11.485 ÷ 2.8.
45. 48.6 ÷ 2.7.
46. 14.2 ÷ 7.25.
47. 4.44 ÷ 8.85.

Divide to the nearest thousandth and test:

48. $237.66 ÷ 17.
49. $70.616 ÷ .009.
50. 2.456 ÷ 1.235.
51. $6,934.25 ÷ .01.
52. $3,692.73 ÷ .27.
53. 3.25 ÷ 8.65.

Add:

54. $\frac{4}{5} + \frac{7}{8} + \frac{2}{3}$.
55. $\frac{3}{4} + \frac{5}{6} + \frac{1}{2}$.
56. $\frac{1}{12} + \frac{4}{9} + \frac{3}{8}$.
57. $\frac{9}{10} + \frac{2}{3} + \frac{1}{4}$.
58. $\frac{1}{2} + \frac{2}{7} + \frac{3}{4}$.
59. $\frac{4}{9} + \frac{3}{5} + \frac{1}{2}$.
60. $\frac{2}{9} + \frac{1}{8} + \frac{2}{5}$.
61. $\frac{1}{10} + \frac{2}{9} + \frac{5}{8}$.
62. $\frac{1}{2} + \frac{1}{4} + \frac{3}{5}$.
63. $\frac{1}{15} + \frac{5}{18} + \frac{3}{4}$.
64. $\frac{5}{6} + \frac{4}{5} + \frac{1}{18}$.
65. $\frac{4}{9} + \frac{2}{7} + \frac{5}{6}$.

Subtract:

66. $\frac{5}{8} - \frac{1}{3}$.
67. $\frac{4}{9} - \frac{1}{4}$.
68. $\frac{6}{7} - \frac{1}{2}$.
69. $\frac{5}{6} - \frac{1}{3}$.
70. $\frac{3}{8} - \frac{1}{9}$.
71. $\frac{4}{7} - \frac{2}{9}$.
72. $\frac{11}{12} - \frac{1}{4}$.
73. $\frac{10}{17} - \frac{2}{7}$.
74. $\frac{24}{25} - \frac{2}{3}$.
75. $\frac{18}{19} - \frac{3}{4}$.
76. $\frac{13}{18} - \frac{2}{9}$.
77. $\frac{12}{13} - \frac{4}{5}$.
78. $\frac{14}{17} - \frac{1}{11}$.
79. $\frac{13}{19} - \frac{1}{12}$.
80. $\frac{15}{21} - \frac{3}{24}$.
81. $\frac{17}{35} - \frac{3}{11}$.

82. $\begin{array}{r} 7\frac{1}{12} \\ \underline{\frac{7}{18}} \end{array}$ **83.** $\begin{array}{r} 21\frac{3}{5} \\ \underline{18\frac{1}{10}} \end{array}$ **84.** $\begin{array}{r} 5\frac{1}{5} \\ \underline{3\frac{1}{6}} \end{array}$ **85.** $\begin{array}{r} 14\frac{7}{9} \\ \underline{3\frac{3}{4}} \end{array}$ **86.** $\begin{array}{r} 41\frac{5}{18} \\ \underline{12\frac{1}{24}} \end{array}$

Multiply:

87. $\frac{6}{7} \times \frac{2}{3}$.
88. $\frac{5}{6} \times \frac{1}{7}$.
89. $\frac{2}{3} \times \frac{5}{6}$.
90. $\frac{8}{9} \times \frac{1}{5}$.
91. $\frac{7}{6} \times \frac{5}{8}$.
92. $\frac{3}{8} \times \frac{1}{7}$.
93. $\frac{4}{9} \times \frac{2}{7}$.
94. $\frac{4}{11} \times \frac{2}{3}$.
95. $\frac{9}{10} \times \frac{3}{5}$.
96. $\frac{5}{13} \times \frac{4}{7}$.
97. $\frac{9}{22} \times \frac{1}{8}$.
98. $\frac{6}{17} \times \frac{2}{7}$.
99. $\frac{11}{12} \times \frac{5}{9}$.
100. $\frac{13}{17} \times \frac{3}{8}$.
101. $\frac{4}{9} \times 12$.
102. $179\frac{3}{4} \times 14$.
103. $276\frac{1}{8} \times 23$.
104. $199\frac{1}{7} \times 35$.
105. $267\frac{4}{9} \times 28$.
106. $100\frac{2}{3} \times 98$.
107. $340\frac{1}{5} \times 79$. **108.** $40\frac{5}{8} \times 21\frac{3}{4}$. **109.** $105\frac{9}{10} \times 55\frac{5}{9}$.

Divide:

110. $\frac{8}{9} \div 4$.
111. $\frac{1}{8} \div 3$.
112. $\frac{4}{7} \div 6$.
113. $\frac{3}{4} \div 5$.
114. $\frac{2}{5} \div 7$.
115. $\frac{5}{6} \div 6$.
116. $\frac{8}{17} \div \frac{1}{3}$.
117. $\frac{5}{12} \div \frac{1}{9}$.
118. $\frac{3}{14} \div \frac{1}{6}$.
119. $\frac{9}{22} \div \frac{2}{5}$.
120. $\frac{11}{13} \div \frac{5}{7}$.
121. $\frac{10}{11} \div \frac{4}{9}$.
122. $\frac{9}{13} \div \frac{4}{5}$.
123. $\frac{24}{25} \div \frac{2}{7}$.
124. $\frac{13}{17} \div \frac{8}{9}$.
125. $\frac{11}{13} \div \frac{4}{7}$.
126. $16\frac{3}{8} \div 12\frac{3}{4}$. **127.** $48\frac{11}{12} \div 36\frac{5}{6}$. **128.** $24\frac{3}{8} \div 7\frac{2}{9}$.

Express as mixed numbers:

129. $\frac{13}{7}$.
130. $\frac{40}{3}$.
131. $\frac{65}{8}$.
132. $\frac{205}{21}$.
133. $\frac{101}{11}$.
134. $\frac{186}{19}$.
135. $\frac{564}{18}$.
136. $\frac{863}{18}$.
137. $\frac{29}{4}$.
138. $\frac{384}{15}$.
139. $\frac{782}{28}$.
140. $\frac{425}{30}$.
141. $\frac{35}{8}$.
142. $\frac{25}{6}$.
143. $\frac{17}{2}$.

Reduce to improper fractions:

144. $112\frac{1}{3}$.
145. $87\frac{3}{4}$.
146. $40\frac{9}{10}$.
147. $106\frac{7}{12}$.
148. $29\frac{8}{27}$.
149. $56\frac{11}{32}$.
150. $201\frac{5}{12}$.
151. $180\frac{9}{32}$.
152. $200\frac{17}{32}$.
153. $604\frac{4}{5}$.
154. $501\frac{2}{3}$.
155. $133\frac{2}{3}$.

Add:

156.
$$\begin{array}{lll} 5 \text{ rd.} & 16 \text{ ft.} & 3 \text{ in.} \\ \underline{8} & \underline{20} & \underline{10} \end{array}$$

157.
$$\begin{array}{lll} 10 \text{ mi.} & 120 \text{ rd.} & 12 \text{ ft.} \\ \underline{16} & \underline{200} & \underline{9} \end{array}$$

158.
$$\begin{array}{lll} 16 \text{ gal.} & 3 \text{ qt.} & 1 \text{ pt.} \\ \underline{20} & \underline{2} & \underline{1} \end{array}$$

159.
$$\begin{array}{ll} 186 \text{ lb.} & 12 \text{ oz.} \\ \underline{95} & \underline{15} \end{array}$$

Subtract:

160.	100 sq. yd.	8 sq. ft.	100 sq. in.
	56	6	110

161.	148 cu. yd.	24 cu. ft.	1000 cu. in.
	75	26	908

Multiply:

162. 10 gal. 3 qt. 1 pt.
15

163. 3 bu. 2 pk. 7 qt.
18

164. 10 hr. 40 min. 20 sec.
35

165. 208 lb. 12 oz.
125

166. 12 rd. 2 yd. 2.5 ft.
25

167. 20 mi. 60 rd. 10 ft.
64

Divide:

168. 12)16 rd. 10 ft. 8 in.

169. 12 ft. 8 in. ÷ 2 ft. 4 in.

170. 15)50 lb. 10 oz.

171. 100 lb. 12 oz. ÷ 4 lb. 14 oz.

172. 10 sq. yd. 8 sq. ft. 100 sq. in. ÷ 3 sq. yd. 4 sq. ft.

Find the greatest common divisor of:

173. 32, 36, 48, 128.

174. 108, 1728, 720.

175. 24, 256, 144.

176. 40, 880, 711.

Find the least common multiple of:

177. 12, 18, 80.

178. 32, 72, 108.

Reduce to the form of a common fraction:

179. .0065.

180. 100.125.

181. .1275.

182. .8025.

183. .000075.

184. .20011.

185. 6.25.

186. .9009.

Reduce to a decimal of not more than 5 places:

187. $\frac{2}{3}$.

188. $\frac{5}{8}$.

189. $\frac{8}{9}$.

190. $\frac{7}{16}$.

191. $\frac{16}{25}$.

192. $\frac{5}{22}$.

193. $\frac{15}{32}$.

194. $\frac{13}{48}$.

195. $\frac{9}{11}$.

196. $\frac{5}{64}$.

197. $\frac{112}{129}$.

198. $\frac{1}{1001}$.

II

Solve:

1. A windmill grinds 5.2 bu. of corn per hour for 9 hr. and 45 min. How many bushels does it grind in this time?

2. When the air-pressure is 15 lb. per square inch, how many pounds pressure are there on a surface containing 8.75 sq. in.?

3. At 15 lb. per square inch, what is the air-pressure on the total surface of a book 4 in. by 6 in. by 1 in.?

4. How many cards 3 in. by 6 in. can be cut from 50 sheets of cardboard 18 in. by 24 in. in size?

5. In taking an account of stock, a merchant found that he had 4 pieces of silk costing 98 cts. per yard. The lengths of the pieces were: 7 yd. 1 ft. 10 in.; 12 yd.; 6 yd. 8 in.; 3 yd. 2 ft. Find the total number of yards and the cost of the whole.

6. Four cars together carry 240,676 lb. of granite. What is the average number of pounds per car?

7. In 1884 the export from the United States of iron and steel and articles manufactured from them amounted to $21,900,000; in 1900 the value of the same exports was $121,900,000. What was the difference between the values of the exports for the two years given?

8. The price of wheat was: Monday, $\$.67\frac{1}{2}$; Tuesday, $\$.68\frac{3}{4}$; Wednesday, $\$.66\frac{1}{8}$; Thursday, $\$.67\frac{5}{8}$; Friday, $\$.67\frac{3}{8}$; Saturday, $\$.66\frac{7}{8}$. Find the average price for the week.

9. From New York to Boston is 217 mi. How long does it take a train traveling 42 mi. per hour to go from New York to Boston? Express the result to .01 of an hour.

10. A train goes 12 mi. in 16.4 min. What is the speed per minute to the nearest .01 of a mile?

11. What fraction of 1 sq. yd. are 12 sq. in.? 144 sq. in.? 156 sq. in.? 288 sq. in.?

12. How many window-curtains $\frac{3}{2}$ yd. long can be cut from a piece of goods 30 yd. long?

13. How many are 20% of 300? Of 500? Of 1,700?

14. How many are 60% of 400? Of 4,000? Of 6,400?

15. How many are 5% of 700? Of 100? Of 900?

16. How many are 100% of 100? Of 900? Of 70? Of 3?

17. How many are 150% of 100? Of 200? Of 400?

18. How many are 200% of 100? Of 200? Of 80? Of 5?

19. A workman received $12 per week of 6 working days. His wages were raised to $14.40 per week. How much was the increase per day? How many per cent?

20. A mailing clerk was to address circulars according to a certain list. After having addressed 3,200, he had copied $\frac{4}{7}$ of the names on the list. How many names were there?

21. If a savings-bank pays interest at 2% semiannually, what is the interest on a deposit of $100 for $\frac{1}{2}$ yr.? Add the interest to the deposit and find the interest for the next $\frac{1}{2}$ yr.

22. What is the cost of 1 dozen pairs of lace curtains at 98 cts. a pair?

23. What is the cost of 15 yd. of muslin at 11 cts. a yard and 24 yd. of gingham at 9 cts. a yard?

24. What is the cost of 5 lb. of spring chicken at 18 cts. a pound and 13 lb. of pork at 11 cts. a pound?

25. What is the cost of 101 boxes of soap at $2.75 a box?

26. What is the cost of 3 shirt-waists at 99 cts. each and 4 pairs of gloves at 98 cts. a pair?

27. What is the cost of 5 lb. of coffee at 29 cts. a pound and 9 lb. at 19 cts. a pound?

28. How much do 11 yd. of carpet cost at 79 cts. per yard?

29. What is the area of a square 21 in. on a side?

30. How many miles does a train run in 18 hr. at the rate of 31 mi. per hour?

31. How many miles does an automobile travel in 11 hr. at the rate of 21 mi. per hour?

32. What is the weight of a load of apples containing 22 bbl., if each barrel of apples weighs 130 lb.?

33. How many square rods are there in a field 17 rd. long and 9 rd. wide?

34. 78% of potatoes is water. How many pounds of water are there in 100 lb. of potatoes? In 500 lb.? In 250 lb.?

35. 1% of flour is fat. How many pounds of fat are there in 200 lb. of flour? In 150 lb.? In 500 lb.? In 625 lb.?

36. 37% of mutton is fat. How many pounds of fat are there in 500 lb. of mutton?

37. The distance from New York to San Francisco is about 3,300 mi. By how many inches is this distance represented on a map whose scale is 1 in. for every 100 mi.? From New York to Chicago is about 1000 mi. What is the distance on the map between these places?

38. If a barometer reads 30 in. on 3 days in a week, 29.5 in. on 2 other days, and 30.1 in. on 2 others, what is the average height for the week?

39. If the thermometer registers 70° one day, 75° the next, and 84° the next, what is the average temperature for the three days?

40. Two men engage in business; one furnishes $40,000, and the other $60,000. What is the ratio of their capitals? They gain $6,000 in one year, which is shared in the ratio of their capital. How many dollars does each receive?

41. The average daily paid circulation of a newspaper is 95,000. If the average receipts for one day are $1,900, what is the price per copy?

42. How many feet of inch boards will be necessary to build a sign-board 40 ft. long and 12 ft. high? What will the lumber cost at $6 per thousand feet?

43. If a fan having 16 rays opens out so that the outer rays form a straight line, how many degrees are there between any two adjacent rays?

44. A house costing $24,000 rents for $1,200. The taxes and expenses are $300 per year. What is the annual rate per cent of income on the investment?

45. How many years, months, and days were there from the assassination of President Lincoln, April 14, 1865, to that of President McKinley, Sept. 7, 1901?

46. The silkworm larvæ from 1 oz. of eggs produce, under ordinary conditions, about 130 lb. of cocoons. With special care the product may be increased by $33\frac{1}{3}\%$. To how many pounds would the total be equal?

47. How many acres are there in a farm $1\frac{1}{2}$ mi. long and $\frac{3}{4}$ of a mile wide?

Find the interest at 4% on:

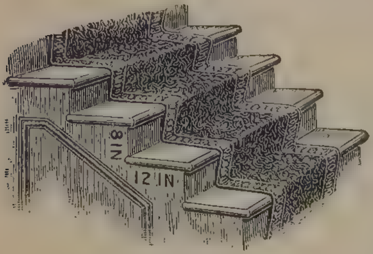
48. $1000 for 2 yr. 6 mo.

49. $500 for 5 yr. 3 mo.

50. $285 for 4 yr. 4 mo.

51. $675.50 for 3 yr. 11 mo.

52. How many yards of carpet $\frac{3}{4}$ yd. wide must be bought to cover the floor of a room $3\frac{3}{4}$ yd. by 5 yd., the strips running the long way, and $1\frac{1}{2}$ yd. being allowed for matching?



53. There are 14 steps to the stairs shown in the picture. What did the carpet cost at 60 cts. a yard?

54. The "East Room" at the White House is 82 ft. × 40 ft. Find the cost of carpeting it with carpet 27 in. wide at $3 a yard.

55. Suppose that in your State 70% of the area is under cultivation and that there are 4 insect-eating birds per acre. If each bird eats 12 injurious insects per day, how many insects are killed weekly by the birds in your State?

56. Before the Spanish-American War, Spanish exports to Cuba amounted to 136 million dollars, to Porto Rico 44 million dollars, to the Philippines 49 million dollars; in 1901 the exports to these places fell to 66 million, 13 million, and 27 million dollars, respectively. What was the per cent of reduction in each case?

57. In Minnesota and in the Dakotas 15 million acres are given to raising wheat. If, by using a new variety of seed, the present yield of 27 bu. per acre is increased 2 bu. an acre, what will be the annual crop in these 3 States? At 75 cts. per bushel, what will be the increase in the value of the wheat-crop by the use of the new seed?

58. In a Fall River cotton-mill 58 hr. are considered a week's work; the average wage paid is $7.50 a week, which is increased $\frac{1}{10}$ by the profit-sharing plan. What is the complete average wage per day? Per hour?

59. Of the 800 employees in a soap factory 325 are boys; the average wage for men is $10 a week, and for boys $5.50. What is the amount of the weekly pay-roll?

60. In a house using 6 Welsbach burners and 6 common burners, the gas costs 3 cts. an evening for each common burner and $1\frac{3}{4}$ cts. for each Welsbach. How much less does the gas for the Welsbach burners cost during a month?

61. If pure air contains 20.04% of oxygen, how many cubic inches of oxygen are there in 1 cu. ft. of air? If 70.02% of pure air is nitrogen, how many cubic inches of nitrogen are there in 1 cu. ft. of air? If the remainder is carbonic-acid gas, what rate per cent of air is carbonic acid gas?

62. If sound travels 1000 ft. per second when the temperature is 32° Fahr., how many seconds after firing will the report of a cannon be heard at a point 4 mi. distant?

63. Sound travels 4 times as fast in water as in air. According to the rate given in Exercise 62, how many seconds will it take sound to travel a mile in water? How long will it take it to travel $11\frac{9}{32}$ mi.?

64. A railroad train crosses a bridge $\frac{1}{2}$ mi. long at a speed of 15 mi. per hour. In how many minutes does the engine pass over the bridge?

65. How many bricks $4\frac{1}{4}$ in. by 9 in. are required to pave a passageway $4\frac{1}{4}$ ft. wide and 120 ft. long? What is the cost of the bricks at $6.75 per thousand?

66. How many feet are there in a mile? How many inches? If an inch on a map represents 100 mi., 3 mi. are represented by what part of an inch? Make a problem involving map measurement and time.

67. If the area of the island of Cuba is 46,000 sq. mi. and that of New York State 49,000 sq. mi., find the ratio of the latter to the former.

68. Make problems from the following data:

NAME.	DATE OF BIRTH.	NAME.	DATE OF BIRTH.
James Madison....	Mar. 16, 1751.	Geo. Washington..	Feb. 22, 1732.
J. G. Whittier.....	Dec. 17, 1807.	H. W. Longfellow.	Feb. 27, 1807.

69. It has been estimated that during some storms the waves on Lake Michigan strike the shore with a force equal to 6,000 lb. per square foot and average 20 ft. in height. With what force does such a wave strike a wall 100 ft. long and 30 ft. high?

70. In a certain total eclipse the sun was completely covered at 2 o'clock 47 min. 23 sec. and reappeared at 2 o'clock 50 min. 48 sec. What was the length of the total eclipse?

71. If slates cost $12.40 per thousand, and if 4 slates are required per square foot, what is the cost of slates for a roof 54 ft. along the ridge and 20 ft. down the slope?

72. The total liabilities of all the business houses failing in a given year were $45,376,472, $\frac{1}{8}$ of which belonged to grocers. How many dollars were their liabilities?

73. What will be the cost of plastering a 10-room house, the rooms averaging 4 yd. by 5 yd. and 3 yd. high, at 15 cts. per square yard?

On account of the extra work around windows and doors, masons seldom make any allowance for these areas in estimating the cost of the work.

74. Find the average rainfall for San Diego for the six months given in the table:

PLACE.	DEC.	JAN.	FEB.	MAR.	APR.	MAY.
San Diego, Cal. . . .	2.34	1.96	2.36	1.47	0.87	0.36
San Francisco.	5.11	4.98	3.72	3.28	2.14	0.68
Sitka, Alaska.	10.11	9.75	10.51	10.02	6.24	4.94
Salt Lake City.	1.59	1.52	1.38	1.92	2.36	1.78
New York.	3.33	3.89	3.76	4.10	3.40	3.05

Similarly, find the average rainfall for each of the other places named.

75. When the wind blows 15 mi. per hour the power of a windmill is sufficient to grind about 46½ bu. per hour. How many bushels would it grind in 87 days of 10 hr. each?

76. Three brothers owned an apartment building jointly, all having equal shares. One sold ⅔ of his share to his cousin. What fraction of the building did the cousin own?

77. They sold the building for $18,000. How much did each receive?

78. Find in square feet the area of a rectangle the length of whose base is ⅔ ft. and whose altitude is ¾ ft.

79. What is the ratio of the population of Tennessee, 2 millions, to that of the United States, 76 millions? Of Texas, 3 millions, to that of the United States?

80. If ⅛ of wheat flour is water, how much water is there in a barrel (196 lb.) of flour?

81. On Oct. 1 a gas meter read 34,800 cu. ft.; on Nov. 1 it read 39,400 cu. ft. How many cubic feet of gas passed through the meter in the interval? What did it cost at $1.20 per thousand feet?

82. A screw with 9 threads to the inch is being driven into a board. How many turns are necessary to drive the screw $1\frac{1}{3}$ in. into the board?

83. What is the weight of 10 baskets of coal each weighing 84.7 lb.?

84. A house cost $2,175; the lot on which it stands cost .2 as much. How many dollars did the lot cost?

85. From a log 35 ft. long a length of 17 ft. 8 in. was cut. How long was the piece that was left?

86. There are 100 small cartridges in a box. How many are there in $\frac{25}{100}$ of a box? In $\frac{1}{4}$ of a box? In $\frac{75}{100}$ of a box? In $\frac{3}{4}$ of a box?

87. A rectangular desk is 5 ft. 8 in. long and 28 in. wide. How many feet of molding are required to go once around the desk?

88. A letter-carrier walks $\frac{23}{8}$ mi. on his first trip, $\frac{29}{8}$ mi. on the second, and $\frac{27}{8}$ mi. on the third. How far does he walk altogether? Express the result as a mixed number.

89. At $1.25 per thousand feet, what is the cost of 5,000 ft. of gas?

90. How many acres are there in a rectangular piece of land 1 mi. wide and 15 mi. long?

91. Find the cost of $4\frac{1}{2}$ lb. of steak at 18 cts. per pound, 3 doz. cookies at 12 cts. a dozen, $2\frac{3}{8}$ lb. of butter at 24 cts. per pound.

92. The expenses of conducting the Library of Congress in 1 year were: $384,185 for cost of service; $91,300 for books; $45,000 for furniture; $95,000 for printing and binding. What were the total expenses?

93. Find the cost of 550 ft. of joists at $22 per thousand feet.

94. $276 was paid for nails at 2.3 cts. per pound. How many pounds were bought? How many tons?

95. At 1.9 cts. per mile, the fare from New York to Chicago by a certain route would be $17.35. Find the distance.

96. How many rods of fence will enclose a quarter-section?

97. How many board feet of 6-inch boards are required to make a fence 4 boards high around a quarter-section?

98. How many steps $\frac{5}{6}$ of a yard long does a man take in walking 5 mi.?

99. A man wishes to plant a hedge about his lot, which is 101 ft. by 200 ft; the hedge is set in 1 ft. from the boundary of the lot; his plants cost $5 a hundred and are set out 3 per foot; he pays a man 2 cts. apiece for setting out the plants. What does it cost him to plant the hedge?

100. Draw a rectangle of length 10 in. and width 5 in. Shade 60% of it. Shade 95% of it.

101. From a rectangular piece of cloth 8 in. wide and $7\frac{1}{2}$ in. long are cut without waste 2 rectangular pieces 2 in. wide and $7\frac{1}{2}$ in. long, one rectangular piece $2\frac{1}{2}$ in. wide and 4 in. long, one right-angled triangle with base 4 in. and altitude 4 in., and one trapezoid with bases 1 in. and 5 in. What is the altitude of the trapezoid, and what is its area?

102. A boy stands so as to face the east and turns to the left through 90°. What point does he face? If he turns through 180°, what point does he face? Through 270°? If he turns to the right through 90°? Through 180°?

103. Through how many degrees does the hour-hand of a watch turn between 12 o'clock and 4 o'clock? Between 2 o'clock and 7 o'clock?

104. If a man receives 75 cts. a 4-foot cord for cutting wood and 50 cts. a 4-foot cord for sawing it into 16-inch stove wood, what is the cost of cutting 125 cords of stove wood?

105. A commission merchant sold 150 bu. of peaches at $2.25 per bushel; his rate of commission was 4½%. What was his commission?

106. What is meant by a 2½% grade? A 6% grade? A 7½% grade?

107. At a special sale a firm offered goods as follows:

Marked price..................	$3.75	$14	$59	$87	$99
Discount....................	10%	16⅔%	20%	3%	5½%

Find the reduced price.

108. Three ships leave port on the same day, January 1; the first returns regularly every 20 days, the second every 22 days, and the third every 28 days. What is the least number of days after January 1 before all three boats will leave port again on the same day?

109. A foundation wall is 45 ft. by 120 ft., outside measurement, 18 in. thick, and 8 ft. deep. Find the number of cubic yards of masonry in the wall?

110. How many rolls of paper 18 in. wide, which cuts five 8-foot strips to the roll, is required to paper the side walls of a room 12 ft. by 18 ft., allowing 15% of the surface for windows and doors?

111. Bulky goods are usually shipped by freight. The charge is made per 100 lb., and articles are grouped in different classes according to bulk and character, with different rates. What is the freight charge on a shipment of 125 lb. from Chicago to Boston at 82 cts.?

112. On a shipment of 248 lb.? Of 850 lb.? Of 1,240 lb.?

113. What is the freight charge on 1,275 lb. from New York to Galveston at $1.65?

114. Between Boston and Chicago, and frequently between other places, freight shipments weighing less than 100 lb. are charged as 100 lb., irrespective of weight. By express the rates are as follows:

Rate per 100 pounds, $2.50; for smaller weights:

Weight.....	1 lb.	2 lb.	3 lb.	4 lb.	5 lb.	7 lb.	10 lb.
Charge.....	$0.25	$0.30	$0.45	$0.50	$0.55	$0.65	$0.75
Weight.....	15 lb.	20 lb.	25 lb.	30 lb.	35 lb.	40 lb.	50 lb.
Charge.....	$0.80	$0.90	$1.00	$1.10	$1.20	$1.25	$1.25

Parcels weighing 50 lb. and over are charged at the rate of 2½ cts. per pound. Weights between those named in the table are charged for at the rate for the next higher weight.

This being the case would it be cheaper to send a 6-pound parcel by freight or by express?

115. Which would be cheaper in the case of 10-pound parcel? A 30-pound parcel? A 38-pound parcel? A 70-pound parcel? A 100-pound parcel?

116. In the preceding exercises no account has been taken of the fact that express charge includes calling for the parcel and delivering it, and that the freight charge includes neither. If 25 cts. is paid for cartage at each end of the route when a parcel is shipped by freight, what is the largest weight that can be sent more cheaply by express than by freight?

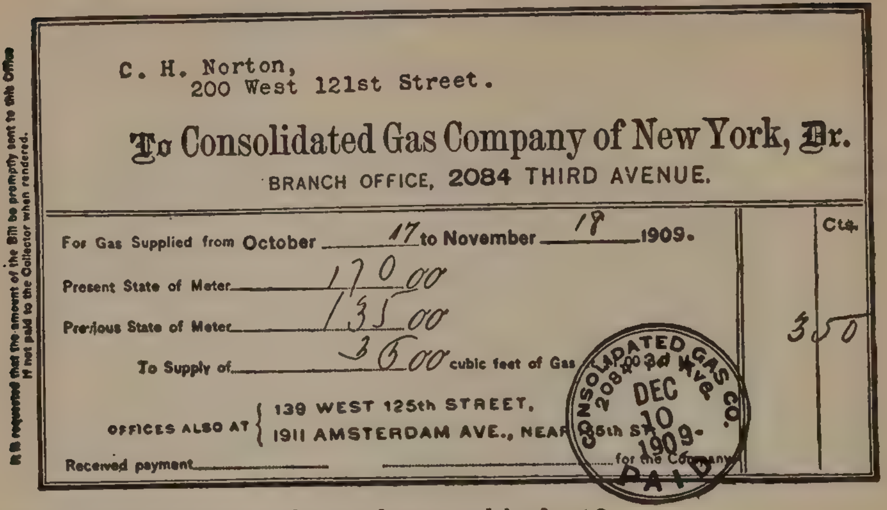
117. Mr. Henderson of Boston orders a 25-pound lot of paper from Chicago; it is sold f. o. b. (delivered *free on board*) in Chicago; the cartage in Boston will cost Mr. Henderson 30 cts. Is it cheaper to have the paper sent by express or by freight and how much cheaper?

118. When the freight rate is 95 cts. per 100 lb. and the cartage at each end is 40 cts., what is the total cost of transportation of a box of books weighing 350 lb.?

119. When the freight rate is 60 cts. per 100 lb. and shipments weighing less than 100 lb. are charged as 100 lb., how much is saved by making one shipment of three 90-pound packages instead of a separate shipment for each?

120. If convenient, ascertain the freight rates from your city to several others and make problems about them.

121. The following is the ordinary form of a gas bill:



N.B. It is requested that the amount of the Bill be promptly sent to this Office if not paid to the Collector when rendered.

C. H. Norton,
200 West 121st Street.

To Consolidated Gas Company of New York, Dr.
BRANCH OFFICE, 2084 THIRD AVENUE.

For Gas Supplied from October 17 to November 18 1909.
Present State of Meter 17000
Previous State of Meter 13500
To Supply of 3600 cubic feet of Gas
3 | 50

OFFICES ALSO AT { 139 WEST 125th STREET,
1911 AMSTERDAM AVE., NEAR 5th ST.

Received payment ________ for the Company

CONSOLIDATED GAS CO.
2084 3d Ave.
DEC
10
1909.
PAID

What was the rate charged per cubic foot?

122. The burners in a house consumed the following number of feet of gas in each month named:

Jan.	*Feb.*	*Mar.*	*Apr.*	*May*	*June*
2,800	3,000	2,800	2,400	1,600	1,800

How many thousand feet were consumed in the six months? What was the cost at $1 per 1000 ft.?

123. Meters for measuring gas are usually subject to error. If the meter which measured the gas billed in Exercise 122 read 2.5% too much, what correction should be made in the gas bill?

124. Find the amount of the following gas bill at 80 cts. per 1000 ft.:

Previous state of meter 106,900 ft.
Present state of meter 129,500 ft.

125. If an ordinary gas-burner consumes 7 ft. of gas per hour, and a Welsbach burner consumes 4 ft. per hour, how much would two burners, one of each kind, consume in 10 hr.?

126. When gas costs $1 per 1000 ft., how much is saved per hour by the use of a Welsbach burner? Per day of 6 hr.? Per month of 30 da. at the same rate?

PART II

CHAPTER VIII

FUNDAMENTAL PROCESSES WITH INTEGERS AND DECIMALS

NOTATION AND NUMERATION

261. The method of writing numbers is called **notation**, and that of reading them **numeration.**

262. Two **systems of notation** have been explained: The **Arabic** or **Hindu** system of notation, with the symbols:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9;

and the **Roman** system, with the symbols:

I for 1. X for 10. C for 100. M for 1 000.
V for 5. L for 50. D for 500.

The Roman numbers from 1 to 10 are written:

I, II, III, IV, V, VI, VII, VIII, IX, X.

In Roman numerals the multiples of 10 from 10 to 100 are written:

X, XX, XXX, XL, L, LX, LXX, LXXX, XC, C.

263. For convenience in reading, numbers are set off into **periods** of three digits each. The last period at the left may have fewer than three digits.

A comma (*separatrix*) or a larger space than that between the other figures may be used to separate two periods.

Thus, 8,569,240,007 or 8 569 240 007.

264. The first four periods of integers are named **units, thousands, millions, billions.**

Thus, $2,630,400,521 is read: *Two billion six hundred thirty million four hundred thousand five hundred twenty-one dollars.*

265. The decimal places in order to the right from the decimal point are named: **tenths, hundredths, thousandths, ten thousandths,** etc.

The decimal is read as if it were an integer, then the name of the last order is added.

Thus, .0256 is read: *Two hundred fifty-six ten-thousandths.*
Decimal places beyond the fourth are rarely used.

In an integer the decimal point belongs at the right, but it is seldom written.

266. The following table shows the names of the periods and places from millionths to billions:

Hundred-billions	**Ten-billions**	**Billions**	**Hundred-millions**	**Ten-millions**	**Millions**	**Hundred-thousands**	**Ten-thousands**	**Thousands**	**Hundreds**	**Tens**	**Units**	*Decimal Point*	**Tenths**	**Hundredths**	**Thousandths**	**Ten-thousandths**	**Hundred-thousandths**	**Millionths**
5	0	7	2	3	7	6	8	9	5	2	1	.	0	2	6	7	4	5
Billions			**Millions**			**Thousands**			**Units**				**Thousandths**			**Millionths**		

Numbers of more than four periods are very rarely needed. The three periods next in order are called, in the system most used, **trillions, quadrillions, quintillions.**

ORAL EXERCISES

1. Read the number in the table above. How many periods are there in the integral part? In the decimal part? How many orders in each period?

2. Read the numbers in the table showing the number of pounds of freight that passed through the various waterways in a recent year:

WATERWAYS.	POUNDS.	TONS.
Suez........................	8,560,284,000	( )
Welland.....................	869,596,000	( )
Sault Ste. Marie............	15,062,594,000	( )
Detroit River...............	25,845,696,000	( )

3. State the number of tons, or the tonnage, for each waterway named in Exercise 2.

Read:

4. Some light waves are .00011 in. long; others are .00001 in. long.

5. Sea water is 1.0249 times as heavy as fresh water.

6. The length of a circle is nearly 3.14159 times the length of its diameter.

7. Light waves whose lengths are greater than .000029 in. or less than .000013 in. are not visible to the eye.

8. In a recent year some of the imports and exports of the United States were:

IMPORTS.		EXPORTS.	
Sugar........	3,031,915,875 lb.	Cotton........	3,500,778,763 lb.
Coffee........	1,091,004,252 lb.	Wheat........	154,856,102 bu.

9. The yearly earnings of the national banks of the United States for eight years were:

(1)................	$87,276,836	(5)................	$112,936,426
(2)................	81,853,797	(6)................	105,909,385
(3)................	106,581,476	(7)................	127,526,836
(4)................	109,881,530	(8)................	152,235,434

WRITTEN EXERCISES

Write in figures:

1. Two million, eighty-three thousand, seventeen.

2. Five million, one hundred six thousand, two hundred.

3. Seventeen million, three hundred thousand, six hundred twenty-five.

4. Three hundred seventy-five thousand four hundred six and one hundred twenty-five thousandths.

5. Six billion, eight hundred five million, four hundred fifty-six thousand, two hundred seventy-nine.

Write the following numbers in words:

6. 3,547,000.275.

7. 6,000,001.33.

8. 11,111,101.111.

9. 22,202,200.002.

10. 77,000,070.077.

11. 126, 379, 008.

12. 96, 439, 109, 100.

13. 436, 701.005.

14. 19,000,000.000019.

15. 125, 000.125.

16. Write 200 million in figures. 400 million. 790 million.

17. Write 1000 million in figures. How many periods are there in the number? What is the name of the fourth period? Read the number.

ADDITION

267. Addition is the process of counting two or more numbers into an equivalent single number.

ORAL EXERCISES

1. What is the result of addition called?

2. What are the numbers added called?

3. When numbers are written for adding, how should the corresponding orders stand?

4. How may the work of addition be tested?

WRITTEN EXERCISES

1. Add 200, 1,760, 25.7, and 2.005. Also, 16.025, 30.008, 9.2693, and .0011.

2. Find from the table below the total number of people working in the lumber business in the United States in a certain year:

In furniture factories....	58,719	In saw mills............	100,087
" cooper shops.........	37,226	" planing mills.........	61,600
" piano factories.......	6,220	" other factories.......	82,390

3. Find the total amount of receipts of a school system from the following school funds:

Instruction fund............................	$440,976.12
Building fund................................	120,000.00
Repair fund..................................	25,046.75
Library fund.................................	5,138.70
Contingent fund.............................	144,411.50

4. Recently the governmental departments of a certain city had on hand the following sums:

Department of education......................	$81,000.00
Department of public works..................	69,374.62
Department of charities and correction........	24,600.00
Department of public safety..................	21,817.92
Legislative department.......................	12,000.00
Department of law...........................	8,000.00
Comptroller's office (estimated)..............	5,000.00
Treasurer's office............................	3,000.00
Department of assessment and taxation.......	395.00
Fire marshal.................................	400.00
Civil service commission.....................	300.00
Park commission.............................	300.00
Mayor's office...............................	300.00
Municipal court..............................	25.00
Police court.................................	25.00

Find the total amount on hand.

Casting Out Nines

268. Dividing the sum of the digits of a number by 9 is called **casting out the nines.**

EXAMPLE: Add the digits in the number 2,639. Divide the sum by 9. The remainder, 2, is the excess.

The remainder after the 9's are cast out is called the **excess.**

ORAL EXERCISES

Cast out the nines from the following numbers:

1. 40,372.
2. 56,432.
3. 172,360.
4. 125,678.
5. 3,061.
6. 3,136.
7. 9,910.
8. 14,037.
9. 72,320.
10. 125,963.
11. 1,265,384.
12. 69,327,436.

269. Addition may be tested by casting out **nines.**

(1)	EXCESS.
26731	1
37654	7
64385	8

(2)	
2635	7
3094	7
4265	8
9994	4

In the first example the sum of the excesses of the *addends* is 8, and the excess of the *result* is 8; hence the work checks.

In the second example the sum of the excesses is 22, whose excess is 4. The excess of the result, 9,994, is 4. Hence the work checks.

The process is the same when the addends contain a decimal part.

WRITTEN EXERCISES

Add, and test the results by casting out nines:

1. 63450
85709
36245

2. 8275
3268
7457

3. 76327
94308
86347

4. 1403
3399
2401

5. 26932
26872
26462

6. 1127
8943
1794

7. 260032
310032
187625

8. 4083
986
77

9.	11.	13.	15.
180060	163.006	347798	198.221
19760	29.376	543297	768.993
4330	197.263	936720	231.795
308699	90.008	399444	550.117

10.	12.	14.	16.
12.7	40.30	90.80	3.125
14.3	17.25	12.60	0.625
40.8	16.83	30.17	3.315
90.6	40.36	45.96	8.055

17. The following are the approximate amounts of the contracts for constructing the Williamsburgh Bridge from Manhattan to Brooklyn:

For the Manhattan tower foundation	$373,462.70
" " Brooklyn " "	485,082.70
" " Manhattan anchorage	797,770.00
" " Brooklyn "	771,778.00
" " Manhattan approach	1,464,000.00
" " Brooklyn "	947,000.00
" " suspended structure	1,123,400.00
" " steel towers and end spaces	1,221,726.00
" " minor work	1,500,000.00

Find the total cost of the bridge. Test the accuracy of your work by casting out nines.

SUBTRACTION

270. Subtraction is the process of finding how much one number exceeds another.

ORAL EXERCISES

1. What is the result of subtraction called?

2. In a case of subtraction what is the number subtracted called? What is the number from which another is taken called?

3. How is the work of subtraction tested?

4. When numbers are subtracted, how are their corresponding orders written? When decimal numbers are subtracted, how are the decimal points made to stand?

WRITTEN EXERCISES

Subtract and test:

1.	7893	**4.**	8654.23	**7.**	5002.17	**10.**	1000.00
	2469		799.86		673.43		.99
2.	84.33	**5.**	126.003	**8.**	909.12	**11.**	2000.00
	23.99		46.098		508.20		9.87
3.	8703	**6.**	763.45	**9.**	406.38	**12.**	1111.110
	2604		97.23		289.		99.992

Solve:

13. The value of the clay pottery made in a recent year was:

Ohio..............	$10,519,138	West Virginia.......	$1,166,464
New Jersey........	6,192,959	All other States......	(?)
Pennsylvania.......	1,876,263	United States.......	24,127,453

What was the total value of the pottery produced in Ohio, New Jersey, Pennsylvania, and West Virginia? In all other States?

14. The following table shows the loans and deposits for certain New York banks for one week:

NAME OF BANK.	TOTAL LOANS.	SHORT-TIME LOANS.	DEPOSITS.
Liberty National...........	$12,795,400	$2,401,500	$10,976,800
N. Y. Produce Exchange....	6,250,400	1,895,900	7,533,500
New Amsterdam National...	4,447,400	678,200	4,945,600
State Bank................	14,290,000	2,223,000	15,179,000
Fourteenth Street Bank.....	6,945,400	896,400	7,261,800

In which bank did the loans exceed the deposits? By how many dollars?

15. Find the difference between the loans and deposits for each of the other banks.

16. Find the difference between the total loans and the short-time loans in the case of each bank.

17. From the following table find the increase in the number of post offices from 1865 to 1880. From 1865 to 1908.

FISCAL YEAR.	NUMBER OF POST OFFICES.	EXTENT OF POST ROUTES IN MILES.	REVENUE OF THE DEPARTMENT.	EXPENDITURE OF THE DEPARTMENT.
1865	20,550	142,340	$14,556,159	$13,694,728
1870	28,492	231,232	19,772,221	23,998,837
1875	35,547	277,873	26,791,360	33,611,309
1880	42,989	343,888	33,315,479	36,542,804
1885	51,252	365,251	42,560,844	49,533,150
1890	62,401	427,991	60,882,097	65,930,717
1895	70,064	456,026	76,983,128	86,790,172
1896	70,360	463,313	82,499,208	90,626,296
1897	71,022	470,032	82,665,462	94,077,242
1898	73,570	480,462	89,012,618	98,033,523
1899	75,000	496,948	95,021,384	101,632,160
1900	76,688	500,982	102,354,579	107,740,268
1901	76,945	511,808	111,631,193	115,554,920
1902	76,215	507,540	121,848,047	124,785,697
1903	74,169	506,268	134,224,443	138,784,488
1904	71,131	406,818	143,582,624	152,362,117
1905	68,131	486,805	152,826,585	167,399,169
1906	65,600	478,711	167,932,782	178,449,778
1907	62,663	463,406	183,585,005	190,238,288
1908	61,158	450,738	191,478,662	208,351,886

18. Make and solve five similar problems.

19. Make and solve similar problems about the extent of post routes.

20. Make and solve problems about the amounts of revenue. Also about the differences between revenue and expenditure.

MULTIPLICATION

271. Multiplication is a short method of finding the result of repeating a number as an addend.

Example: What is the cost of 825 blankets at $6 each? The solution is evidently 825 × $6.

$$\begin{array}{r} 6 \\ \underline{825} \\ 30 \\ 120 \\ \underline{4800} \\ 4950 \end{array}$$

Since the multiplication at the left is awkward, we use the plan at the right.

We can do this because the product of two factors is the same no matter which factor is taken as the multiplier.

$$\begin{array}{r} 825 \\ \underline{6} \\ 4950 \end{array}$$

1. In the example below, which number is the multiplicand? The multiplier? The product?

$$\begin{array}{rr} & 8.421 \\ & \underline{7.63} \\ .03 \times 8.421 = & .25263 \\ .6 \times 8.421 = & 5.0526 \\ 7 \times 8.421 = & \underline{58.947} \\ & 64.25223 \end{array}$$

272. *The decimal point in the product is located by setting off from the right as many decimal places as there are in both multiplicand and multiplier.*

The work of multiplication should be tested by repeating the calculation.

WRITTEN EXERCISES

1. What is the cost of 963 chairs at $5 each?
2. Find the cost of 1,045 stands at $11 each.
3. Find the cost of 964 carpet sweepers at $3 each.
4. Find the cost of 733 rugs at $4 each.

5. Find the numbers that would fill out the last column in this table:

NUMBER.	BOOKS.	PRICE (EACH).	TOTAL COST.
500	Spellers	$.15	( )
250	Readers	.35	( )
365	Classics	.60	( )
783	Latin Grammars	1.10	( )
1260	Dictionaries	3.50	( )
876	Geographies	1.25	( )
253	Physics	1.30	( )
765	Geometries	1.25	( )
980	Algebras	1.12	( )
1343	Chemistries	1.18	( )
25500	Primers	.45	( )
54200	Grammars	.75	( )

Multiply:

6. 6.30 by 86.
7. 1280 by 17.
8. 15.07 by 29.
9. 46.3 by 10.8.
10. 19.7 by 999.

11. 724 by 86.
12. 8.46 by 1.33.
13. 943 by 6.09.
14. 712 by 50.8.
15. 8.63 by 8.77.

16. 402 by 4.2.
17. 87.5 by .29.
18. 300 by 7.6.
19. 41.8 by .27.
20. 609 by 4.1.

Short Processes of Multiplication

ORAL EXERCISES

1. How is an integer multiplied by 10? By 100? By 1000?

2. How is a decimal multiplied by 10? By 100? By 1000?

3. What part of 10 is 5? What is a short way to multiply a whole number by 5?

4. What part of 100 is 25? What is a short way to multiply a whole number by 25?

5. 75 is what part of 100? If two ciphers are annexed to a whole number, and $\frac{3}{4}$ of the result is taken, by what is the number multiplied?

6. 125 is what part of 1000? How may a whole number be multiplied easily by 125? Illustrate the method. How may a whole number be multiplied easily by 1.25?

7. Multiply by 10: 18; 180; 1,800; 63; 630; 876; 806.

8. Multiply by 100: 6; 60; 600; 13; 130; 106; 66; 660.

9. Multiply by 1000: 3; 4; 5; 50; 23; 20; 145; 1,635.

10. Multiply by 10: .3; 9.4; 3.05; 16.67; 1.009; 90.09.

11. Multiply by 100: .06; .005; 6.05; 12.005; 12.635.

12. Multiply by 1000: .008; 2.005; 2.63; 46.73; 965.5.

13. Multiply by 5 without writing the figures:
36; 18; 28; 49; 70; 136; 43; 476.

14. Multiply by 25 without writing the figures:
16; 17; 24; 40; 13; 19; 116.

15. Multiply by 75 without writing the figures:
12; 16; 40; 24; 20; 28; 56.

Find the cost of:

16. 16 lb. of coffee at 25 cts. a pound.

17. 36 cans of baking powder at 25 cts. a can.

18. 84 cans of corn at 5 cts. each.

19. 124 bottles of olives at 25 cts. a bottle.

20. 82 doz. oranges at 50 cts. a dozen.

21. 24 cans of cherries at 25 cts. a can.

22. 136 Arithmetics at 45 cts. each.

23. 48 chairs at $1.25 each.

24. 96 caps at 75 cts. each.

25. 112 doz. teaspoons at $1.25 a dozen.

26. 320 pairs of skates at $1.25 a pair.

27. 444 pairs of slippers at 75 cts. a pair.

273. 9 = 10 minus what number? How many times a number must be taken from 10 times the number to leave 9 times the number? State a short way to multiply a number by 9.

$$\text{Thus, } 9 \times 181 = \left\{\begin{array}{r} 1810 \\ 181 \\ \hline 1629 \end{array}\right.$$

99 = 100 minus what number? How many times a number must be taken from 100 times the number to leave 99 times the number? State a short way to multiply by 99.

WRITTEN EXERCISES

1. Multiply by 9: 23; 54; 76; 97.

2. Find the cost of 47 packages of macaroni at 9 cts. each.

3. Find the cost of 381 packages of oatmeal at 9 cts. each.

4. Multiply by 99: 14; 27; 59; 84.

5. Find the cost of 52 shirt-waists at 99 cts. each.

6. Find the cost of 16 pairs of curtains at 99 cts. a pair.

Suggestion: 99 = 100 minus what number?

7. Multiply by 999: 4; 51; 84; 26; 718.

8. Multiply by 98: 7; 15; 36; 49; 72.

Suggestion: 98 = 100 minus what number?

9. Find the cost of 12 horses at $98 each.

10. Multiply by 199: 3; 9; 52; 69; 81; 46.

11. Find the cost of 42 bbl. of flour at $4.99 a barrel.

12. Multiply by 198: 6; 12; 93; 47.

13. Find the cost of 7 rugs at $1.98 each.

14. Multiply by 298: 3; 16; 39; 57.

15. Find the cost of 16 doz. handkerchiefs at $2.98 a dozen.

16. Find the cost of 17 bags of sugar at $1.99 a bag.

17. Multiply by 399: 26; 52; 3; 17.

18. How much did a milliner receive for 24 hats at $6.98?

274. When several factors are to be combined, the amount of work to be done sometimes depends upon the order in which the factors are taken.

EXAMPLE: Multiply $5 \times 7 \times 6$.

Which of the following is the easiest to do without writing if we begin at the left and multiply by the second number and then by the third:

(1) $5 \times 7 \times 6 = (\quad)$, or (2) $7 \times 6 \times 5 = (\quad)$,
or (3) $5 \times 6 \times 7 = (\quad)$?

ORAL EXERCISES

Multiply:

1. $2 \times 9 \times 5$.
2. $11 \times 18 \times 5$.
3. $2 \times 17 \times 3 \times 5$.
4. $7 \times 4 \times 5$.
5. $5 \times 28 \times 3$.
6. $8 \times 13 \times 5$.
7. $14 \times 4 \times 5$.
8. $82 \times 2 \times 5$.
9. $25 \times 84 \times 8$.

275. For rapid oral work, it is often easier to multiply successively by the factors of the multiplier.

EXAMPLE: Multiply 14×15.

From left to right, which is the easier to do without writing:

(1) $14 \times 15 = (\quad)$, or (2) $2 \times 15 \times 7 = (\quad)$?

ORAL EXERCISES

1. 16×45.
2. 24×50.
3. 38×50.
4. 25×14.
5. 18×35.
6. 36×50.
7. 150×18.
8. 162×15.
9. 12×45.
10. 22×45.
11. 35×16.
12. 25×84.

Solve:

13. A teamster hauled 24 cu. ft. of earth at a load. How many cubic feet did he haul in 45 loads?

14. There are 12 pairs of shoes in a case. How many pairs are there in 25 cases?

15. Which is the easier to multiply, 15×18 or $5 \times 18 \times 3$?

276. Example: What is the cost of 468 books at 15 cts. each?

Solution: (1) Since, to multiply by 15 is to multiply by 1½ times 10,

(2) Then, 468 × 15 = 468 × 1½ × 10 = 7,020.

What part of a number is added to the number to multiply it by 1½? How is a number multiplied by 10?

ORAL EXERCISES

Tell why the following are true, with any number as multiplicand:

TO MULTIPLY BY:	ADD TO MULTIPLICAND:	MOVE DECIMAL POINT:
1. 15.	½ of itself.	1 place to the right.
2. 13⅓.	⅓ " "	1 " " " "
3. 133⅓.	⅓ " "	2 places " "
4. 112½.	⅛ " "	2 " " " "

TO MULTIPLY BY:	SUBTRACT FROM MULTIPLICAND:	MOVE DECIMAL POINT:
5. 87½.	⅛ of itself.	2 places to the right.
6. 66⅔.	⅓ " "	2 " " " "

WRITTEN EXERCISES

What is the cost of:

1. 96 yd. of silk at $1.12½ per yard?

2. 56 yd. of carpet at 87½ cts. per yard?

3. 72 bu. of potatoes at 66⅔ cts. per bushel?

4. 66 yd. of ribbon at 13⅓ cts. per yard?

5. 45 yd. of lace at 15 cts. per yard?

6. 28 yd. of velvet at $1.33⅓ per yard?

7. 14 yd. of gingham at 13⅓ cts. per yard?

8. 64 screw drivers at 87½ cts. each?

9. 25 rolls of wall paper at 87½ cts. per roll?

Powers

277. A **power** of a number is the result of taking the number two or more times as a factor.

Thus, 2 × 2, or 4, is the second power of 2; 5 × 5, or 25, is the second power of 5; 4 × 4 × 4, or 64, is the third power of 4, and so on.

ORAL EXERCISES

1. What other name has the second power? The third power?
2. What power of 3 is 3 × 3 × 3 × 3, or 81?
3. What power of 5 is 5 × 5 × 5 × 5 × 5, or 3,125?
4. What is the square, or second power, of 3? Of 5?
5. What is the cube, or third power, of 2? Of 3? Of 5?
6. What is the area of a square whose side is 4 in.?
7. What is the volume of a cube whose edge is 4 in.?
8. 100 is what power of 10? 1000 is what power of 10?

278. The product of two like factors, as 2 × 2, 3 × 3, 4 × 4, 5 × 5, is written for brevity 2^2, 3^2, 4^2, 5^2. Similarly, 2 × 2 × 2, 3 × 3 × 3, 4 × 4 × 4, and so on, may be written 2^3, 3^3, 4^3, and so on.

279. Powers of 10 larger than 1000 are expressed more simply by the use of exponents.

Thus, 10,000 is 10^4; 50,000 is 5×10^4; 85,000 is 85×10^3.

280. The small figure written above is called an **exponent** and shows how often the number above which it stands is taken as factor in the product.

ORAL EXERCISES

1. What exponent indicates the square of a number?

2. What number is expressed by each of the following: 1^2; 2^2; 3^2; 4^2; 5^2; 6^2; 7^2; 8^2; 9^2; 10^2?

3. If s is the side of a square, what product represents the area of the square?

4. What exponent is used to indicate the cube of a number? Indicate the cube of: 5; 6; 7; 8; 10; 15.

5. If e is the edge of a cube, what product represents the volume of the cube?

6. What is the number expressed by: 1^3; 2^3; 3^3; 4^3; 5^3?

WRITTEN EXERCISES

1. What is the value of: $2^3 \times 3^2$? 5×4^3? $3^2 \times 10^3$?

2. Write the factors of each of the following numbers, and express any powers by the use of exponents: 25; 50; 75; 18; 27; 63; 72; 90.

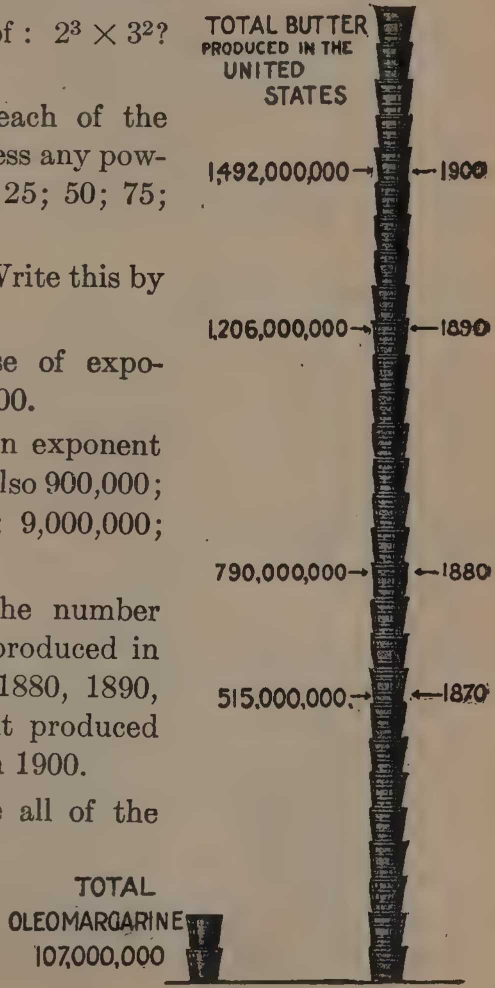
3. $600 = 6 \times 10 \times 10$. Write this by use of an exponent.

4. Write similarly by use of exponents: 300; 900; 1000; 5,000.

5. Write 300,000, using an exponent to express the power of 10. Also 900,000; 950,000; 895,000; 1,000,000; 9,000,000; 129,000,000; 1,236,000,000.

6. The diagram shows the number of million pounds of butter produced in the United States in 1870, 1880, 1890, and 1900. Read the amount produced in 1870; in 1880; in 1890; in 1900.

7. Using exponents, write all of the numbers in the diagram expressing amounts of butter; also that relating to oleomargarine.



DIVISION

281. **Division** is the inverse of multiplication. It is the process by which, if the product and one factor are given, we find the other factor.

282. In every division the divisor multiplied by the quotient and increased by the remainder, if any, equals the dividend; or

Divisor × Quotient + Remainder = Dividend.

Thus, in 47 ÷ 5, the quotient is 9, and the remainder 2; and 47 = 9 × 5 + 2.

ORAL EXERCISES

Find the numbers to fill the blanks:

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
Dividend:	28	46	72	( )	( )	40
Divisor:	4	( )	12	2.5	6	8
Quotient:	( )	23	( )	8	9.5	( )

	7.	**8.**	**9.**	**10.**	**11.**	**12.**	**13.**
Dividend:	46	52	( )	( )	96	128	256
Divisor:	8	( )	6	7	( )	( )	16
Quotient:	( )	4	18	12	12	25	( )
Remainder:	( )	0	0	4	0	3	( )

WRITTEN EXERCISES

Perform the following divisions:

1. 17.55 ÷ .39.

2. 3.465 ÷ 6.3.

3. 2846 ÷ 24.

4. 1599 ÷ 39.

5. 3216.84 ÷ 16.

6. 96048 ÷ 48.

7. 190.9 ÷ .83.

8. 284.9 ÷ 3.7.

9. 96064 ÷ 32.

10. 6334 ÷ 31.

11. 7683 ÷ 29.

12. 6.25 ÷ 1.25.

13. 1.909 ÷ 2.3.

14. 64.48 ÷ .31.

15. 14641 ÷ 11.

16. 28346 ÷ 29.

17. 186.75 ÷ 25.

18. 70.185 ÷ 3.5.

283. *Multiplying or dividing both dividend and divisor by the same number does not change the quotient.*

Example: In a recent year the pay-roll of the city of New York contained the names of 43,000 persons. The total amount of the pay-roll was about $38,000,000. Find within one cent the average compensation of each employee.

Divide both dividend and divisor by 1000 to simplify the work.

The decimal point of the quotient will stand over the decimal point of the dividend, if the first figure of the quotient is placed over the last figure of the first partial dividend. Show that this plan is followed in the work at the right.

How may the result be tested?

$$\begin{array}{r}
883.72 \\
43\overline{)38000.000} \\
\underline{344} \\
360 \\
\underline{344} \\
160 \\
\underline{129} \\
310 \\
\underline{301} \\
90 \\
\underline{86}
\end{array}$$

WRITTEN EXERCISES

Find the price of each to the nearest cent if:

1. 36 sheep cost $107.

2. 48 chairs cost $60.

3. 160 baskets cost $157.

4. 74 books cost $27.

5. 52 hats cost $93.

6. 23 watches cost $195.

7. 24 handkerchiefs cost $1.92.

8. 17 bbl. of flour cost $99.

9. 16 cans of corn cost $1.24.

10. 46 pairs of gloves cost $51.

11. 28 ostrich plumes cost $94.

12. 15 mandolins cost $146.25.

Find the average cost per mile of each ship canal; test your work:

	CANAL.	LENGTH.	COST.
13.	Suez	90 miles	$100,000,000.
14.	Russian	16	10,000,000.
15.	Manchester	35.5	75,000,000.
16.	Kaiser Wilhelm	61	40,000,000.
17.	Elbe and Trave	41	6,000,000.

18. The table shows approximately the population and assessed value of property in New York City from 1870 to 1900:

YEAR.	POPULATION.	ASSESSED VALUE.
1870..................	1,500,000	1,300,000,000
1880..................	1,900,000	1,400,000,000
1890..................	2,500,000	2,100,000,000
1900..................	3,400,000	3,600,000,000

Find to the nearest cent the value of the property for each person (*per capita*).

19. The total consumption of wool, as shown in the table, for any year when divided by the consumption *per capita* gives the total population for the corresponding year. Find the total population of the United States for each year mentioned. Omit the decimal part of each result.

YEAR.	TOTAL CONSUMPTION.	CONSUMPTION OF WOOL PER CAPITA.
1840...	76,710,602	4.49
1850...	129,354,978	5.58
1860...	213,832,799	6.80
1870...	305,770,597	7.93
1880...	427,184,530	8.52
1890...	548,167,332	8.75
1900...	483,865,236	6.34

Short Processes of Division

ORAL EXERCISES

1. How is a number ending in zero divided by 10? In an integer where is the decimal point supposed to be?

2. How is a number ending in two zeros divided by 100?

3. How many places is the decimal point moved to the left in dividing a number by 10? In dividing by 100? In dividing by 1000?

4. State the quotient of each of the following numbers when divided by 10: 20; 70; 50; 130; 180; 1,900.

5. State the quotient when each of the following numbers is divided by 100: 200; 400; 700; 1,300; 19,000.

6. Divide by 10: 16; 130; 1.3; 12.5; 7.25.

7. Divide by 100: 160; 26.3; 7.60; 183.6; 1,900.7.

8. Divide by 1000: 285.6; 1,234.5; 10,000; 10.36.

Divide by 20: **9.** 63.4. **11.** 1.75. **13.** 5.6.
10. 18.4. **12.** 40.3. **14.** .104.

Divide by 30: **15.** 2.7. **17.** 1.96. **19.** 38.52.
16. .69. **18.** 30.3. **20.** 27.06.

Divide by 200: **21.** 64.8. **23.** 126. **25.** 7.26.
22. 19.1. **24.** 420. **26.** .42.

Divide by 600: **27.** 2.46. **29.** 720. **31.** 82.2.
28. 954. **30.** 660. **32.** 17.4.

284. Since $15 = \frac{3}{2} \times 10$, to divide by 15 is to multiply by $\frac{2}{3}$ and divide by 10. To multiply by $\frac{2}{3}$ is to subtract $\frac{1}{3}$ of the multiplicand.

Example: How many sewing machines at $15 each can be bought for $285?

Solution: (1) $\frac{1}{3}$ of 285 = 95. (2) 285 − 95 = 190.
(3) 190 ÷ 10 = 19.

The solution is given merely to indicate the mode of thinking. The problems are to be solved without written calculation.

ORAL EXERCISES

Divide by 15: **1.** 4.5. **3.** 1.05. **5.** 7.5.
2. 46.5. **4.** 900. **6.** 5.25.

7. What part of a number taken from it leaves $\frac{8}{9}$ of the number?

8. How many yards of silk at $1.12½ a yard can be bought for $900?

Solution: (1) $1.12\frac{1}{2} = \frac{9}{8}$ of 1. (2) $900 \div \frac{9}{8} = 900 \times \frac{8}{9} = 800$.

ORAL EXERCISES

Tell why the following are true, with any number as dividend:

TO DIVIDE BY:	SUBTRACT FROM DIVIDEND:	MOVE DECIMAL POINT:
1. 15.	⅓ of itself.	1 place to the left.
2. 13⅓.	¼ " "	1 " " " "
3. 133⅓.	¼ " "	2 places to the left.
4. 112½.	⅑ " "	2 " " " "

TO DIVIDE BY:	ADD TO DIVIDEND:	MOVE DECIMAL POINT:
5. 87½.	⅐ of itself.	2 places to the left.
6. 66⅔.	½ " "	2 " " " "

WRITTEN EXERCISES

Find how many of each article can be bought for the amount given:

1. Yards of carpet at $.87½ per yard........	$ 70.00.
2. Atomizers at 33⅓ cts. each...............	40.00.
3. Concert phonographs at $66⅔............	600.00.
4. Parlor phonographs at $12½ each........	350.00.
5. Teakettles at $1.33⅓ each...............	60.00.
6. Bookcases at $15 each..................	855.00.
7. Pairs of dumb-bells at $1.12½ per pair...	20.25.
8. Tennis rackets at $1.33⅓ each..........	24.00.
9. Tents at $13⅓ each......................	120.00.
10. Dictionaries at $15 each................	135.00.
11. Chairs at $1.33⅓ each..................	80.00.
12. Land at $66⅔ an acre..................	10,000.00.
13. Wagons at $87½ each....................	6,300.00.
14. Wood at $1.12½ a cord..................	27.00.

FACTORS AND MULTIPLES

Divisibility

285. All integers divisible by 2 are called **even** numbers. All integers that are not even are called **odd.**

286. Various tests for divisibility are stated in the following table:

A number is divisible:

By 2, if it ends in 0, 2, 4, 6, 8.
By 3, if the sum of its digits is divisible by 3.
By 4, if the number represented by the two digits at the right is divisible by 4.
By 5, if it ends in 0 or 5.
By 6, if divisible by 2 and by 3.
By 8, if the number represented by the three digits at the right is divisible by 8.
By 9, if the sum of its digits is divisible by 9.
By 10, if it ends in 0.
By 12, if it is divisible by 3 and by 4.

For 7 and 11, actual division is the best test.

ORAL EXERCISES

Which of the following are divisible by 3? By 4?

1. 128.
2. 585.
3. 276.
4. 3,284.
5. 648.
6. 950.
7. 87,000.
8. 11,656.

Which of the following could be set in 3 equal rows? In 4?

9. 126 cabbage plants.
10. 872 fruit trees.
11. 126 strawberry plants.
12. 432 raspberry bushes.
13. 1,706 tomato plants.
14. 1000 pansies.
15. 2,103 onion sets.
16. 2,468 rose bushes.

17. Name some multiples of 5. Is 45 divisible by 5? Is 50? 73? 90? 99? 109? 160?

18. Which of the following numbers are divisible by 2? By 3? By 4? By 5?

16 21 48 35 52 17 64
128 400 536 279 1,436 3,687 1,904

19. Which of the numbers above are divisible by 6? By 12?

Prime Factors

287. An integer that is divisible only by itself and 1 is called a **prime number.** If divisible by any other number it is called a **composite number.**

288. A factor that is a prime number is called a **prime factor.**

Every composite number can be expressed as a product of prime factors.

The prime factors may be found by dividing the number by any prime factor, then by dividing the quotient similarly by any of its prime factors, and so on, until a prime quotient is reached. The original number is the product of all the divisors and the last quotient. Thus, $60 = 2 \times 2 \times 3 \times 5$.

WRITTEN EXERCISES

Find the prime factors of:

1. 46.
2. 108.
3. 125.
4. 38.
5. 2,240.
6. 4,030.
7. 211.
8. 8,451.
9. 111.
10. 144.
11. 625.
12. 51.
13. 1,896.
14. 777.
15. 1,008.
16. 1,916.
17. 37.
18. 276.
19. 605.
20. 1000.
21. 1,900.
22. 813.
23. 6,990.
24. 2,433.
25. 9,333.
26. 1,964.
27. 1,728.
28. 1,600.
29. 1,908.
30. 1,452.
31. 4,771.
32. 7,007.

33. Express all the numbers to 100 as products of prime factors. Make a list of the prime numbers below 100.

Common Factors

289. A factor or divisor common to two or more integers is called a **common factor** or **common divisor** of the integers.

Thus, 3 is a common divisor of 9, 15, 33, and 60.

290. The largest integer that will divide without a remainder two or more given integers is called the **greatest common divisor** of those integers.

Thus, 12 is the greatest common divisor of 24, 84, and 360 because no larger integer is a common divisor of them all.

291. *To find the greatest common divisor of two or more numbers, divide the numbers by their smallest common prime factor. Divide the quotients obtained by their smallest common prime factor, and repeat the process until the quotients have no common prime factor. The greatest common divisor is the product of the prime divisors thus found.*

The greatest common divisors of large numbers not easily factored are not required in elementary arithmetic.

Example: Find the g. c. d. of 24, 84, and 360.

$$\begin{array}{r|rrr} 2 & 24 & 84 & 360 \\ \hline 2 & 12 & 42 & 180 \\ \hline 3 & 6 & 21 & 90 \\ \hline & 2 & 7 & 30 \end{array}$$

The numbers are first divided by 2, the smallest common prime factor.

The quotients are then divided by 2, their smallest common prime factor.

The new quotients are then divided by 3, their smallest common prime factor.

The quotients thus obtained have no common factor. The g. c. d. is the product of the successive prime divisors, or 2 × 2 × 3 = 12.

WRITTEN EXERCISES

Find the g. c. d. of:

1. 26, 52, 130.
2. 18, 108, 756.
3. 70, 490, 238.
4. 38, 95, 209.
5. 36, 60, 1728.
6. 45, 265, 85.
7. 144, 1728, 84.
8. 90, 990, 195.
9. 25, 175, 625.
10. 21, 105, 315.
11. 49, 700, 749.
12. 85, 1000, 585.

Common Multiples

292. A number that is divisible by two or more numbers is called a **common multiple** of those numbers. The smallest number that can be divided by each of them is called their **least common multiple (l. c. m.).**

12, 24, and 36 are common multiples of 4, 6, and 12; and 12 is the least common multiple.

293. *To find the least common multiple of two or more numbers, factor the given numbers; multiply all of their prime factors, taking each one the greatest number of times that it occurs in any of the given numbers.*

EXAMPLES:

1. Find the l. c. m. of 28 and 42.

$28 = 2 \times 2 \times 7$, and $42 = 2 \times 3 \times 7$. The l. c. m. of 28 and 42 is $2 \times 2 \times 3 \times 7 = 84$. The l. c. m. contains 2 twice, because it is found in 28 twice. It contains 3 and 7 each once, because they occur once only in one or both of the given numbers.

2. Find the l. c. m. of 24, 56, 108.

$24 = 2 \times 2 \times 2 \times 3.$
$168 = 2 \times 2 \times 2 \times 3 \times 7.$
$108 = 2 \times 2 \times 3 \times 3 \times 3.$

The smallest number divisible by 24 must contain the factors $2^3 \times 3$. To be divisible by 168 it must also contain the factor 7. To be divisible by 108 it must contain, in addition to the above, the factor 3^2. Therefore the l. c. m. of 24, 168, and 108 is $2^3 \times 3 \times 7 \times 3^2 = 2^3 \times 3^3 \times 7$, or 1512.

WRITTEN EXERCISES

Find the l. c. m. of:

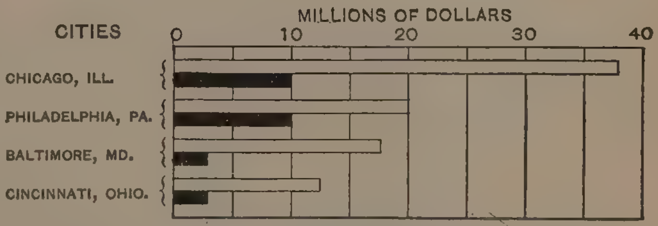
1. 24, 56.
2. 36, 56, 96.
3. 22, 121, 77.
4. 46, 18, 108.
5. 18, 15, 27.
6. 55, 165, 121.
7. 39, 169, 85.
8. 12, 144, 1,728.
9. 256, 196, 180.
10. 25, 125, 625.
11. 360, 120, 90.
12. 48, 144, 1,728.

GRAPHICAL PROBLEMS

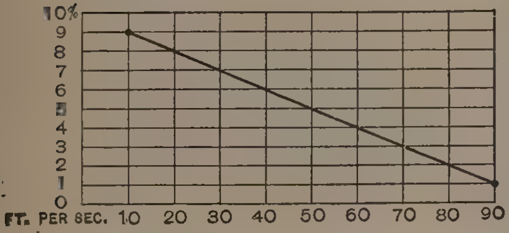
294. Number pictures, or graphs, enable us to show relations between given numbers.

ORAL EXERCISES

1. In the diagram the white strips represent the number of million dollars' worth of men's clothing recently manufactured during one year in the cities named; the black strips represent similarly the value of women's clothing. What was the value of Chicago's product of each kind? Of Philadelphia's? Of Baltimore's? Of Cincinnati's? Of Chicago's and Philadelphia's together?



2. When water flows through a ditch, some of it soaks into the soil. For soil of a certain character the diagram shows what per cent of the water entering the ditch soaks into the soil during the passage through the ditch at different rates of flow.



For example, when 10 cu. ft. of water enter during a second, 9% of it soaks into the soil.

What per cent is lost when 20 cu. ft. enter per second? 40 cu. ft.? 80 cu. ft.? 100 cu. ft.?

3. When 10 cu. ft. enter per second, how many cubic feet soak into the soil? How many cubic feet are delivered at the other end of the ditch per second? Per minute?

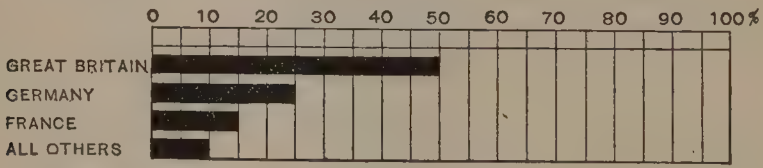
4. Make and solve 5 similar problems.

WRITTEN EXERCISES

1. Represent by a diagram as explained on page 257:

CITY.	MEN'S CLOTHING.	WOMEN'S CLOTHING.
New York City........	$110,000,000	$105,000,000
Boston...............	15,000,000	5,000,000
St. Louis.............	10,000,000	5,000,000

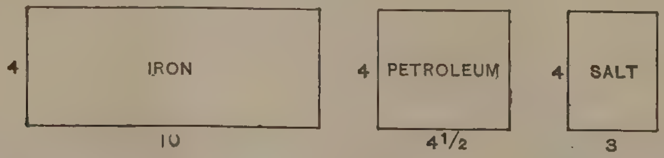
2. The following diagram shows by percentages how the 6,450,000 bales of cotton exported by the United States in one year were distributed:



How much was shipped to Great Britain? To France? To Germany? To all other places? The amount exported to Germany was what part of the amount shipped to Great Britain? Make and answer three similar questions from the diagram.

3. Indicate graphically that all of the spindles spinning the cotton were divided thus: Great Britain, 47 million; the European Continent, 34 million; the United States, 22 million; and the East Indies, 5 million.

4. These rectangles represent for a recent year the world's production of the minerals named:



Compute the areas of these figures. Each number found is the number of million tons of the mineral named. What amount of each mineral was produced?

5. In a recent year the acreage of corn, wheat, and oats and the number of bushels grown in the United States were:

	ACRES.	BUSHELS.	BUSHELS PER ACRE.
Corn.......	83,328,000	2,105,103,000	( )
Wheat.....	42,504,000	( )	12.3
Oats.......	( )	809,280,000	30.0

Find the numbers to fill the blanks in the table.

6. If 1 sq. in. represents 1000 acres, how long must a rectangle 2 ft. wide be to represent the acreage of corn? Of wheat? Of oats?

7. Find the average yield of potatoes per acre from the table for each group of states and for the whole country:

GROUP OF STATES.	ACRES.	TONS OF POTATOES.
New England.............	180,000	505,000
Middle..................	410,000	1,440,000
Southern................	240,000	420,000
Central.................	1,710,000	2,710,000
Pacific.................	110,000	290,000

8. Using $\frac{1}{8}$ in. to represent 5,000 tons, draw on the blackboard parallel lines to represent these yields.

9. Using 1 in. to represent a million pounds, draw parallel lines to represent the yield of tobacco for each state below. Represent the acreage similarly, letting 1 in. equal 10,000 acres.

STATES.	POUNDS.	ACRES.
Virginia...............	73,000,000	110,000
North Carolina.........	70,000,000	120,000
Wisconsin..............	50,000,000	40,000
Tennessee..............	34,000,000	43,000
Connecticut............	25,000,000	14,000

REVIEW

ORAL EXERCISES

1. What is meant by numeration? By notation? What is meant by Roman notation?

2. Name the four periods into which integers of 12 places may be separated for reading.

3. Name the two periods into which decimals of 6 places may be separated for reading.

4. State and illustrate a way of testing the work of addition. State another way.

5. State and illustrate a way of testing the work of subtraction.

6. State and illustrate two short processes in the work of multiplication.

7. State and illustrate two short processes in the work of division.

8. How may the number of decimal places in a product be determined? In a quotient?

9. State five tests of divisibility.

10. Define greatest common divisor. Also, least common multiple.

11. What is an exponent? Illustrate the use of exponents.

12. State a principle in division that enables us to make the divisor a whole number before performing the division.

13. What is meant by "casting out nines"? What use can be made of this process?

14. Define: odd number; even number; prime number· composite number.

15. How is the greatest common divisor of two or more numbers found? How is their least common multiple found?

16. How is division tested?

WRITTEN EXERCISES

1. Write by use of exponents: $3 \times 3 \times 2$; $3 \times 3 \times 2 \times 2$; $3 \times 3 \times 3$.

2. Write as a product of factors, using exponents where applicable: 50; 200; 1000; 5,000; 8,000.

3. 9,945 tons of iron ore were unloaded from the lake steamer Wolvin in $4\frac{1}{2}$ hours. How many tons were unloaded per hour?

4. A large cattle ranch in Texas contains 700,000 acres. What is the land worth at $7.85 an acre?

5. The table shows the number and value of the railway cars manufactured in the United States in a recent year:

KIND OF CAR.	NUMBER.	TOTAL VALUE.
Passenger coach	181	$957,526
Sleeping car	194	2,767,061
Dining car	37	404,503
Freight car	116,590	62,161,013

Find the average value of a car of each kind (to the nearest $100). Also to the nearest cent.

6. The woodland area of the United States is about $\frac{1}{3}$ of the total area, 3,025,600 sq. mi. How many square miles are woodland?

7. How many acres of woodland are there in the United States? (640 acres = 1 sq. mi.)

8. The Government and many of the states have established forest preserves as follows:

1891	13,457,680 acres	1898–1901	7,050,089 acres
1893	4,443,000 acres	1902–1904	15,995,196 acres

Find the total acreage of these preserves. How many more acres were established between 1902 and 1904 than in 1891? How many more between 1898 and 1901 than in 1893?

9. In 1900 there were 33,035 establishments manufacturing from lumber $611,611,524 worth of products. What was the average output of each factory?

10. These factories employed 283,260 workmen at an average wage of $370 a year. Find their total earnings.

11. In 1900 the lumber mills sawed 35,084,166,000 board feet, valued at $566,832,984; this was an increase of $\frac{1}{4}$ over the value of the product in 1890. What was that value?

12. It is estimated that trees containing 21,239,000,000 ft. of spruce are now standing in Maine and increasing at the rate of 600 million feet a year. If none are destroyed, how many feet will there be in 25 yr?

13. Taking the annual capacity of the Maine pulp mills to be 275 million feet, how much may be used for other purposes without exceeding the natural increase of the timber?

14. A farmer kept the following account of a field of corn:

EXPENSES PER ACRE.				RETURNS.	
Plowing.......	$2.50	Seeding......	$1.65	50 bu. at....	$.45
Harrowing.....	2.50	Cultivating...	4.50	Fodder.....	8.00
Fertilizing.....	.80	Harvesting...	8.00		

Find the total returns, expenses, and net profit per acre. What was the net profit from 25 acres of corn?

15. The street railways of New York City carried, in all, 1,036,000,000 passengers in a recent year. Taking the population as 3,400,000, what was the average number of times that each inhabitant used the street railways?

16. What is the cost of 7 yd. of cloth at $\$1.33\frac{1}{3}$ a yard?

17. A Washington lumber mill cuts 87 thousand feet of lumber every 10 hr. How many feet will it cut in 26 days of 10 hr. each? If an ordinary 10-room house requires 7 thousand feet of lumber, how many such houses will one day's product build?

18. Some orange groves net their owners $875 an acre. At this rate what is the profit on 55½ acres?

19. The value of the furniture manufactured annually in this country is $80,000,000. There are about 1,500 factories. What is the average output for each factory?

20. The large wheat farms of Minnesota and the Dakotas contain from 3,000 to 10,000 acres of land. How many bushels of wheat would a farmer harvest from 4,625 acres yielding 15 bu. an acre?

21. What would this wheat bring at 60 cts. a bushel?

22. The grain was threshed in the field by 10 threshing machines, each having a capacity of 400 bu. a day. How many days did it take to thresh the wheat?

23. The grain was loaded on cars carrying 900 bu. each. How many cars were required for the crop?

24. 4½ bu. of wheat are used to make a barrel of flour. How many barrels would this crop make?

25. A ranch had 100,000 cattle; its owner employed 300 cowboys. How many was this for every 1000 cattle?

26. For grazing, cattle are grouped in herds of from 125 to 200 each. How many herds of 125 each are there in 100,000 cattle?

27. What would 1,200 ponies for herding the cattle cost at $29.75 each?

28. How many cattle cars carrying 19 steers each are needed to carry 6,099 steers?

29. 6,099 steers averaged 800 lb. in weight and sold at 5 cts. a pound. How many dollars was this?

30. On a ranch in Kansas a blizzard froze $\frac{1}{24}$ of a herd of 15,000 cattle. How many were frozen?

31. About 20 million dollars' worth of beef is exported annually; of this the British Isles take .99. What is the value of the beef exported to all the rest of the world?

32. Find the cost of growing 5.5 acres of beets when the cost per acre of preparing the soil is \$1.55; of seed and planting, \$3.25; and of cultivating and care, \$31.

33. If it costs \$230.20 to grow and harvest a 5.5-acre crop of beets, what is the cost of harvesting alone?

34. If the beets from 1 acre sell for \$56.04 at \$4.38 a ton, what is the yield from each acre?

35. At this rate how many tons were grown on the 5.5 acres? What was the total profit? What was the profit on each acre?

36. If the total receipts for sugar, molasses, and pulp is \$11.10 per ton of beets, and if the total cost of these products is \$7.84 per ton, what is the profit to the manufacturer?

37. Represent graphically the coal production of the following countries in a recent year:

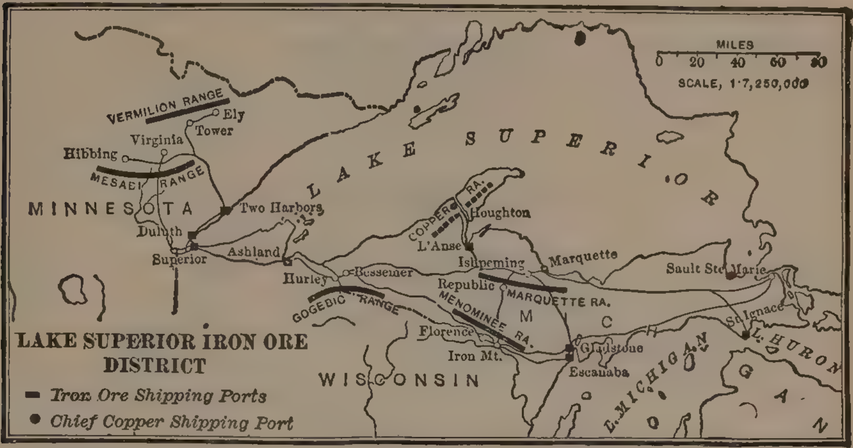
COUNTRY.	MILLION TONS.	COUNTRY.	MILLION TONS.
United States..	266	France........	32
British Isles....	223	Belgium.......	22
Germany......	153	Russia.........	16
Austria........	40	All others......	25

38. Taking the total of Exercise 37 to represent the world's annual consumption and 650,000,000,000 to be the total deposits, how many years would it take to exhaust the supply?

39. Taking the average production of honey per colony of bees to be 35 lb. a season and the selling price of the honey to be 9 cts., what is the value of the product of an apiary of 200 colonies? If it takes $\frac{1}{3}$ of this amount to meet incidental expenses and to pay the interest on the investment, what is the profit of the apiary for one season?

40. The area over which a colony of bees generally ranges is equal to a square 5 mi. on a side. How many acres is this?

41. A good bee pasture should average 1 honey-bearing plant per square foot. How many honey-bearing plants are there in 25 sq. mi. of such pasture?



42. The total production of iron ore in the Lake Superior district has reached 12,000,000 tons in one year. What was the average production for each of the five "ranges"?

43. Much of the ore is taken with steam shovels, averaging 1,800 tons a day. How many days would it take 25 such shovels to load 8,000,000 tons of ore?

44. What was the cost of mining this output at $23\frac{1}{4}$ cts. a ton?

45. The other 4,000,000 tons were mined underground. What was the cost of mining at $89\frac{1}{2}$ cts. a ton?

46. 3,000,000 tons have been shipped in one year from the ten ports of Lake Michigan and Lake Superior. Find the average tonnage per port.

47. The iron and steel produced in the United States in a recent year was 33 million tons. This was about $\frac{11}{27}$ of the amount produced in all countries. How many million tons were there in the world's production?

48. The amount of iron used by the people of the United States averages 300 lb. *per capita*. What was the consumption in 1900 when the population was 76 million?

CHAPTER IX

FRACTIONS

REDUCTION

295. One or more of the equal parts of a quantity, thought of as a unit, is called a **fraction.**

Thus, $\frac{1}{21}$, $\frac{2}{3}$, $\frac{8}{5}$, $\frac{11}{3}$ are fractions.

296. Any one of the equal parts of a unit, or thing, is called a **fractional unit.**

Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{100}$ are fractional units.

ORAL EXERCISES

1. How may $\frac{2}{4}$ be changed to $\frac{4}{8}$? $\frac{1}{3} = \frac{(\ )}{6}$? $\frac{2}{3} = \frac{(\ )}{6}$?

2. What effect is produced upon the value of a fraction by multiplying or dividing both terms by the same number?

3. How may a fraction be changed to an equal fraction with a different denominator?

4. $\frac{1}{2} = \frac{(\ )}{8}$?

5. $\frac{1}{3} = \frac{(\ )}{6}$?

6. $\frac{1}{2} = \frac{(\ )}{6}$?

7. $\frac{1}{3} = \frac{(\ )}{9}$?

8. $\frac{1}{2} = \frac{(\ )}{10}$?

9. $\frac{1}{2} = \frac{(\ )}{18}$?

10. $\frac{9}{12} = \frac{(\ )}{4}$?

11. $\frac{15}{24} = \frac{(\ )}{8}$?

12. 3 lb. and 4 oz. are how many ounces? In what unit are they expressed before adding?

13. How must numbers be expressed before they can be added or subtracted?

14. What is the fractional unit in each of the following fractions: $\frac{2}{3}$? $\frac{5}{7}$? $\frac{9}{11}$? $\frac{13}{27}$? $\frac{99}{100}$? $\frac{17}{34}$? $\frac{29}{58}$? $\frac{43}{49}$?

15. Fractions expressed in the same fractional unit are alike in which term?

16. What change must be made in fractions with different denominators in order to make them fractions having the same fractional unit?

297. *To reduce fractions to equal fractions having the least common denominator, find the l. c. m. of the given denominators and change each fraction to an equal fraction having the least common multiple as denominator.*

Thus, in $\frac{2}{3}$, $\frac{11}{12}$, $\frac{7}{15}$, the least common multiple of the denominator is 60.

$$60 \div 3 = 20 \qquad 60 \div 12 = 5 \qquad 60 \div 15 = 4$$

$$\frac{20 \times 2}{20 \times 3} = \frac{40}{60} \qquad \frac{5 \times 11}{5 \times 12} = \frac{55}{60} \qquad \frac{4 \times 7}{4 \times 15} = \frac{28}{60}$$

WRITTEN EXERCISES

Find the least common denominator of:

1. $\frac{3}{5}$, $\frac{6}{7}$, $\frac{1}{10}$.

2. $\frac{1}{7}$, $\frac{2}{3}$, $\frac{8}{9}$.

3. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{5}{6}$.

Reduce each of the following sets of fractions to fractions having the least common denominator:

4. $\frac{1}{8}$, $\frac{1}{15}$, $\frac{3}{4}$.

5. $\frac{2}{3}$, $\frac{5}{8}$, $\frac{9}{16}$.

6. $\frac{3}{40}$, $\frac{19}{20}$, $\frac{9}{10}$.

7. $\frac{11}{12}$, $\frac{7}{84}$, $\frac{5}{108}$.

8. $\frac{5}{12}$, $\frac{5}{144}$, $\frac{5}{1728}$.

9. $\frac{1}{16}$, $\frac{3}{7}$, $\frac{1}{4}$.

10. $\frac{3}{20}$, $\frac{5}{44}$, $\frac{2}{11}$.

11. $\frac{6}{35}$, $\frac{7}{70}$, $\frac{13}{140}$.

12. $\frac{1}{40}$, $\frac{1}{60}$, $\frac{1}{33}$.

13. $\frac{9}{40}$, $\frac{9}{500}$, $\frac{9}{1000}$.

14. $\frac{5}{9}$, $\frac{1}{8}$, $\frac{11}{12}$.

15. $\frac{5}{6}$, $\frac{7}{21}$, $\frac{9}{28}$.

16. $\frac{5}{12}$, $\frac{11}{96}$, $\frac{7}{104}$.

17. $\frac{10}{13}$, $\frac{2}{39}$, $\frac{6}{91}$.

18. $\frac{7}{105}$, $\frac{6}{84}$, $\frac{9}{21}$.

298. *To reduce an improper fraction to a whole or a mixed number, divide the numerator by the denominator.*

Thus, $\frac{24}{6} = 24 \div 6 = 4$. Also, $\frac{38}{5} = 38 \div 5 = 7\frac{3}{5}$.

299. *To reduce a mixed number to an improper fraction, multiply the whole number by the denominator of the fraction, add the numerator, and place the result over the given denominator.*

Thus, $9\frac{5}{8} = \frac{8 \times 9 + 5}{8} = \frac{77}{8}$.

WRITTEN EXERCISES

Reduce to a whole or a mixed number:

1. $\frac{18}{5}$.
2. $\frac{26}{15}$.
3. $\frac{85}{13}$.
4. $\frac{76}{25}$.
5. $\frac{32}{13}$.
6. $\frac{89}{17}$.
7. $\frac{63}{14}$.
8. $\frac{100}{19}$.
9. $\frac{48}{17}$.
10. $\frac{93}{13}$.
11. $\frac{121}{20}$.
12. $\frac{263}{25}$.
13. $\frac{840}{50}$.
14. $\frac{73}{33}$.
15. $\frac{201}{16}$.
16. $\frac{7\text{[illegible]}}{40}$.
17. $\frac{824}{75}$.
18. $\frac{624}{35}$.
19. $\frac{483}{75}$.
20. $\frac{960}{88}$.

Reduce to an improper fraction:

21. $5\frac{8}{9}$.
22. $15\frac{3}{4}$.
23. $26\frac{1}{2}$.
24. $73\frac{5}{8}$.
25. $19\frac{2}{7}$.
26. $41\frac{3}{5}$.
27. $18\frac{2}{3}$.
28. $45\frac{7}{8}$.
29. $19\frac{5}{8}$.
30. $46\frac{9}{10}$.
31. $7\frac{13}{15}$.
32. $8\frac{3}{25}$.
33. $72\frac{9}{50}$.
34. $86\frac{4}{75}$.
35. $12\frac{3}{35}$.

ADDITION AND SUBTRACTION

300. *Fractions having the same denominators are added or subtracted by adding or subtracting their numerators and placing the result over the common denominator. If the fractions have unequal denominators, reduce them to fractions having the l. c. d.*

WRITTEN EXERCISES

Add or subtract the following fractions as indicated:

1. $\frac{3}{11}+\frac{7}{55}$.
2. $\frac{12}{17}+\frac{9}{34}$.
3. $\frac{9}{64}+\frac{7}{16}$.
4. $\frac{12}{17}-\frac{9}{34}$.
5. $\frac{3}{11}-\frac{7}{55}$.
6. $\frac{9}{16}+\frac{11}{32}$.
7. $\frac{12}{23}+\frac{9}{46}$.
8. $\frac{5}{96}-\frac{1}{32}$.
9. $\frac{12}{23}-\frac{9}{46}$.
10. $\frac{11}{32}-\frac{3}{16}$.
11. $\frac{4}{9}+\frac{8}{45}$.
12. $\frac{7}{20}+\frac{19}{45}$.
13. $\frac{25}{72}-\frac{5}{36}$.
14. $\frac{4}{9}-\frac{8}{45}$.
15. $\frac{7}{16}-\frac{9}{64}$.
16. $\frac{4}{9}-\frac{8}{45}-\frac{4}{15}$.
17. $\frac{4}{5}+\frac{7}{8}-\frac{9}{16}$.
18. $\frac{6}{7}+\frac{25}{48}-\frac{8}{9}$.
19. $\frac{4}{5}+\frac{5}{8}-\frac{5}{16}$.
20. $\frac{7}{20}-\frac{3}{40}-\frac{7}{60}$.
21. $\frac{5}{3}-\frac{4}{9}+\frac{7}{12}$.
22. $\frac{25}{64}-\frac{3}{32}+\frac{5}{18}$.
23. $\frac{12}{15}+\frac{7}{30}-\frac{9}{10}$.
24. $\frac{3}{4}+\frac{1}{3}+\frac{5}{12}-\frac{2}{3}+\frac{5}{6}-\frac{7}{9}+\frac{4}{8}-\frac{3}{8}+\frac{7}{12}+\frac{8}{9}-\frac{3}{16}-\frac{1}{12}$.

Solve:

25. A banker owns $\frac{1}{16}$ of the stock of a certain bank, another banker owns $\frac{3}{8}$, and still another owns $\frac{1}{5}$. What part of the whole is not owned by these three men together?

26. A farm is $\frac{2}{3}$ of a mile on one side, $\frac{7}{8}$ of a mile on another side, $\frac{5}{6}$ of a mile on the third side, and $\frac{11}{12}$ of a mile on the fourth side. How far is it around the farm?

301. A number consisting of an integer and a fraction is called a **mixed number.**

302. *In adding or subtracting mixed numbers, the integers and fractions may be treated separately and the results combined, or the mixed numbers may first be reduced to improper fractions and then added or subtracted.*

Examples:

1. Add $17\frac{2}{5}$, $5\frac{1}{3}$, 10, $8\frac{1}{2}$.

Solution:

$$\begin{aligned} 17\tfrac{2}{5} &= 17\tfrac{12}{30} \\ 5\tfrac{1}{3} &= 5\tfrac{10}{30} \\ 10 &= 10 \\ 8\tfrac{1}{2} &= 8\tfrac{15}{30} \\ \hline \text{Sum} &= 40\tfrac{37}{30} = 41\tfrac{7}{30} \end{aligned}$$

If the fractions are expressed in different fractional units, what change must be made in the fractions? How is the work completed?

2. Subtract $5\frac{3}{4}$ from $17\frac{1}{4}$.

Solution:

$$\begin{aligned} 17\tfrac{1}{4} &= 16\tfrac{5}{4} \\ 5\tfrac{3}{4} &= 5\tfrac{3}{4} \\ \hline \text{Diff.} &= 11\tfrac{2}{4} = 11\tfrac{1}{2}. \end{aligned}$$

If the fraction in the minuend is less than the fraction in the subtrahend, what change must be made? Subtract the integers and fractions separately. How are the results combined?

WRITTEN EXERCISES

1. To what kind of fraction may a mixed number be changed? Change $19\frac{2}{3}$ to a fraction.

2. Change $7\frac{5}{8}$ and $15\frac{3}{4}$ to improper fractions and find their sum.

Add or subtract as indicated:

3. $\frac{9}{15} + 1\frac{1}{5}$.

4. $1\frac{1}{5} - \frac{9}{15}$.

5. $\frac{8}{9} + \frac{7}{45} + \frac{1}{3}$.

6. $\frac{8}{9} - \frac{7}{45} - \frac{1}{3}$.

7. $40\frac{3}{4} - 17\frac{5}{12}$.

8. $170\frac{5}{22} - 7\frac{7}{66}$.

9. $40\frac{3}{4} + 17\frac{5}{12}$.

10. $84\frac{17}{18} + 12\frac{5}{36}$.

11. $\frac{7}{25} + 3\frac{2}{5}$.

12. $3\frac{2}{5} - \frac{7}{25}$.

13. $\frac{11}{12} + \frac{3}{4} + \frac{7}{16}$.

14. $\frac{11}{12} - \frac{3}{4} - \frac{1}{16}$.

15. $52\frac{5}{8} - 12\frac{3}{16}$.

16. $84\frac{21}{29} - 7\frac{1}{58}$.

17. $75\frac{4}{5} + 1\frac{8}{45}$.

18. $7\frac{5}{8} + 120\frac{7}{16}$.

19. $\frac{9}{32} + 2\frac{7}{16}$.

20. $2\frac{7}{16} - \frac{9}{32}$.

21. $\frac{5}{9} + \frac{5}{27} + \frac{5}{36}$.

22. $\frac{25}{27} - \frac{5}{9} - \frac{5}{36}$.

23. $75\frac{4}{5} - 1\frac{8}{45}$.

24. $84\frac{17}{18} - 12\frac{5}{36}$.

25. $84\frac{21}{29} + 7\frac{1}{58}$.

26. $40\frac{23}{24} + 91\frac{35}{36}$.

27. $20\frac{8}{9} + 119\frac{7}{18}$.

28. $100\frac{9}{10} + 999\frac{17}{20}$.

29. $146\frac{2}{3} - 123\frac{5}{18}$.

30. $17\frac{1}{6} + \frac{5}{6} + 4\frac{5}{9}$.

31. $705\frac{11}{12} - 126\frac{3}{4} - \frac{2}{3}$.

32. $12\frac{6}{7} + 18\frac{3}{14} + \frac{9}{28}$.

33. $145\frac{2}{3} + 9\frac{1}{6} + 3\frac{4}{5}$.

34. $40\frac{3}{25} + 1\frac{4}{5} + 9\frac{7}{10}$.

35. From the sum of $5.86\frac{7}{8}$ and $1.45\frac{5}{16}$ take $2.93\frac{7}{32}$.

36. From a roll of cloth containing $17\frac{5}{8}$ yd., $9\frac{3}{5}$ yd. are cut. How many yards remain in the roll?

37. Two garments, one requiring $3\frac{4}{5}$ yd. and another $5\frac{2}{3}$ yd. are cut from a piece of cloth containing $10\frac{1}{2}$ yd. How many yards are left?

38. Five cars contained the following loads of coal:

$30\frac{5}{8}$ T. $40\frac{3}{10}$ T. $49\frac{4}{5}$ T. $36\frac{3}{4}$ T. $45\frac{1}{8}$ T.

Find the total weight of the 5 loads.

39. The prices of wheat for 6 days were:

Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
$86\frac{1}{8}$¢	$85\frac{3}{8}$¢	$88\frac{3}{4}$¢	$90\frac{5}{8}$¢	89¢	$87\frac{7}{8}$¢

On which day was it the lowest? How much above the lowest price was the price for each day?

40. The rear wheel of a wagon was $4\frac{5}{12}$ ft. high, and the front wheel $3\frac{5}{24}$ ft. high. How much higher was the rear wheel than the front one?

MULTIPLICATION

303. *The product of two or more fractions is a fraction whose numerator is the product of the given numerators and whose denominator is the product of the given denominators.*

EXAMPLE: Multiply $\frac{7}{8}$ by $\frac{9}{11}$.

$$\tfrac{7}{8}\times\tfrac{9}{11}=\frac{7\times 9}{8\times 11}=\tfrac{63}{88}.$$

To find the product of an integer and a fraction, write the integer with denominator 1 *and proceed as with two fractions.*

EXAMPLE: Multiply 12 by $\frac{5}{8}$.

$$12\times\tfrac{5}{8}=\tfrac{12}{1}\times\tfrac{5}{8}=\tfrac{60}{8}=7\tfrac{1}{2}.$$

304. *To multiply mixed numbers, first reduce them to improper fractions.*

If only one factor is a mixed number, it may be easier to multiply directly. For example, multiply 196 by $15\frac{3}{4}$.

$$\begin{array}{lcr} & & 196\\ & & 15\tfrac{3}{4}\\ \hline \tfrac{3}{4}\times 196 & = & 147\\ 5\times 196 & = & 980\\ 10\times 196 & = & 196\\ \hline 15\tfrac{3}{4}\times 196 & = & 3087 \end{array}$$

305. Canceling is useful in finding products of fractions.

EXAMPLE: $\frac{2}{3}\times\frac{9}{16}\times\frac{8}{15}\times\frac{30}{23}$. By canceling, this becomes

$$\frac{\cancel{2}}{\cancel{3}}\times\frac{\overset{3}{\cancel{9}}}{\underset{\cancel{2}}{\cancel{16}}}\times\frac{\cancel{8}}{\cancel{15}}\times\frac{\overset{2}{\cancel{30}}}{23}=\frac{1}{1}\times\frac{3}{1}\times\frac{1}{1}\times\frac{2}{23}=\frac{6}{23}.$$

WRITTEN EXERCISES

Multiply (canceling when possible):

1. $24\times\frac{3}{4}$.

2. $\frac{2}{3}\times 5$.

3. $\frac{3}{4}\times 48$.

4. $\frac{5}{6}\times\frac{9}{25}$.

5. $\frac{2}{9}\times 18$.

6. $\frac{2}{3}\times\frac{5}{8}$.

7. $\frac{7}{8}\times\frac{9}{16}$.

8. $\frac{4}{5}\times 50\frac{1}{5}$.

9. $\frac{5}{7} \times \frac{2}{3}$. **11.** $\frac{3}{16} \times \frac{4}{9}$. **13.** $\frac{5}{8} \times \frac{3}{10}$. **15.** $5\frac{1}{2} \times 12\frac{3}{8}$.
10. $\frac{3}{5} \times \frac{7}{8}$. **12.** $\frac{9}{16} \times 4\frac{2}{3}$. **14.** $16 \times 2\frac{2}{7}$. **16.** $80\frac{1}{3} \times \frac{5}{27}$.

17. $\frac{4}{5} \times \frac{5}{6} \times \frac{9}{28} \times \frac{3}{10}$. **20.** $\frac{5}{6} \times \frac{7}{8} \times \frac{9}{5} \times \frac{40}{3}$.
18. $1\frac{1}{3} \times 5\frac{1}{2} \times 7\frac{7}{8}$. **21.** $\frac{6}{5} \times 3\frac{3}{5} \times 5\frac{5}{8}$.
19. $4\frac{1}{2} \times 7\frac{3}{10} \times 11\frac{6}{7}$. **22.** $40\frac{1}{3} \times 66\frac{2}{3} \times 15\frac{5}{8}$.

23. Find the product of $\frac{3}{4}$, $\frac{5}{9}$, $\frac{9}{10}$, and $\frac{4}{15}$, and cancel where possible.

Solve:

24. Rose has $\frac{2}{3}$ of a yard of ribbon and uses $\frac{7}{8}$ of this. What part of a yard does she use?

25. Mr. Morgan received $\frac{3}{5}$ of the income of a farm; he spent $\frac{5}{12}$ of his share. What part of the whole income from the farm did he spend?

26. When a certain engineer shuts off steam and puts on the brakes, the driving wheels of his engine still revolve $16\frac{1}{2}$ times before the engine comes to a standstill. If the driving wheels are $20\frac{2}{3}$ ft. in circumference, how far from the place where the brakes were applied is the engine stopped?

27. Find the cost of $5\frac{7}{8}$ yd. of cloth at $\frac{2}{3}$ of a dollar a yard.

28. A teamster drew 20 loads of $\frac{8}{9}$ of a cubic yard each every day for $4\frac{3}{4}$ days. How many cubic yards did he draw in all?

29. Some blocks of granite weighed $4\frac{9}{16}$ tons each. What was the weight of 128 blocks?

30. When cotton is worth \$$53\frac{2}{5}$ per bale, what is the value of 186 bales?

31. A garden was $16\frac{8}{9}$ yd. long and $5\frac{11}{18}$ yd. wide. How many square yards in the area of the garden?

32. How many miles does a train travel in $6\frac{5}{8}$ hr. at the rate of $57\frac{13}{32}$ mi. per hr.?

33. A drawing was made on the scale of $\frac{3}{32}$; that is, every length on the plan was $\frac{3}{32}$ of the actual length in the building.

Find the lengths in the drawing corresponding to the following lengths in the building: 40½ ft.; 30⅓ ft.; 25⅕ ft.; 20⅖ ft.; 18½ ft.; 36⅙ ft.

34. The coffee berry grows on a tree. The average yield of a coffee tree is 6 lb. of salable coffee in one season; the average productive life of a tree is 30 yr. Find the production of 1 coffee tree during its life, and the value of the coffee at 15¾ cts. a pound.

35. What would be the value of the coffee from 48,000 such trees in one season at 14¼ cts. a pound?

36. Different countries produce different grades of coffee:

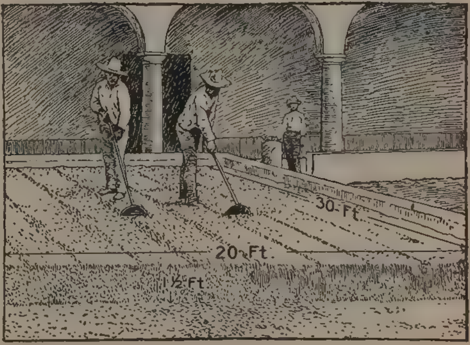
COUNTRY.	WHOLESALE PRICE.	COUNTRY.	WHOLESALE PRICE.
Mexico	15¾¢	Venezuela	13½¢
Costa Rica	14¼	Colombia	13½
Guatemala	14½	Brazil	9¼

What would the coffee mentioned in Exercise 35 be worth at the price named for each country?

37. The coffee crop of Brazil often amounts to 11,000,000 sacks of 132¾ lb. each. How many pounds is the yield?

38. What is the value of this product according to Exercise 36?

39. 70¾ lb. of coffee occupy about 1 cu. ft. How many pounds are there in the bed of dried coffee shown in the picture?


30 Ft.
20 Ft.
½ Ft.

40. Venezuela averages 55,000 tons of coffee a year, $\frac{9}{10}$ of which is exported. What is the value of the exports according to Exercise 36?

DIVISION

306. *The quotient of any two fractions may be found by inverting the divisor and multiplying; or*

307. *The quotient of any two fractions may be found by changing them to fractions having a common denominator and then dividing the numerators of these fractions.*

Which of these processes is used in (1) below? In (2)?

$$(1)\quad \frac{2}{3} \div \frac{4}{5} = \frac{2\times 5}{3\times 5} \div \frac{3\times 4}{3\times 5} = \frac{\cancel{2}\times 5}{3\times \underset{2}{\cancel{4}}} = \frac{5}{6}.$$

$$(2)\quad \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{\cancel{2}\times 5}{3\times \underset{2}{\cancel{4}}} = \frac{5}{6}.$$

308. *To divide an integer by a fraction, write the integer with 1 as a denominator and proceed as in the case of two fractions.*

Example: Divide 28 by $\frac{4}{7}$.

$$\frac{28}{1} \div \frac{4}{7} = \frac{\overset{7}{\cancel{28}}}{1} \times \frac{7}{\cancel{4}} = \frac{49}{1}, \text{ or } 49.$$

309. *To divide a fraction by an integer, divide the numerator or multiply the denominator by the integer.*

Thus, $\frac{8}{9} \div 4 = \frac{2}{9}$. Also, $\frac{3}{8} \div 5 = \frac{3}{40}$.

310. *To divide mixed numbers, change them to improper fractions and divide as in the case of two fractions.*

ORAL EXERCISES

Divide:

1. $\frac{1}{2} \div 5$.
2. $\frac{3}{4} \div 9$.
3. $\frac{4}{5} \div 5$.
4. $\frac{2}{3} \div 8$.
5. $\frac{2}{3} \div 7$.
6. $\frac{6}{7} \div 8$.
7. $\frac{3}{5} \div 6$.
8. $\frac{4}{7} \div 12$.

Solve:

9. How many breadths of carpet $\frac{3}{4}$ of a yard wide are needed to cover a room 15 ft. wide?

10. How many breadths of plush $1\frac{1}{2}$ yd. wide are needed to make a curtain 45 ft. wide?

11. How many yards are there in a roll of carpet costing \$6 at $\frac{3}{4}$ of a dollar a yard?

12. Into how many pieces $\frac{5}{6}$ of a yard long can 15 yd. of ribbon be divided?

13. How many strips of carpeting $\frac{3}{4}$ yd. wide and 8 yd. long are required to cover a room 6 yd. wide and 8 yd. long?

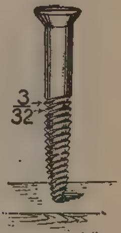
WRITTEN EXERCISES

Divide:

1. $\frac{1}{2} \div \frac{1}{3}$.
2. $8 \div \frac{7}{16}$.
3. $15 \div \frac{1}{15}$.
4. $10 \div \frac{5}{64}$.
5. $\frac{7}{6} \div \frac{7}{12}$.
6. $\frac{1}{15} \div \frac{5}{8}$.
7. $\frac{52}{64} \div \frac{5}{8}$.
8. $\frac{9}{10} \div \frac{3}{100}$.
9. $\frac{1}{4} \div \frac{2}{9}$.
10. $400 \div \frac{3}{20}$.
11. $6\frac{2}{3} \div 4\frac{5}{6}$.
12. $12\frac{1}{5} \div 3\frac{3}{20}$.
13. $16 \div \frac{4}{7}$.
14. $25 \div \frac{5}{8}$.
15. $\frac{5}{12} \div \frac{3}{32}$.
16. $\frac{4}{7} \div \frac{12}{7}$.
17. $\frac{2}{5} \div \frac{1}{3}$.
18. $\frac{1}{40} \div \frac{13}{80}$.
19. $\frac{70}{125} \div \frac{35}{12}$.
20. $\frac{50}{3} \div \frac{25}{6}$.
21. $7\frac{1}{8} \div 12\frac{1}{4}$.
22. $20\frac{1}{12} \div 5\frac{7}{24}$.
23. $18 \div \frac{3}{32}$.
24. $40 \div \frac{7}{12}$.
25. $\frac{2}{3} \div \frac{1}{3}$.
26. $25 \div \frac{5}{48}$.
27. $\frac{7}{64} \div \frac{7}{8}$.
28. $\frac{9}{16} \div \frac{5}{32}$.
29. $\frac{90}{49} \div \frac{2}{7}$.
30. $900 \div \frac{9}{10}$.
31. $9\frac{1}{10} \div 3\frac{3}{20}$.
32. $6\frac{1}{25} \div 12\frac{3}{5}$.

Solve:

33. A certain screw is $3\frac{1}{4}$ in. long, the threads are $\frac{3}{32}$ in. apart; in one revolution the screw is driven a distance equal to that between two threads. How many revolutions must be made to drive the screw its whole length?



34. A screw is $2\frac{1}{2}$ in. long and the threads are $\frac{1}{8}$ in. apart. How many complete turns are required to drive it in?

35. A screw has threads $\frac{5}{64}$ in. apart. How many turns are needed to drive it $2\frac{1}{8}$ in.?

COMPLEX FRACTIONS

311. If a fraction occurs in the numerator or denominator of a fraction or in both, the latter fraction is called a **complex fraction.**

Thus, $\frac{1\frac{1}{3}}{5}$, $\frac{3}{5\frac{1}{2}}$, $\frac{3\frac{1}{2}}{5\frac{2}{3}}$, $\frac{6-\frac{1}{2}}{12\frac{1}{3}+4\frac{3}{5}}$ are complex fractions.

Such fractions furnish drill in operations but they have no place in practical arithmetic.

312. *Complex fractions are simplified by performing the operations indicated in the numerator and the denominator, and then reducing the fraction to lower terms if possible.*

EXAMPLES:

1. Simplify $\frac{5\frac{2}{3}}{6\frac{1}{2}}$.

(1) $5\frac{2}{3} = \frac{17}{3}$.

(2) $6\frac{1}{2} = \frac{13}{2}$.

(3) $\frac{17}{3} \div \frac{13}{2} = \frac{34}{39}$.

(4) Therefore, $\frac{5\frac{2}{3}}{6\frac{1}{2}} = \frac{34}{39}$.

2. Simplify $\frac{6-\frac{1}{2}}{12\frac{1}{3}+4\frac{3}{5}}$.

(1) $6 - \frac{1}{2} = 5\frac{1}{2} = \frac{11}{2}$.

(2) $12\frac{1}{3} + 4\frac{3}{5} = 16\frac{14}{15} = \frac{254}{15}$.

(3) $\frac{11}{2} \div \frac{254}{15} = \frac{165}{508}$.

(4) Therefore, $\frac{6-\frac{1}{2}}{12\frac{1}{3}+4\frac{3}{5}} = \frac{165}{508}$.

WRITTEN EXERCISES

Simplify:

1. $\frac{2\frac{2}{3}}{8\frac{5}{6}}$.

2. $\frac{4 \times 3\frac{1}{5}}{7 \div 2\frac{1}{2}}$.

3. $\frac{5\frac{1}{2} \div 7\frac{1}{8}}{6\frac{4}{5} \times 3\frac{1}{2}}$.

4. $\frac{5\frac{3}{5}}{8\frac{9}{10}}$.

5. $\frac{7-6\frac{2}{3}}{\frac{2}{3}\text{ of }\frac{3}{4}}$.

6. $\frac{40\frac{9}{10}}{20\frac{3}{20}}$.

7. $\frac{126\frac{11}{30}}{42\frac{25}{90}}$.

8. $\frac{7.6+3\frac{1}{2}}{9-6\frac{2}{3}}$.

9. $\frac{9-3\frac{3}{4}+7\frac{1}{8}}{\frac{4}{5}\text{ of }\frac{7}{12}}$.

REVIEW

ORAL EXERCISES

1. What principle is employed when a fraction is changed to an equal fraction having a different denominator? Illustrate.

2. What is a fractional unit? Illustrate your answer.

3. How must fractions be expressed before they can be added or subtracted?

4. What is meant by the least common denominator of several fractions? Illustrate.

5. What is the least common denominator of $\frac{1}{3}$, $\frac{2}{5}$, $\frac{4}{7}$? Of $\frac{1}{12}$, $\frac{3}{8}$, $\frac{5}{6}$?

6. What is a mixed number? Explain two ways of adding mixed numbers. Illustrate each way.

7. Explain two ways of subtracting mixed numbers.

8. How may one fraction be multiplied by another?

9. What is the product of $\frac{7}{8}$ and $\frac{4}{5}$? Of $\frac{3}{5}$ and $\frac{5}{8}$? Of $\frac{4}{9}$ and $\frac{3}{5}$?

10. How may one fraction be divided by another? State another way.

Add:

11. $\frac{5}{6} + \frac{2}{5}$.

12. $\frac{1}{2} + \frac{1}{3} + \frac{5}{6}$.

13. $\frac{4}{5} + \frac{3}{10}$.

14. $\frac{7}{8} + \frac{3}{4}$.

15. $\frac{1}{2} + 1\frac{1}{2} + \frac{3}{2}$.

16. $\frac{1}{4} + 1\frac{1}{4} + 2\frac{5}{8}$.

Subtract:

17. $\frac{5}{6} - \frac{2}{3}$.

18. $\frac{9}{10} - \frac{2}{3}$.

19. $\frac{7}{8} - \frac{1}{5}$.

20. $\frac{8}{9} - \frac{2}{3}$.

21. $\frac{4}{5} - \frac{1}{3}$.

22. $\frac{7}{12} - \frac{1}{3}$.

Multiply:

23. $\frac{8}{9} \times \frac{2}{3}$.

24. $\frac{3}{4} \times \frac{5}{6}$.

25. $3\frac{3}{4} \times \frac{1}{2}$.

26. $1\frac{1}{2} \times \frac{3}{4}$.

27. $\frac{7}{8} \times \frac{2}{3}$.

28. $2\frac{1}{2} \times \frac{2}{5}$.

29. $\frac{7}{12} \times \frac{5}{3}$.

30. $\frac{4}{5} \times \frac{8}{9}$.

31. $6 \times 2\frac{1}{3}$.

Divide:

32. $\frac{8}{9} \div \frac{2}{3}$.

33. $\frac{5}{6} \div \frac{3}{4}$.

34. $\frac{7}{8} \div \frac{3}{4}$.

35. $\frac{5}{12} \div \frac{5}{3}$.

36. $1\frac{1}{2} \div \frac{2}{3}$.

37. $\frac{5}{8} \div \frac{1}{4}$.

38. On a certain day the following were the prices of grain per bushel on the New York market. The months indicate when the grain is to be delivered to the purchaser. The prices are those prevailing at the opening and at the closing of the market; also the highest and the lowest on that day.

WHEAT	Opening.	Highest.	Lowest.	Closing.
May........	95	95	95	95
July........	$91\frac{1}{2}$	$91\frac{11}{16}$	$91\frac{5}{16}$	$91\frac{5}{8}$
Sept........	$83\frac{7}{8}$	$84\frac{1}{8}$	$83\frac{3}{4}$	$83\frac{7}{8}$
Dec........	84	84	$83\frac{1}{2}$	$83\frac{5}{8}$
CORN				
May........	$59\frac{3}{4}$	$60\frac{5}{8}$	$59\frac{1}{4}$	60
July........	$53\frac{3}{8}$	$53\frac{7}{8}$	$53\frac{1}{4}$	$53\frac{7}{8}$
Sept........	$52\frac{1}{4}$	$52\frac{5}{8}$	$52\frac{1}{4}$	$52\frac{5}{8}$

What was the difference between the opening and closing prices of July wheat? Between the highest and lowest prices?

39. Answer the same questions for September wheat.

40. How much was gained on 1000 bu. of July wheat bought at the opening price and sold at the closing price?

41. Make and solve five other problems from the prices in the table above.

WRITTEN EXERCISES

Perform the operations and simplify:

1. $1\frac{1}{2} + \frac{5}{6} + \frac{2}{3} + \frac{7}{12}$.

2. $4\frac{1}{4} + \frac{7}{8} + \frac{3}{4} + \frac{9}{16}$.

3. $2\frac{3}{4} + \frac{5}{12} + \frac{1}{6} - 1\frac{1}{2} - \frac{5}{8}$.

4. $\frac{3}{5} + 5\frac{9}{20} - 3\frac{1}{2} - 1\frac{2}{5}$.

5. $9 - 1\frac{1}{2} - 2\frac{3}{4} - 3\frac{7}{8}$.

6. $3\frac{3}{4} + 1\frac{1}{3} - \frac{5}{12} - \frac{3}{8}$.

7. $5\frac{1}{2} + 6\frac{2}{3} - \frac{5}{6} - 2\frac{1}{6}$.

8. $4\frac{9}{10} + 3\frac{8}{15} - 1\frac{7}{30} - \frac{11}{15}$.

9. $11\frac{2}{3} - 2\frac{5}{6} + 3\frac{2}{3} - \frac{7}{30}$.

10. $\frac{13}{24} + \frac{16}{27} - \frac{7}{54} + \frac{19}{12}$.

Multiply:

11. $\frac{7}{8} \times \frac{3}{4}$.

12. $\frac{8}{9} \times \frac{5}{16}$.

13. $5\frac{1}{2} \times 8\frac{17}{24}$.

14. $7\frac{5}{8} \times 14\frac{11}{15}$.

15. $\frac{7}{12} \times \frac{3}{16}$.

16. $\frac{14}{15} \times \frac{13}{32}$.

17. $19\frac{7}{16} \times 21\frac{25}{32}$.

18. $43\frac{7}{15} \times 20\frac{3}{5}$.

19. $\frac{19}{24} \times \frac{7}{12}$.

20. $\frac{18}{25} \times \frac{4}{15}$.

21. $12\frac{13}{24} \times 8\frac{7}{12}$.

22. $20\frac{9}{16} \times 15\frac{1}{32}$.

Divide:

23. $\frac{3}{8} \div \frac{1}{5}$.

24. $\frac{22}{27} \div \frac{11}{3}$.

25. $5\frac{7}{8} \div 3\frac{14}{15}$.

26. $\frac{5}{9} \div \frac{4}{10}$.

27. $\frac{13}{6} \div \frac{26}{3}$.

28. $19\frac{6}{25} \div 4\frac{3}{5}$.

29. $\frac{24}{25} \div \frac{1}{100}$.

30. $\frac{57}{8} \div \frac{19}{4}$.

31. $40\frac{7}{12} \div 10\frac{3}{24}$.

Perform the operations and simplify:

32. $2\frac{1}{2} + 5\frac{5}{8} - \frac{3}{4}$ mulitiplied by $2\frac{1}{3} \div 4\frac{1}{2}$.

33. $\frac{7}{16} + 1\frac{3}{4} - \frac{5}{8}$ divided by $\frac{5}{8} \times 2\frac{3}{4}$.

34. $4\frac{1}{2} + 2\frac{2}{3} - 3\frac{1}{6}$ multiplied by $3\frac{2}{3} \div 2\frac{1}{2}$.

35. $5\frac{1}{8} - 1\frac{7}{12} + 3\frac{19}{24}$ divided by $4\frac{1}{8} \times 7\frac{3}{4}$.

Solve:

36. A rug $8\frac{1}{4}$ ft. by $10\frac{1}{2}$ ft. is placed on a floor $10\frac{3}{4}$ ft. by 13 ft. What is the area of the uncovered part of the floor?

37. What is the cost of a metal ceiling $18\frac{1}{2}$ yd. long and $10\frac{1}{4}$ yd. wide at $1.87 per sq. yd.?

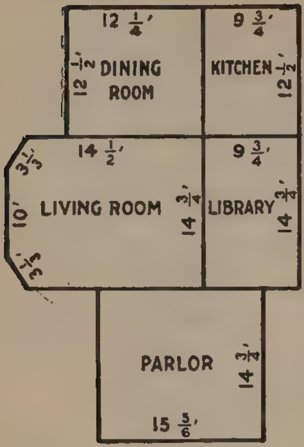
38. A $17\frac{1}{2}$-acre field of corn yielded 48 bu. to the acre. At $62\frac{1}{2}$ cts. per bushel what was the value of the yield? Various expenses amounted to $\frac{5}{16}$ of this sum. What was the profit?

39. If wheat had been grown in the field mentioned in Exercise 38, the profit would have been $\frac{1}{6}$ more. Find the profit from wheat.

40. A half-eagle, or 5-dollar gold piece, weighs 129 grains; $\frac{9}{10}$ of its weight is pure gold. How many grains of gold are there in a half-eagle? How many grains of cheaper metal?

41. The 5-cent piece weighs 77.16 grains; $\frac{1}{4}$ of each coin is nickel, and the rest copper. How many grains of each kind are there in a 5-cent piece?

42. Find the number of feet of baseboard needed for each room in the plan, making no allowance for openings.



43. How many feet more are needed for the parlor than for the library?

44. Find the total length of the house, counting the thickness of each wall (outer or partition) as $\frac{1}{2}$ ft.

45. Make and solve 3 other problems about the plan.

46. The silver dollar weighs 412.5 grains. $\frac{9}{10}$ of its weight is pure silver. How many grains of pure silver are there in one dollar?

47. A builder bought a lot in New York City, and later found that the lot did not include a narrow strip along the street. This strip was $\frac{7}{8}$ in. wide and 40 ft. long. What was its area in square inches? He bought the strip for $50. How much was this per square inch? Per square foot?

48. At the rate found in Exercise 47, what would be the cost of a city lot 25 ft. by 100 ft.?

49. To allow for shrinkage, patterns for cast-iron articles are made $\frac{1}{8}$ in. longer *per foot* than the article. Similarly, for width and thickness. What are the length and the width of a pattern for a cast-iron article $3\frac{1}{2}$ ft. long and 8 in. wide?

50. The allowance per foot for shrinkage in other castings is:

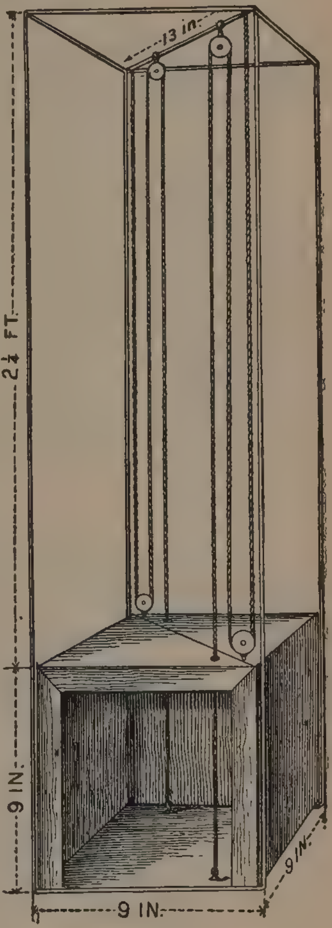
Brass	$\frac{3}{16}$ in.	Tin	$\frac{1}{12}$ in.
Lead	$\frac{1}{8}$ in.	Zinc	$\frac{3}{16}$ in.

A casting of lead is to be 18 in. long and 15 in. wide. What are the length and the breadth of the pattern?

51. Make and solve 4 other problems about patterns for castings.

52. Each of 15 pupils in manual training made a weaving loom. The base frame of each loom required two pieces of moulding $1\frac{1}{2}$ ft. long and two pieces $\frac{3}{4}$ ft. long. How many feet were used for all of the looms? How many feet were left out of six 12-foot strips?

53. Harold made a model of an elevator; the framework was made of stiff wire. Find from the given dimensions how many feet of wire were used. Find how many feet of string he used for the cables.



54. He noticed that he worked on the model $1\frac{3}{4}$ hr. on each of 3 days, $1\frac{1}{2}$ hr. on each of 2 days, and $\frac{3}{4}$ hr. on each of 5 days. How long did it take him to make it? How many days of 8 hr. each?

55. A class in domestic science made 16 cup-custards. For each cup they allowed 1 egg, $1\frac{1}{2}$ oz. of sugar, and $\frac{1}{2}$ pt. of milk. At 18 cts. a dozen for eggs, $5\frac{1}{2}$ cts. a pound for sugar, and 6 cts. a quart for milk, what did all of these materials cost?

56. A class of girls made 5 sachet bags, 8 calendars, and 7 letter cases for an exhibition. Each sachet bag required $1\frac{1}{2}$ ft. of ribbon, each calendar $\frac{3}{4}$ ft., and each letter case $1\frac{2}{3}$ ft. How many feet of ribbon did they use? How many yards?

57. The boys made bent iron candlesticks, each requiring $2\frac{3}{8}$ yd. of iron. How many feet of iron did they use in making 6 candlesticks?

CHAPTER X

DENOMINATE NUMBERS

313. Numbers considered apart from particular things are called **abstract numbers.** Numbers that refer to particular things are called **concrete numbers.**

Thus, 2, 3, $\frac{5}{8}$, and $1\frac{1}{2}$ are abstract numbers; while 2 words, 3 apples, $\frac{5}{8}$ ft., and $\$1\frac{1}{2}$ are concrete numbers.

314. Concrete numbers used in weights and measures are commonly called **denominate numbers.**

Thus, 8 lb., 12 gal. 3 qt., 2 bu. 2 pk. 6 qt., and $3.75 are denominate numbers.

The tables of denominate numbers are given on pages 449 and 450.

REDUCTION

315. In solving problems it is often necessary to reduce a denominate number of several denominations to one number of lower denomination. This is called **reduction descending.**

Example: How many lamps holding 1 pt. can be filled from 4 gal. 3 qt. 1 pt. of kerosene?

Solution: (1) 4 gal. = 4 × 4 qt., or 16 qt.
(2) 16 qt. + 3 qt. = 19 qt.
(3) 19 qt. = 19 × 2 pt., or 38 pt.
(4) 38 pt. + 1 pt. = 39 pt.
(5) Therefore, 4 gal. 3 qt. 1 pt. = 39 pt.

The problem now becomes "How many lamps holding 1 pt. can be filled from 39 pt.?" The result is evidently 39 lamps.

316. *A number of several denominations may be reduced to one of the lowest denomination by beginning with the highest, multiplying this by the number of units in the next lower denomination, adding to this result any such units that the number may contain, and repeating the process to the lowest denomination.*

WRITTEN EXERCISES

Reduce to the lowest denominations:

1. 5 T. 200 lb. 8 oz.
2. 17 gal. 3 qt. 1 pt.
3. 6 gross 9 doz. 10 units.
4. 5 cu. ft. and 1,020 cu. in.
5. 50 cu. yd. 8 cu. ft. 5 cu. in.
6. 1 mi. 160 rd. 2 ft.
7. 5 da. 20 hr. 6 min. 30 sec.
8. 40° 20′ 45″.
9. 12 bu. 3 pk. 7 qt.
10. 10 rd. 12 ft. 8 in.
11. 4 reams, 20 quires, 18 sheets.
12. 10 sq. yd. 8 sq. ft. 40 sq. in.

317. In solving problems containing denominate numbers, it is often necessary to change from lower to higher denominations. This is called **reduction ascending.**

EXAMPLE: Clarence picked 205 qt. of cherries. How many bushels and quarts did he pick?

Here it is necessary to express 205 qt. as bushels and quarts.

Solution: (1) 1 bu. = 32 qt.
(2) 205 ÷ 32 = 6 and 13 remaining.
(3) Therefore, 205 qt. = 6 bu. and 13 qt.

Another plan is this:

(1) 1 qt. = $\frac{1}{32}$ bu.
(2) 205 qt. = 205 × $\frac{1}{32}$ bu. = $6\frac{13}{32}$ bu. = 6 bu. 13 qt.

The second solution is preferable if the result is wanted in one denomination, as $6\frac{13}{32}$ bu.

318. *A number of one denomination may be reduced to the next higher denomination by dividing it by the number of the given units that make one of the next denomination.*

WRITTEN EXERCISES

1. A milk wagon carried 125 pint-bottles. How many gallons, quarts, and pints was this?

2. An oil tank contained 16,526 pt. How many gallons, quarts, and pints was this?

3. A load of berries contained 327 one-quart boxes. How many bushels, pecks, and quarts in the load?

Express:

4. 875 pt. as gallons, quarts, pints.

5. 986 pt. as bushels and quarts.

6. 4,956 lb. as tons and pounds.

7. 4,763 min. as days, hours, minutes.

8. 5,690 sec. as hours, minutes, seconds.

9. 15,760 gr. avoirdupois as pounds, ounces, grains.

10. 4,325 in. as rods, feet, inches.

11. 10,378 ft. as miles and feet.

12. 5,890 sq. in. as square yards, square feet, square inches.

13. 15,763 cu. in. as cubic yards, cubic feet, cubic inches.

Reduction with Decimals

319. The custom is growing of expressing quantities in one denomination only, using fractions if necessary. Decimal fractions are usually the most convenient.

Thus, 1.5 ft. would be used instead of 1 ft. 6 in.

EXAMPLES:

1. How many hours, minutes, and seconds are there in .48 da.?

Solution: (1) .48 × 24 hr. = 11.52 hr.
(2) .52 × 60 min. = 31.2 min.
(3) .2 × 60 sec. = 12 sec.
(4) Therefore, .48 da. = 11 hr. 31 min. 12 sec.

2. 1 ft. is what part of a yard? Express decimally as parts of a yard: 1 ft.; 2 ft.; .4 ft.; $1\frac{1}{2}$ ft.; $\frac{3}{4}$ ft.; $\frac{1}{2}$ ft.

3. Express decimally 7 yd. 2 ft. 11 in. as a number of yards.

Solution: (1) $11 \times \frac{1}{12}$ ft. = .92 ft.
(2) 2 ft. + .92 ft. = 2.92 ft.
(3) $2.92 \times \frac{1}{3}$ yd. = .97 yd.
(4) 7 yd. + .97 yd. = 7.97 yd.
(5) Therefore, 7 yd. 2 ft. 11 in. = 7.97 yd.

WRITTEN EXERCISES

1. Express in hours and minutes: 1.3 hr.; 2.7 hr.; 4.9 hr.; 12.6 hr.; .5 hr.

2. Add the numbers as they are given in Exercise 1. Also when changed to hours and minutes. Which way is shorter?

3. How many yards, feet, and inches are there in 7.8 yd.? In 5.25 yd.?

4. How many square rods are there in 1.7 acre?

5. How many square feet are there in 9.25 sq. yd.?

6. How many yards are there in 3.75 in.?

7. How many ounces are there in $12.37\frac{1}{2}$ lb.?

8. Express decimally, as a number of feet: 5 ft. 8 in.; 9 ft. 5 in.

9. Express decimally as gallons: 2 gal. 3 qt. 1 pt.; 5 gal. 1 qt. 1 pt.

10. Express decimally as square feet: 6 sq. ft. 72 sq. in.; 15 sq. ft. 96 sq. in.

11. Express decimally as square yards: 1 sq. yd. 8 sq. ft. 16 sq. in.

12. Express decimally as cubic yards: 4 cu. yd. 9 cu. ft. 144 cu. in.

13. Express decimally: $6\frac{3}{4}$ ft.; $2\frac{4}{5}$ ft.; $7\frac{13}{20}$ ft.; $8\frac{23}{25}$ ft.

14. Add the numbers of Exercise 13 as given. Add the numbers expressed decimally. Which way is the shorter?

ADDITION AND SUBTRACTION

320. Example: Add 120 lb. 12 oz., 119 lb. 4 oz., and 118 lb. 8 oz.

Add the numbers in the first column at the right. How many pounds and how many ounces besides is this? Add the second column, including the 1 lb. from the first.

120 lb.	12 oz.
119	4
118	8
358 lb.	8 oz.

321. *To add denominate numbers, arrange the addends in columns of like denomination and add the numbers in each column. If the sum in any column is equal to, or greater than, the number of units in the next higher denomination, reduce it and add the higher units to the next column.*

Example: Subtract 17 hr. 54 min. from 25 hr. 36 min.

Add 1 hr. to the 36 min. of the right column and subtract 54 min. Take 1 hr. from 25 hr. of the left column and subtract 17 hr. from 24 hr.

25 hr.	36 min.
17	54
7 hr.	42 min.

322. *To subtract denominate numbers, arrange the numbers in columns of like denomination, and subtract. Where the minuend in any denomination is smaller than the subtrahend, increase it by the equivalent of one unit of the next higher denomination.*

WRITTEN EXERCISES

Add:

1.

7 gal.	3 qt.	1 pt.
8	2	
7	1	1

2.

6 pk.	4 qt.	1 pt.
3	2	
6	7	1

3.

10 mi.	80 rd.	10 ft.
20	20	15
15	100	8

4.

7 yd.	2 ft.	2 in.
10	1	10
15		8

5. 12 da. 10 hr. 15 min. 5 sec.
18 20 45 55

6. 40 bu. 3 pk. 6 qt.
8 2 5
20 1 3

7. 3 cu. yd. 20 cu. ft. 10 cu. in.
10 18 100
15 12 1000

8. 40 A. 80 sq. rd. 3 sq. yd.
60 100 10

9. 4 wk. 6 da. 20 hr.
12 3 10
15 5 12

10. 39 sq. yd. 2 sq. ft. 40 sq. in.
15 6 25
10 5 100

Subtract:

11. 15 wk. 5 da. 12 hr.
4 6 20

12. 20 mi. 20 rd. 15 ft.
10 80 10

13. 15 yd. 2 ft. 2 in.
7 1 10

14. 15 cu. yd. 12 cu. ft. 100 cu. in.
12 18 1000

15. 40 bu. 3 pk. 6 qt.
15 1 7

16. 30 sq. yd. 5 sq. ft. 25 sq. in.
15 6 100

Solve:

17. A lot has a frontage of 18 yd. 2 ft. 6 in., and another a frontage of 19 yd. 1 ft. 9 in. Find the total frontage.

18. The Empire State Express traveled from Rome to Albany in 2 hr. 49 min. 36 sec. and from Albany to New York City in 2 hr. 58 min. 50 sec. How long was it in traveling from Rome to New York?

19. An eclipse began at 8 hr. 38 min. 6 sec. P.M. and ended at 8 hr. 55 min. 51 sec. P.M. How long did it last?

20. From a 10-gallon can of milk 21 qt. and 1 pt. were sold. How much was left?

21. The best watches permit one to observe fifths of a second. A bicycle rider began a mile at 2 hr. 17 min. $23\frac{4}{5}$ sec. P.M. and ended it at 2 hr. 19 min. $56\frac{1}{5}$ sec. P.M. How long did it take him to ride the mile?

MULTIPLICATION

323. EXAMPLE:

A square lot measures 5 yd. 2 ft. 7 in. on a side. What is its perimeter?

5 yd. 2 ft. 7 in.
4
20 yd. 8 ft. 28 in.
23 yd. 1 ft. 4 in.

(1) 4 × 7 in. = 28 in. = 2 ft. 4 in.
(2) 4 × 2 ft. = 8 ft.
(3) 8 ft. + 2 ft. = 10 ft. = 3 yd. 1 ft.
(4) 4 × 5 yd. = 20 yd.
(5) 20 yd. + 3 yd. = 23 yd.
(6) Therefore, the perimeter is 23 yd. 1 ft. 4 in.

324. *To multiply a compound number, multiply each denomination separately and reduce the results to units of higher denomination, if possible.*

WRITTEN EXERCISES

Multiply:

1. 4 gal. 3 qt. 1 pt.
5

2. 3 pk. 7 qt. 1.5 pt.
15

3. 2 bu. 6 pk. 1.5 qt.
10

4. 10 mi. 41 rd. 3.75 ft.
16

5. 10 sq. rd. 20 sq. ft. 12 sq. in.
20

6. 3 cu. yd. 20 cu. ft. 12 cu. in.
3.25

7. 1 da. 16 hr. 20 min. 16 sec.
2.5

8. 40 A. 12 sq. rd. 5 sq. yd.
62

9. Multiply 5 yd. 2 ft. 7 in. by 4. Then express 5 yd. 2 ft. 7 in. as a number of feet and a fraction of a foot and multiply by 4.

10. A wire fence 6 wires high is 20 yd. 2 ft. 9 in. long. How many feet of wire are needed for the fence? (Find the result in two ways.)

11. Express decimally to the nearest hundredth of a bushel 4 bu. 3 pk., 2 bu. 1 pk.; 6 bu. 4 qt.; 8 bu. $1\frac{1}{2}$ pk.

12. Multiply the numbers as given in Exercise 11 by 10.

13. Multiply the decimal equivalents of the numbers of Exercise 11 by 10. Which way of multiplying is shorter?

14. Express decimally in gallons 4 gal. 3 qt. 1 pt. and multiply the result by 5.

15. Express decimally in miles 10 mi. 41 rd. 3.75 ft. and multiply by 16.

DIVISION

325. EXAMPLES:

1. How many 5-inch badges can be cut from 5 yd. 2 ft. 11 in. of ribbon?

Solution: (1) 5 yd. 2 ft. 11 in. = 215 in.
(2) 215 in. ÷ 5 in. = 43.
(3) Therefore, 43 badges can be made.

2. Divide 6 bu. 3 pk. 4 qt. by 4.5.

Solution:

$$\begin{array}{r} 1 \text{ bu. } 2 \text{ pk. } 1\frac{7}{9} \text{ pt.} \\ 4.5\,\overline{)\; 6 \text{ bu. } 3 \text{ pk. } 4 \text{ qt.}} \end{array}$$

(1) 6 bu. ÷ 4.5 = 1 bu. and 1.5 bu. remainder.
(2) 1.5 bu. + 3 pk. = 9 pk.
(3) 9 pk. ÷ 4.5 = 2 pk.
(4) 4 qt. (8 pt.) ÷ 4.5 = $1\frac{7}{9}$ pt.
(5) Therefore, the quotient is 1 bu. 2 pk. $1\frac{7}{9}$ pt.

326. *To divide a compound number by an abstract number, divide the highest denomination by the given divisor; reduce the remainder, if there is one, to the next lower denomination and add it to the units of that denomination. Repeat the division as before.*

If the divisor is a compound number, reduce both divisor and dividend to the same denomination and divide.

WRITTEN EXERCISES

1. Divide by 5: 207 lb. 8 oz.; 2 tons 600 lb.; 5 bu. 2 pk. 4 qt.

Divide:

2. 12 bu. 2 pk. 2 qt. by 5.
3. 10 gal. 3 qt. 1 pt. by 1.5.
4. 12 mi. 80 rd. 12 ft. by 16.
5. 140 cu. yd. 25 cu. ft. by 18.
6. 160 sq. ft. 50 sq. in. by 2.5.
7. 40 tons 1,500 lb. by 200.
8. 20 hr. 30 min. 15 sec. by 15.
9. 4 da. 20 hr. 40 min. by 20.
10. 45 bu. 2 pk. 6 qt. by 30.
11. 1 T. 500 lb. 12 oz. by 24 lb.
12. 40 yd. 2 ft. 10 in. by 20.
13. 4 gro. 9 doz. 8 units by 6.5 doz.

Solve:

14. The perimeter of a square is 20 rd. 14 ft. 4 in. What is the length of a side?

15. A rectangular lot with a frontage of 36 yd. 2 ft. 10 in. was divided into 5 equal lots. What was the frontage of each of these lots?

16. A cellar contained 50 cu. yd. 18 cu. ft. of earth, which was drawn away in loads of 1 cu. yd. 3 cu. ft. each. How many loads were there?

17. An elevator contains 5,000 bu. 3 pk. of grain. How many bushels would there be in each car, if this grain were loaded in 8 equal carloads?

18. How many boards 5 yd. 1 ft. long are needed to build a fence 4 boards high and 320 ft. long?

19. Some strips of moulding were 1 yd. 1 ft. 3 in. long. How many did it take to go around a room 12 ft. by 15 ft.?

20. A tram car carried 3 cu. yd. and 15 cu. ft. of earth. How many such carloads were there in 9,681 cu. ft. of earth?

21. How many times can a stove's oil tank holding 7 pt. be filled from a 5-gallon can of gasoline?

22. How many bottles containing 1 oz. and 3 dr. can be filled from 77 dr. of medicine?

EQUIVALENTS

Comparison of Liquid Measure and Cubic Measure

327. A gallon contains 231 cu. in.

WRITTEN EXERCISES

1. One cubic foot of water contains how many cubic inches? How many gallons of water in 1 cu. ft. of water?

2. A tank contains 400 gal. of water. How many cubic feet of water in the tank?

3. An oil car has a capacity of 360 cu. ft. What is its capacity in gallons?

4. How many cubic feet in the capacity of a 5-gallon can?

5. How many barrels ($31\frac{1}{2}$ gal.) of water will a cistern hold that is 8 ft. by 10 ft. by 6 ft.?

Comparison of Dry Measure and Cubic Measure

328. A bushel contains 2150.42 cu. in.

WRITTEN EXERCISES

1. How many cubic inches in 5 bu.?

2. How many cubic inches in 3 pk.?

3. A bin contained 10,752 cu. in. How many bushels did it hold?

4. How many cubic feet in a bushel measure?

5. A car 32 ft. long, $7\frac{1}{2}$ ft. wide, 8 ft. high, is filled uniformly to $\frac{1}{2}$ of its height with wheat. How many bushels in the car? (Use 1.24 cu. ft. = 1 bu.)

6. Using $1\frac{1}{4}$ cu. ft. for a bushel, find the number of bushels in the capacity of a grain elevator 40 ft. long, 40 ft. wide, and 100 ft. high.

7. How many cubic inches in 1 qt. dry measure? How many cubic inches in 1 qt. liquid measure? Which is the larger? By how many inches?

Comparison of Weights

329. Precious metals are weighed by troy weight.

The pound troy contains 5,760 grains, or 12 oz. of 480 grains each. The ordinary pound (*the pound avoirdupois*) contains 7,000 grains.

EXAMPLE: How many pounds avoirdupois does a gold nugget weigh that weighs 39 oz. troy?

Solution: (1) 39×480 gr. = 18,720 gr.
(2) $18{,}720 \div 7{,}000 = 2\frac{4720}{7000} = 2\frac{118}{175}$.
(3) Therefore, the nugget weighs $2\frac{118}{175}$ lb.

WRITTEN EXERCISES

1. A bullet weighed 240 gr. troy. What part of an ounce avoirdupois was this?

2. A pound troy is what part of a pound avoirdupois?

3. A carat used to weigh diamonds is 3.168 gr. How many grains in a 3-carat diamond? What part of an ounce avoirdupois does a 3-carat diamond weigh?

4. How many grains in one ounce avoirdupois?

5. An ounce avoirdupois is what part of an ounce troy?

LATITUDE AND LONGITUDE

330. PREPARATORY.

The location of a point on the earth's surface may be indicated by stating its distance north or south of the equator and east or west of a given meridian.

331. Distance in degrees north or south of the equator is called **latitude.**

Latitude 42° N. means 42° north of the equator.

332. Distance measured in degrees east or west of a given meridian is called **longitude.**

The meridian of Greenwich, near London, is usually taken. That of Washington is sometimes used. To say that a place is in longitude 87° W. means that it is 87° west of the meridian of Greenwich.

If you walk directly north, how does your latitude change? Your longitude? If you walk east? West?

333. Angles are measured in **degrees.** 1 degree is divided into 60 equal parts called **minutes,** and 1 minute is divided into 60 equal parts called **seconds.**

334. Degrees, minutes, and seconds are denoted by °, ′, and ″, respectively.

Thus, 12 degrees, 14 minutes, and 17 seconds would be written 12° 14′ 17″.

335. EXAMPLES:

1. Add 30° 16′ 23″ and 37° 50′ 19″.

30° 16′ 23″
37 50 19
68° 6′ 42″

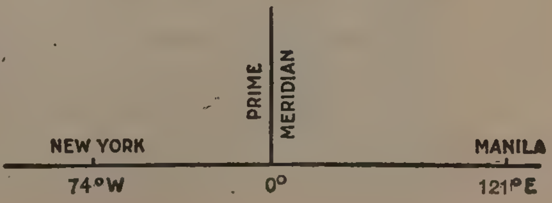
Numbers expressed in degrees, minutes, and seconds are best added by columns, and the usual tests applied to the work of each column. Similarly for subtraction.

2. The longitude of New York is 73° 58′ 25.5″ west, and that of Chicago is 87° 36′ 42″ west. What is the difference in longitude between these places?

87° 36′ 42″
73 58 25.5
13° 38′ 16.5″

3. Find from the figure the difference in longitude between Manila and New York City.

When one place is in east longitude and the other in west, the difference in longitude between the two places is found by adding.



WRITTEN EXERCISES

1. Find the difference in longitude between Berlin and each of the other places in the first column of the table:

PLACE.	LONGITUDE.	PLACE.	LONGITUDE.
Berlin, Germany	13° 23′ 43.5″ E.	Manila, Philippines	120° 58′ 00″ E.
Dublin, Ireland	6 20 30 W.	Peking, China	116 27 00 E.
Honolulu, Hawaii	157 51 48 W.	New York	73 58 25.5 W.
Paris, France	2 20 15 E.	Chicago	87 36 42 W.
Tokyo, Japan	139 42 30 E.	San Francisco	122 25 40.8 W.

2–11. Make and solve 10 other problems from the table.

LONGITUDE AND TIME

336. Preparatory.

1. How many hours are required by the earth to make one rotation on its axis?

2. In which direction does the earth rotate?

3. When it is sunrise in New York City, is it before or after sunrise in Boston? In Buffalo? In San Francisco?

4. How many hours elapse while a point of the earth's surface turns through 360°? 180°? 90°? 30°? 15°?

5. When it is noon in Chicago, what time is it in a place 15° west of Chicago? In one 15° east of Chicago?

This table shows the relation between *longitude* and *time:*

TABLE

360° of longitude correspond to 24 hr. of time.
15° of longitude correspond to 1 hr. of time.
1° of longitude corresponds to $\frac{1}{15}$ hr., or 4 min., of time.
15′ of longitude correspond to 1 min. of time.
1′ of longitude corresponds to $\frac{1}{15}$ min., or 4 sec., of time.
15″ of longitude correspond to 1 sec. of time.
1″ of longitude corresponds to $\frac{1}{15}$ sec. of time.

337. Examples:

1. What is the difference in time corresponding to a difference of 200° 30′ 15″ of longitude?

Solution: (1) 200×4 min. = 800 min. = 13 hr. 20 min.
(2) 30×4 sec. = 120 sec. = 2 min.
(3) $15 \times \frac{1}{15}$ sec. = 1 sec.
(4) Therefore, 200° 30′ 15″ corresponds to 13 hr. 22 min. 1 sec.

Since a difference of longitude of 200° 30′ 15″ is the same as a difference of 159° 29′ 45″ taken the other way around the earth, another answer to the problem is 10 hr. 37 min. 59 sec.

2. What is the difference in longitude corresponding to a difference in time of 20 hr. 45 min. 10 sec.?

Solution: (1) $20 \times 15° = 300°$.
(2) $45 \times 15' = 675' = 11° 15'$.
(3) $10 \times 15'' = 150'' = 2' 30''$.
(4) Therefore, 20 hr. 45 min. 10 sec. corresponds to 311° 17′ 30″.

Since a difference of more than 12 hr. of time may be reckoned the other way around the earth as a difference of less than 12 hr., this problem has also the answer 48° 42′ 30″.

WRITTEN EXERCISES

Express the difference in time that corresponds to each of the following differences in longitude:

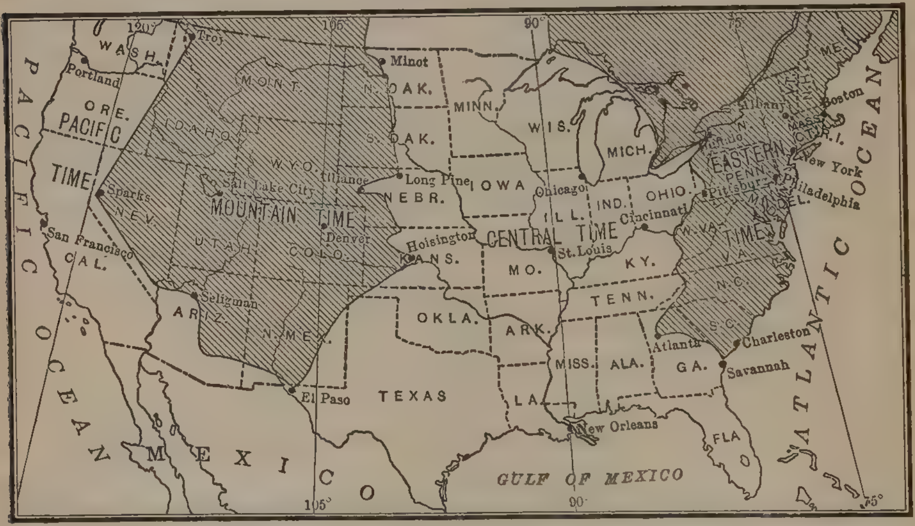
1. 10° 50′ 40″
2. 5° 18′ 15″
3. 8° 9′ 58″
4. 14° 20′ 45″
5. 90° 0′ 50″
6. 12° 30′ 10″
7. 25° 12′ 25″
8. 17° 0′ 19″
9. 180° 40′ 0″

Express the difference in longitude that corresponds to each of the following differences in time:

10. 8 hr. 20 min. 15 sec.
11. 6 hr. 4 min. 40 sec.
12. 5 hr. 54 min. 56 sec.
13. 10 hr. 0 min. 45 sec.
14. 12 hr. 4 min. 10 sec.
15. 23 hr. 20 min. 16 sec. (2 Ans.)

Standard Time

338. All places east of a given point have later time than that point; all west of it have earlier time. As a convenience the country has been divided into sections corresponding to certain meridians, the time of the meridians being taken as the time throughout the sections. This is called **standard time.** The meridians used are 75°, 90°, 105°, 120° west longitude. The map shows the meridians and the corresponding time belts.



ORAL EXERCISES

1. When it is noon, standard time at New York, what is the time at Boston? Denver? Philadelphia? Chicago?

2. San Francisco is how many hours earlier in time than New York?

3. In traveling from New York City to Portland, Oregon, how many times is the watch set back one hour?

4. When it is 12 o'clock midnight at Denver, Colorado, standard time, what is the standard time at New Orleans? New York City? Chicago? Boston? St. Louis?

REVIEW

Reduce:

1. 16 gal. 2 qt. 1 pt. to pints.
2. .3 mi. + 1000 ft. 10 in. to inches.
3. 25 yd. 2 ft. 9 in. to inches.
4. 4⅓ sq. yd. + 8 sq. ft. + 100 sq. in. to square inches.
5. 5 cu. ft. 1125 cu. in. to cubic inches.
6. 40,125 oz. to tons, pounds, and ounces.
7. 63,365 in. to miles, feet, and inches.
8. 4,327 cu. ft. to cubic yards and cubic feet.
9. 16 gal. 3 qt. 1 pt. to gallons expressed decimally.
10. 5 bu. 3 pk. 6 qt. 1 pt. to bushels expressed decimally.

Add:

11. 3 wk. 5 da. 20 hr.
8 4 17

12. 25 gal. 2 qt. 1 pt.
18 3

13. 12 bu. 3 pk. 7 qt.
18 2 5

14. 40 sq. yd. 8 sq. ft. 100 sq. in.
50 2 44

Subtract:

15. 10 gal. 1 qt.
7 3

16. 5 mi. 10 rd. 11 ft.
3 10 13

Multiply:

17. 3 T. 400 lb.
20

18. 12 cu. yd. 19 cu. ft.
3.5

Divide:

19. 17)420 yd. 2 ft.

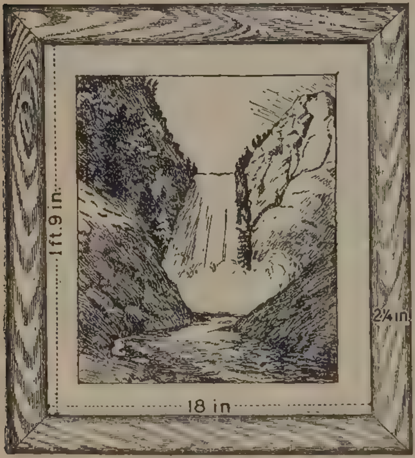
20. 8.2)164 bu. 8.2 qt. 1 pt.

Solve:

21. How many more cubic inches in 10 gal. than in 1 bu.?
22. How many pounds troy make 1 lb. avoirdupois?

23. A bag of wheat weighed 118 lb. 8 oz., another 120 lb. 12 oz., and a third 119 lb. 4 oz. What was the total weight of the 3 bags of wheat?

24. A clerk sold $2\frac{1}{2}$ lb. of paper to one customer, 3 lb. 12 oz. to another, and 4 lb. 4 oz. to another. How many pounds did he sell in all?


1 ft. 9 in.
2¼ in.
18 in

25. How many feet of molding, $2\frac{1}{4}$ in. wide, are needed to make a frame for this picture?

26. The sides of a rectangular court measure 32 ft. 8 in. and 64 ft. 9 in. What is the length of the railing surrounding the court?

27. From a ten-gallon can of gasoline a stove holding $7\frac{1}{2}$ qt. is filled. How much gasoline remains in the tank?

28. During four weeks it stormed 1 wk. 2 da. and 20 hr. How many weeks, days, and hours were not stormy?

29. From a 5-gallon can of kerosene were filled a tank holding 1 gal. 1 qt. and 1 pt., a lamp holding 1 qt. and 1 pt., and two other lamps holding $\frac{3}{4}$ of a pint each. How much kerosene was left in the can?

30. From a farm containing 140 A. and 40 sq. rd., a field containing 17 A. and 120 sq. rd. is sold. How many acres and square rods remain in the farm?

31. Write 40 degrees, 30 minutes, and 59 seconds, using the abbreviations; also 84 degrees, 55 minutes, and 43 seconds.

32. How many seconds are there in $1'$? In $2\frac{1}{2}'$? In $5\frac{3}{4}'$?

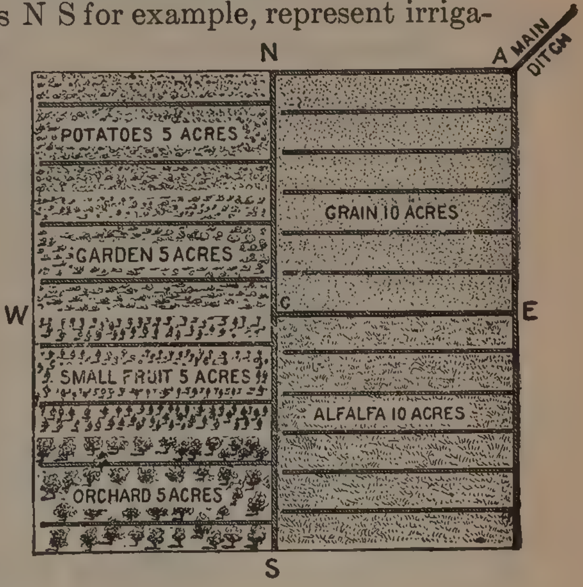
33. How many minutes are there in $1\frac{1}{2}°$? In $4\frac{3}{4}°$?

34. Find the difference in longitude between Paris 2° 20′ 15″ E. and New York 73° 58′ 25″ W.

35. When it is noon on the 90th meridian west, what time is it at 75° W.? At 120° W.? At 105° W.? At 15° E.? At 0°? At 105° E.?

36. The longitude at Chicago is 87° 36′ 42″ west. When it is noon on the 90° meridian, what is the local time in Chicago? What is the standard time in Chicago at the same instant? What is the difference between local time and standard time in Chicago?

37. The picture shows a plan for irrigating a square farm. The double shaded lines, as N S for example, represent irrigation ditches. Taking each ditch running east and west to be 40 rd. long, how many rods of ditches are there in all?



38. Indicate the number of square feet in 1 sq. rd. (1 rd. = $16\frac{1}{2}$ ft.) Find the number of square feet in 1 A.

39. If the ditches on the alfalfa field deliver 2 cu. ft. of water a second, how many hours will it take to deliver a volume of water equal to a lake 6 in. deep over the whole field?

40. If, in excavating the main ditch and the branches for a 10-acre farm, 250 cu. yd. of earth were removed at a cost of 35 cts. a cubic yard, what did the excavating cost?

41. If the surveying cost $12, the boxes and gates $125, what was the total cost of the system?

42. What would it cost to irrigate a farm of $25\frac{1}{2}$ A. for one season at $4.85 per acre?

CHAPTER XI

THE SOLUTION OF PROBLEMS

339. The four essentials in the solution of a problem have already been explained and applied.

1. **Read the problem.**
2. **Plan the solution.**
3. **Make the calculation.**
4. **Test the work.**

The analysis of a problem is the plan of the solution. It may be expressed orally or written in steps.

Example: A dealer buys lamps at $9 a dozen. For how much must he sell them to gain 50 cts. each?

Plan:

(1) $9 ÷ 12 = $( ).
(2) $.75 + $.50 = $( ).

Calculation:

$$\frac{\overset{.75}{\cancel{\$9.00}}}{\cancel{12}} + \$.50 = \$1.25.$$

Oral Analysis: $\frac{1}{12}$ of \$9, or \$.75, is the cost of 1 lamp. \$.75, the cost, plus \$.50, the gain, is \$1.25, the selling price of the lamp.

WRITTEN EXERCISES

Solve the following problems, writing the analysis in step form and the calculation; also give the oral analysis:

1. 3½ lb. of coffee cost $1.12. What was the cost of 1 lb.?

2. A dealer paid $30 for a dozen pairs of shoes; he sold them at a gain of $.75 a pair. Find the selling price a pair.

3. Taking 500 bunches of lath as a carload, how many carloads would 3,500,000 bunches make?

4. How many 2½-gallon buckets of water in 5,180 gal.?

340. There is no single plan that will apply to the solution of all problems of arithmetic. But there are two methods widely used, **unitary analysis** and **the equation.**

UNITARY ANALYSIS

341. Examples:

1. What is the cost of 45 bottles of ink at $3 per hundred?

Analysis: (1) 100 bottles of ink cost $3.
(2) Therefore, 1 bottle costs $\frac{1}{100}$ of $3, or $.03.
(3) Therefore, 45 bottles cost 45 × $.03, or $1.35.

2. If 14 bu. of peaches make 270 qt. of fruit for canning, how many quarts will 63 bu. make?

Analysis: (1) 14 bu. make 270 qt.
(2) Therefore, 1 bu. makes $\frac{1}{14}$ of 270 qt., or $19\frac{2}{7}$ qt.
(3) Therefore, 63 bu. make 63 × $19\frac{2}{7}$ qt., or 1,215 qt.

It is usually better first to indicate all the work, then cancel as much as possible, and multiply last of all. The solution of the second example would thus appear:

$$\frac{\overset{135}{\cancel{270}}}{\underset{2}{\cancel{14}}} \times \overset{9}{\cancel{63}} = 1{,}215.$$

WRITTEN EXERCISES

Analyze and solve as above:

1. If 25 yd. of cloth make 15 vests, how many yards are needed to make 36 vests?

2. If a steamer goes 9 mi. in 48 min., how long will it take to go 51 mi. at the same rate?

3. If the interest on $330 is $22, how much is the interest on $780 for the same time and rate?

4. If 5 printers can set 100 pages in 1 day, how many pages can 12 printers set in 2 days, at the same rate?

5. If 16 men can dig 980 yd. of sewer trench in 8 days, indicate the amount they can dig in 1 day; the amount 1 man can dig in 1 day; in 20 days; the amount that 28 men can dig in 20 days. Find the last named amount.

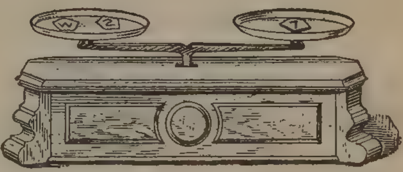
THE EQUATION

Principles

342. PREPARATORY.

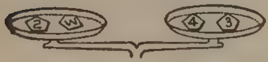
1. How heavy must the weight, w, be in order that the two pans may balance each other?

The condition of balancing may be represented by $w+2=7$.

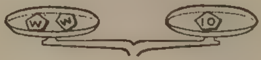


State the condition of balancing for each of the following:

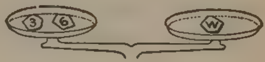
2.



3.



4.



343. An **equation** is formed by two equal numbers connected by the sign of equality. The sign separates the equation into two parts called **members;** that on the left is called the **left** (or **first**) member, and that on the right, the **right** (or **second**) member.

344. PREPARATORY.

1. One side of a balance contains $(x + 2)$ oz., and the other an equal weight of $(2 + 8)$ oz. State the equation that expresses that these two scale pans balance. Write it. What is the value of x?

2. If 2 oz. are taken from each pan, how will the balance be affected? State the equation that expresses the resulting condition.

3. If, instead of 2 oz. being subtracted, 2 oz. had been added to both pans, how would the balance have been affected? State the equation that expresses the resulting condition.

4. If the weight in each pan is doubled, will the balance be affected? If each weight is made $\frac{1}{3}$ as large?

345. *If both members of an equation are increased, diminished, multiplied, or divided by the same number, the resulting members are equal.*

Literal Notation

346. PREPARATORY.

1. If 25 yd. represents the speed of a train per second, what represents its speed per minute? If r represents the speed of a train per second, what represents its speed per minute?

2. If 1000 ft. represents the speed per hour, what represents the speed per minute? If r represents the speed per hour, what represents the speed per minute?

3. If s represents the speed of a train per minute, what represents the distance the train will travel at that rate in 10 min.? In an hour? In 5 hr.? In h hr.?

4. How many seconds will a bicyclist require to ride 3,000 ft. at the rate of 15 ft. a second? At the rate of r ft. a second? How many seconds will he require to ride s ft. at the rate of r ft. a second?

5. If l, b, and h represent respectively the number of feet in the length, breadth, and height of a room, what represents the number of cubic feet in the room?

ORAL EXERCISES

1. When x is the age of one man and y that of another, what does $x + y$ stand for? $x - y$? $2x + y$? $x - 2y$? $\frac{1}{2}x + \frac{1}{2}y$, that is, $\frac{1}{2}$ of x + $\frac{1}{2}$ of y?

Instead of the expression "$x + y$ stands for" or "n stands for" we may use "$x + y$ is," or "n is."

2. If h is the number of dollars a house costs, and l the number of dollars the lot costs, what is $h + l$? What is $h - l$? What does $h = l$ mean? $h = 2l$? What is the meaning of $h = l + 500$? $h = l - 500$?

3. If l is the length of a rectangle and w is its width, what is $l \times w$? What is $2l + 2b$? What does $l = 2b$ mean? What does $l = w + 20$ mean? $w = l - 10$?

Instead of $l \times w$, we may write lw. In general, absence of a sign between letters denotes multiplication.

State how the second equation is found from the first in each case:

1. $x + 10 = a + 5.$
$x + 10 - 5 = a.$

2. $2x + 5 = 20.$
$2x = 15.$

3. $5x = 17.$
$x = 3\frac{2}{5}.$

4. $4y = 12.$
$y = 3.$

5. $2x + 1 = 3.$
$x = 1.$

6. $3x + 2 = 11.$
$x = 3.$

Problems Solved by Equations

347. Problems may often be stated most clearly in the form of equations, using some letter to represent the unknown quantity.

WRITTEN EXERCISES

State as an equation and find the number required:

1. What number increased by 3 equals 12?

Plan: (1) Let n represent the number.
(2) Then $n + 3 = 12$.
(3) Hence, $n = (\quad)$. Why?

2. What number increased by 6 equals 16?

3. What number must be doubled and the result increased by 12 to produce 28?

Plan: (1) Let n stand for the number.
(2) Then $2n + 12 = 28$.
(3) Hence, $2n = 16$. Why?
(4) Hence, $n = (\quad)$.

4. To what number must 16 be added to produce 78?

5. A salesman sold twice as many hats on Tuesday as on Monday, and 6 more on Wednesday than on Tuesday. On Wednesday he sold 16. How many did he sell on Monday?

6. Mary thought of a number; doubled it; added 12 and obtained 58. Of what number did she think?

The equation is $2n + 12 = 58$. Why?

7. In 6 years John will be 4 times as old as James is now. James is now 4 years old. How old is John?

The equation is $x + 6 = 16$. Why?

8. If Mr. Brown's annual salary were increased by $\frac{1}{5}$ of itself, it would lack $500 of being $2,000. What is his salary?

9. $\frac{3}{4}$ of the distance from Ann Arbor to Detroit is 30 mi. Find the distance between these places.

Plan: (1) Let x be the distance.
(2) Then $\frac{3}{4}x$, or $\frac{3x}{4}$, $= 30$.
(3) $3x = (\quad)$.
(4) $x = (\quad)$.
(5) *Test:* $\frac{3}{4}$ of $40 = 30$, as the problem states.

Find the distances between the cities named in Exercises 10–12, if:

10. $\frac{4}{15}$ of the distance from Detroit to Chicago is 76 mi.

11. $\frac{7}{15}$ of the distance from Boston to Cincinnati is 441 mi.

12. $\frac{2}{3}$ of the distance from Albany to Buffalo is 198 mi.

13. Twice a certain number plus 3 is 15. Find the number.

Plan: (1) Let n represent the number; write 2 times n plus 3 in symbols.
(2) Write the equation that shows that this expression is 15.
(3) What does $2n$ equal? What does n equal?
(4) *Test:* 2 times the number found plus 3 should be 15.

14. 3 times a certain number plus 6 is 12. Find the number.

15. John is x yr. old, and James is 4 years older. The sum of their ages is 26 yr. How old is each boy?

16. If a tennis ball rebounds to $\frac{2}{3}$ of the height from which it was dropped, from what height must it be dropped to rebound $3\frac{1}{2}$ ft.?

17. How high will the same ball rise on the second rebound? On the third?

18. A tower and flag staff are together 100 ft. high; the staff is $\frac{1}{4}$ of the height of the tower. Find the height of each.

19. A house and lot is worth \$3,500; the house is worth 6 times as much as the lot. Find the value of each.

20. The part of a bridge pier under water is $\frac{2}{3}$ as high as the part out of water; the whole height is 45 ft. Find the height of each part.

21. Twice the number of pounds of meat consumed annually per inhabitant in Great Britain is 186% of the consumption per inhabitant in the United States. If the latter is 150 lb., what is the consumption in Great Britain?

22. The number of pounds for the United States, as given in the previous problem, falls 39 lb. short of being three times the number for Germany. Find the latter.

23. An average workman should eat daily a certain weight of starchy foods, 16% of that weight of fats, and 20% of that weight of albuminous foods (protein); the total weight of these three foods consumed daily should be at least $1\frac{1}{2}$ lb. What weight of each is required?

24. 18% by weight of wheat is lost (as bran, etc.) in grinding it into flour. How many 60-pound bushels of wheat are used in making 246 lb. of flour?

25. The weight of bread is $133\frac{1}{3}$% of the weight of the flour used to make it. According to Exercise 24, how many 1-pound loaves of bread can be made from 10 bu. of wheat?

26. In the dairies of the United States the milk of some cows is used to make cheese, that of others to make butter; the average amount of butter produced annually per cow is 130 lb; this is 16 lb. less than twice the average amount of cheese. Find the latter amount.

RATIO AND PROPORTION

348. The quotient of two like numbers is often called their **ratio.**

Thus, the ratio of 4 to 2 is $\frac{4}{2}$, or 2. The ratio of 2 to 4 is $\frac{2}{4}$, or $\frac{1}{2}$.

349. The equality of two ratios is called a **proportion.**

Thus, $\frac{1}{2} = \frac{4}{8}$ and $\frac{x}{3} = \frac{3}{9}$ are proportions.

350. Two or more pairs of numbers are said to be **proportional** when their ratios are equal.

Thus, 2, 6; $1\frac{1}{2}$, $4\frac{1}{2}$; 9, 27 are proportional numbers, because $\dfrac{2}{6} = \dfrac{1\frac{1}{2}}{4\frac{1}{2}} = \dfrac{9}{27}$.

351. Consequently a proportion is an equation, and problems involving proportion can be solved by the use of the equation.

ORAL EXERCISES

1. Compare the ratios: $\frac{1}{2}$ and $\frac{2}{4}$; $\frac{3}{4}$ and $\frac{6}{8}$; and $\frac{1}{3}$ and $\frac{3}{9}$.

2. If $\frac{x}{8} = \frac{1}{2}$, what is the value of x? If $\frac{x}{3} = \frac{3}{9}$, what is the value of x? What is x, if $\frac{x}{2} = \frac{3}{5}$?

Find the value of x in each of the following proportions:

3. $\frac{x}{3} = \frac{5}{9}$. **4.** $\frac{x}{5} = \frac{12}{10}$. **5.** $\frac{1}{6} = \frac{x}{7}$. **6.** $\frac{2}{3} = \frac{x}{5}$.

WRITTEN EXERCISES

1. An automobile traveled 70 mi. in 6 hr. How far did it travel in 15 hr. at the same rate?

2. If a sum of money earns $48 interest in 5 yr., how much will it earn in 16 yr. at the same rate per cent?

3. A city whose population was 40,000 had 4,500 children of school age; the total population increased to 48,000, and the number of children of school age increased proportionally. How many children of school age were there then?

Partitive Proportion

352. The method used to divide a quantity into parts proportional to several given numbers is called **partitive proportion.**

EXAMPLE: An inheritance of \$8,000 was divided among 3 heirs in proportion to their ages: 5 yr., 15 yr., and 20 yr. How many dollars did each receive?

Solution: (1) The statement means that each receives the same sum for each year of his age.

(2) Let x = the sum received for 1 year.

(3) Then the first receives $5x$, the second $15x$, and the third $20x$.

(4) Altogether they receive $5x + 15x + 20x = 40x$.

(5) Hence $40x = \$8,000$.

(6) Then, $x = \$200$, $5x = \$1000$, the share of the first, etc.

WRITTEN EXERCISES

1. A invests \$3,000, B \$8,000, and C \$11,000 in a factory. The profits for the year are \$2,000, shared in proportion to the amount of capital invested. What did each receive?

Suggestion: Let p = the profit on \$1000. Then, A receives $3p$, etc.

2. A street was paved at a cost of \$2,400. The cost was assessed on the property owners in proportion to the frontage owned. Mr. A owned 100 ft., Mr. B 300 ft., Mr. C 150 ft., Mr. D 650 ft. How much did each pay?

3. Three men rented an automobile for \$140 per month of 28 days; one used it 1 day each week, another 2 days, and the third 4 days. How many dollars should each pay toward the monthly rental?

4. In 100 cu. in. of air there are approximately 21 cu. in. of oxygen and 79 cu. in. of nitrogen; that is, air is composed of 21 volumes of oxygen to 79 volumes of nitrogen. How many cubic inches of each are there in a cubic foot of air?

REVIEW

ORAL EXERCISES

1. When x dollars buys 10 yd. of silk, what does 1 yd. cost? $\frac{1}{3}$ of a yard? $\frac{2}{3}$ of a yard?

2. When c cts. buys 12 oranges, what does 1 orange cost? If d dozen oranges cost a dollars, what is the cost of 1 orange?

3. If a man can do a piece of work in n days, how much of it can he do in 1 day? In 4 days? In d days? How much of it could 3 men do in a days? In x days? How much could q men do in 1 day? In t days?

4. There are a rows of b trees each in an orchard. How many trees are there?

5. A rectangular pile of bricks is a bricks long, b wide, and c high. How many bricks are there in the pile?

6. Represent the length of a rectangle by l in.; its breadth 12 in. less than its length. What represents its perimeter?

7. State and illustrate some processes that may be performed on both members of an equation without destroying the equality.

8. Clara paid c cts. for ribbon and twice as much for silk, spending in all 48 cts. How much did she pay for each article?

When l = *length,* b = *breadth,* a = *area, and* p = *perimeter of a rectangle, state the meaning of:*

9. $a = lb$. **11.** $\frac{a}{l} = b$. **13.** $\frac{a}{b} = l$.

10. $p = 2b + 2l$. **12.** $p - 2l = 2b$. **14.** $\frac{p}{2} - b = l$.

15. What represents the interest on p dollars for 1 yr. at 6%? For 3 yr.? For t yr.?

16. When c is the cost and r is the rate of gain, what represents the actual gain?

17. When x is the rate of motion of a train per hour, y the number of hours traveled, and d the distance, state the meaning of: $d = xy$; $\frac{d}{y} = x$; $2d = 2xy$; $\frac{1}{2}xy = \frac{1}{2}d$.

WRITTEN EXERCISES

1. One box weighed p lb., and another weighed 10 lb. more than the first; the two together weighed 130 lb. What did each weigh?

2. A man 48 years of age is 3 yr. more than 5 times as old as his youngest child. How old is the latter?

3. If Mr. Brown's annual salary were increased by ⅕, it would lack $600 of being $1,800. What is his salary?

4. A sum of money was divided into 2 parts in the ratio of 1 to 2; the first part is invested at 6%, the other part at 4½%; the annual income is $45. What was the total sum invested?

5. Divide the sum $5,000 into two parts so that the interest on the larger at 3% a year shall equal that on the smaller at 5% a year.

6. Mr. Gray sold his house and lot for $17,670; he invested part of the proceeds at 5½%, and the remainder at 4⅓%; the income from each part was the same. How was the sum divided?

7. The sum $15,000 is invested, part at 4½% and the rest at 5%; the annual income is $687. How is the investment divided?

8. A man bought a tract of land at $75 an acre. He sold ⅓ of it at $84, ¼ at $85, and the remainder at $90 an acre, thus gaining $11,280. How many acres were there in the tract? What per cent of the purchase price did he gain?

9. Two persons save $300 apiece annually; the first had already saved $1,500 when the second began. In how many years after the second began will he have half as much as the first then has?

10. A bookseller bought 656 copies of a book at a discount of 25%; he sold half of them at list price, and the other half in larger quantities at a discount of 10%; he made $209.92. Find the list price.

11. The area of the United States exceeds 5 times that of Alaska by 71,180 sq. mi. What is the area of Alaska?

The area of the United States, excluding Alaska and the island possessions, is 3,025,600 sq. mi. Throughout the following problems, "the area of the United States" means this area.

12. 880 times the area of Porto Rico is 2,800 sq. mi. less than that of the United States. Find the area of Porto Rico.

13. If the area of the Philippine Islands were increased by 2,000 sq. mi., 25 times this area would be 99,950 sq. mi. less than the area of the United States. What is the area of the Philippines?

14. The area of all the other island possessions increased by 134 sq. mi. is twice the area of Porto Rico found in Exercise 12. Find the area of these possessions.

15. The area of France increased by 92,946 sq. mi. and the area of the United States when diminished by 2,560 sq. mi. have the ratio 1 to 10. What is the area of France?

16. The area of the United States diminished by 25,600 sq. mi. is $\frac{3}{4}$ of that of China when diminished by 277,170 sq. mi. What is the area of China?

17. The area of the United States, including Alaska and other possessions, is 3,756,884 sq. mi. This area increased by 243,116 sq. mi., is $\frac{1}{3}$ of that of the British Empire when increased by 713,656 sq. mi. Find the area of the British Empire.

18. The area of Germany diminished by 8,830 sq. mi. is $\frac{1}{15}$ of the area of the United States when diminished by 25,600 sq. mi. What is the area of Germany?

19. Three partners divided $12,000, the gain from one year's business, into parts proportional to the numbers 2, 3, 5. How many dollars did each receive?

CHAPTER XII

PERCENTAGE

353. A **number per cent** means that *number per hundred.*

Thus, "5% of the population are foreign born" means that 5 *persons per hundred* are foreign born.

PERCENTAGE OF NUMBERS

354. The result of applying a rate per cent to a given number is called a **percentage.**

Thus, "5% of 600 bu. of fruit was lost in transit." The rate was 5%, and the percentage 30 bu.

The name *percentage* is applied also to that part of arithmetic which deals with rates per cent.

355. *To find a certain percentage of a number, multiply it by the corresponding number of hundredths.*

Example: 37% by weight of a certain grade of carpet is cotton. How many pounds of cotton in 250 lb. of this carpet?

Solution: 37% of 250 lb., that is, $.37 \times 250$ lb. = 92.5 lb., cotton.

WRITTEN EXERCISES

Find the following percentages:

1. $18\frac{1}{2}\%$ of 435.
2. $27\frac{3}{4}\%$ of 165.
3. 43% of 600.
4. 69% of 40.8.
5. 83% of 17.
6. 47.5% of 5.
7. 13% of 459.
8. $15\frac{1}{2}\%$ of 960.
9. $3\frac{1}{2}\%$ of 2,000.
10. 89% of 169.
11. $14\frac{1}{5}\%$ of 8.5.
12. $12\frac{2}{3}\%$ of 21.
13. 54% of 17.9.
14. 30.6% of .005.
15. $19\frac{3}{4}\%$ of 8.06.
16. $9\frac{8}{3}\%$ of 250.
17. 97% of 780.
18. $78\frac{1}{2}\%$ of 2.25.

Solve:

19. The duty imposed by the United States Government upon Oriental rugs is 42%. How many dollars are collected on each 100 dollars' worth of rugs? How many dollars are collected on a lot worth \$1,575? On a lot worth \$9,763.65?

20. A supply house offered a reduction of 5% on all orders of \$100 or more. What would orders of the following amounts cost the customer:

\$517.98? \$1,563.95? \$850.75? \$1000.50?

21. The timber land of a certain farm was 19% of the whole; the farm contained 168 acres. How many acres were cleared?

356. In solving problems it is often simpler to use the fractional equivalents of rates per cent.

Example: The prices in a certain wholesale catalogue are subject to a reduction of $33\frac{1}{3}$%. What would be the reduction on goods priced at \$690.30 in the catalogue?

Solution: (1) $33\frac{1}{3}$% of a number is $.33\frac{1}{3}$, or $\frac{1}{3}$ of the number.
(2) $\frac{1}{3}$ of \$690.30 = \$230.10, the reduction.
Taking $\frac{1}{3}$ of \$690.30 is much simpler than multiplying it by $.33\frac{1}{3}$.

TABLE OF CORRESPONDING NUMBERS

PER CENT.	FRACTION.	PER CENT.	FRACTION.
5	$\frac{1}{20}$	40	$\frac{2}{5}$
10	$\frac{1}{10}$	50	$\frac{1}{2}$
$12\frac{1}{2}$	$\frac{1}{8}$	60	$\frac{3}{5}$
$16\frac{2}{3}$	$\frac{1}{6}$	$66\frac{2}{3}$	$\frac{2}{3}$
20	$\frac{1}{5}$	75	$\frac{3}{4}$
25	$\frac{1}{4}$	80	$\frac{4}{5}$
$33\frac{1}{3}$	$\frac{1}{3}$	$87\frac{1}{2}$	$\frac{7}{8}$
$37\frac{1}{2}$	$\frac{3}{8}$	90	$\frac{9}{10}$

WRITTEN EXERCISES

1. A spelling test paper containing 76 words was marked 75%. How many words were spelled correctly? How many were spelled incorrectly?

2. In an arithmetic test paper containing 25 problems 80% were marked right. How many were right? How many were wrong?

3. The window surface in a schoolroom should be 20% of the floor surface. How many square feet of window surface should a room 20 ft. square have? A room 30 ft. by 50 ft.? One 20 ft. by 25 ft.?

4. 40% of the air in a schoolroom was changed by ventilation in 12 min. At the same rate in how many minutes would it all be changed? The room contained 4,000 cu. ft. How many cubic feet were changed in 12 min.? In 30 min.? In 1 hr.?

5. Find 20% of: 575; 860; $100.20; $105.80; 1,580.

6. Find $33\frac{1}{3}$% of: 363; 690; $180.66; $360.90; $303.63.

7. Find $16\frac{2}{3}$% of: 192; $246.12; $252.60; $126.36; 108.

8. Find $37\frac{1}{2}$% of: 160; 360; $400.80; $1000.40; $568.40.

9. Find 75% of: 800; $760.40; $1000.80; $1,204.60; 900.

10. Find $87\frac{1}{2}$% of. 160; 360; 400.8; 1000.4; 568.4.

11. Find $66\frac{2}{3}$% of: 363; 690; $180.66; $360.90; $303.63.

12. Find 90% of: 810; 465; 863; $100.20; $1,005.

13. Find 10% of: 196.3; 1805; 17.25; $40.90; 1,762.

14. Find $16\frac{2}{3}$% of: $184; $27.60, 26.9; 4,064; 920.

To Find How Many Per Cent One Number Is of Another

357. *To find how many per cent one number is of another, divide the first by the second and express the quotient in hundredths. The number of hundredths is the number per cent.*

EXAMPLES:

1. 5 is what per cent of 20?

Solution: (1) $\frac{5}{20} = \frac{1}{4} = .25$.
(2) Thus, 5 is .25 of 20, or 25% of 20.

2. 15 is what per cent of 38?

Solution: (1) $15 \div 38 = .39\frac{9}{19}$.
(2) Thus, 15 is $.39\frac{9}{19}$ of 38, or $39\frac{9}{19}\%$ of it.

3. 3 is what per cent of 4,500?

Solution: (1) $3 \div 4{,}500 = .0006\frac{2}{3} = .00\frac{2}{3}$.
(2) Thus, 3 is $\frac{2}{3}$ of 1% of 4,500; that is, $\frac{2}{3}\%$ of it.

WRITTEN EXERCISES

1. 15 is what per cent of 360? Of 375? Of 420?

2. 45 is what per cent of 990? Of 270? Of 360? Of 5?

3. 3 is what per cent of 28? Of 190? Of 95?

4. Ethel answered correctly 10 questions in a list of 11. What per cent was this? What per cent did she miss?

5. Roland solved 11 problems in a list of 13. What per cent did he solve? What per cent did he fail to solve?

6. A term of school contained 100 days; a class of 20 pupils had therefore a possible total attendance of 2,000 days; there were total absences amounting to 40 days. What was the rate per cent of absence? Of attendance?

7. In a recent month the employees of the Chicago post-office handled 48,000,000 pieces of mail matter; of these 100,000 were addressed defectively. How many pieces per million of mail matter were addressed defectively? How many per thousand? Per hundred? Per cent?

8. Of the defectively addressed mail 85,000 addresses were deciphered so as to permit delivery. What per cent were delivered?

To Find a Number From a Given Percentage

358. *To find a number of which the percentage at a certain rate is given, divide the percentage by the rate per cent, and multiply the quotient obtained by 100. Or, divide the given number by the fraction corresponding to the rate per cent.*

EXAMPLES:

1. 200 is 17% of what number?

Solution by analysis: (1) $200 \div 17 = 11\frac{13}{17}$.
(2) $100 \times 11\frac{13}{17} = 1{,}176\frac{8}{17}$.

Explanation: Since 200 is 17%, 1 per cent is the quotient of 200 divided by 17, or $11\frac{13}{17}$. Therefore, 100% is $100 \times 11\frac{13}{17}$, or $1{,}176\frac{8}{17}$.

Use of the equation: (1) Let n be the number sought.
(2) Then, 17% of n, or $.17n$, = 200.
(3) Dividing by .17, $n = \frac{200}{.17} = 1{,}176\frac{8}{17}$.

2. 125 is $66\frac{2}{3}$% of a number. What is the number?

Solution: $125 \div 66\frac{2}{3}\% = 125 \div \frac{2}{3} = 125 \times \frac{3}{2} = 187.5$.

WRITTEN EXERCISES

1. An agent received $25 for the sale of a consignment of goods; this was 5% of the amount of the sale. How many dollars in the amount of the sale?

2. The window space in a room is 20% of the floor space. What is the floor space in a room whose window space is 200 sq. ft.? In one whose window space is 355 sq. ft.?

3. Clarence received a mark of 85% on his spelling examination; he missed 30 words. How many did he spell correctly?

Find the numbers to supply the blanks:

4. 105 is 40% of ( ).

5. 400 is $66\frac{2}{3}$% of ( ).

6. 490 is $87\frac{1}{2}$% of ( ).

7. 800 is 75% of ( ).

8. 144 is $8\frac{1}{3}$% of ( ).

9. 279 is 90% of ( ).

APPLICATIONS OF PERCENTAGE

Gain and Loss

359. Gain or loss in business transactions is commonly expressed by a rate per cent of the cost.

Thus, a merchant bought cloth at \$1 a yard and sold it at \$1.25 a yard, thereby gaining 25%.

WRITTEN EXERCISES

Find the missing numbers:

	ARTICLES.	Cost Per Pound.	Selling Price Per Pound.	Part of Cost Gained.	Gain Per Cent of Cost.
1.	Nails.........	\$.02	\$.04	( )	( )
2.	Butter	.20	.26	( )	( )
3.	Sugar	$.04\frac{1}{2}$	.05	( )	( )
4.	Cinnamon	.90	1.08	( )	( )
5.	Ham..........	.15	( )	( )	20
6.	Cheese	( )	.26	( )	$8\frac{1}{3}$
7.	Nuts..........	( )	.14	$\frac{1}{6}$	( )
8.	Sausage	( )	.12	$\frac{1}{3}$	( )
9.	Veal Cutlets ..	( )	.20	$\frac{1}{4}$	( )

Solve:

10. A bicycle costing \$50 was sold at a gain of 40%. What was the selling price?

11. An automobile was sold for \$650 at a gain of 30%. What was the cost?

If c represents the cost, $1.3\,c = \$650$ represents the cost plus the gain; that is, the selling price.

Find the cost in each case:

	12.	**13.**	**14.**	**15.**
Selling price:	\$240	\$390	\$170	\$1,400
Gain or loss:	20% (gain)	30% (gain)	15% (loss)	40% (gain)

Commission

360. Services in buying or selling for others are often paid by a fixed part of the value of the article sold. Such payment is called **commission.** The rate is usually expressed as a rate per cent.

361. An agent who handles and sells commodities on commission is called a **commission merchant.**

362. A person who sends articles to a commission merchant is called a **shipper, consignor,** or **principal.**

363. The merchandise sent is called a **consignment.**

WRITTEN EXERCISES

Find the commission at the rate given:

CONSIGNMENT.	RATE.
1. 500 crates of berries sold at $1.75 a crate	12½%
2. 300 T. of cabbage sold at $1.25 a ton	8¾%
3. 50 crates of eggs at $5.50 a crate	9½%
4. 200 bbl. of apples sold at $2.60 a barrel	10%
5. 100 forty-pound tubs of butter sold at 23 cts. a pound	5%
6. 48 dressed lambs, 95 lb. each, sold at 10 cts. a pound	5%
7. 250 lb. of butter sold at 25 cts. a pound	4½%
8. 175 baskets of peaches at 85 cts. a basket	10¾%

9. A commission merchant sold a consignment of peaches for $235.60, charging 5½%. What was his commission?

10. Find the commission on 500 lb. of lard sold at 7 cts. a pound, the rate of commission being 3½%.

11. A commission agent sold 500 baskets of peaches at 75 cts. a basket and remitted $356.25. What was his commission in dollars? What was his rate of commission?

12. A commission agent sold some apples on a commission of 15%. His commission amounted to $63.75. For how much were the apples sold?

Commercial Discounts

364. A reduction made in the price of goods is called a **discount.**

Discounts are made for various reasons, as for large amount purchased, prompt payment, because of damage to the goods, or on special sales to attract custom. Standard prices are fixed for various commodities, and published in *price lists.* When these prices are higher than the usual selling prices, and the selling price is specified by the discount allowed from the *list price,* the price after the discount is deducted is called the *net price.*

ORAL EXERCISES

1. A discount of 10% is offered on the following prices. Find the net prices:

Range	$40	Saw	$3.50
Refrigerator	16	Keg of nails	3.00
Gasoline stove	22	200 ft. of rope	16.00
Water heater	12	100 lb. of steel rod	5.00
Bath tub	30	2 dozen door fixtures	27.00

2. A firm offers a discount of 20% on the following prices:

Flags and Pennants.	Silk—One Color.	Silk—Two Colors.	Cashmere.
12 in. by 18 in.	$0.85	$1.15	$0.70
16 in. by 24 in.	1.00	1.50	0.85
24 in. by 36 in.	1.35	2.15	1.15

Find the cost of 1 pennant of each kind.

365. When two or more discounts are quoted, the first discount is taken from the list price, the second discount from the amount left after the first discount has been taken off, and so on.

Example: If the list price is $9, and the discounts are 20%, 10%, and 5%, what is the net price?

Plan: (1) $9.00 – $1.80 = $( ). (2) $7.20 – $.72 = $( ).
(3) $6.48 – $.32 = $( ), net price.

WRITTEN EXERCISES

1. A bill of goods amounted to \$163 (list price). It was bought at 10%, 10%, and 5% off, and the goods were sold at 10% more than the list price. Find the rate per cent of profit.

2. To what single discounts are the following discounts equivalent: 25% and 10%? ⅓ and 25%? 40% and 10% and 5%?

3. Compare the net prices of a carriage listed at \$330, sold at the following discounts:

(*a*) 30%, 20%, and 10%.
(*c*) 20%, 10%, and 30%.
(*b*) 10%, 30%, and 20%.
(*d*) 10%, 20%, and 30%.

What is the similarity in these cases?

4. A bill for \$480 was discounted regularly at 20%, and a further discount of 2% was allowed for cash payment. What was the amount required to settle the bill?

5. The following table shows prices and discounts:

ARTICLE.	PRICE.	DISCOUNT.
Common plow	\$7.50	5%
Sulky-plow	19.50	12½%
Hoe	0.40	5%
Hay-fork	0.60	5%
Truck-wagon	70.00	16⅔%
Runabout	55.00	12½%
Surrey	90.00	20%
Grain-drill	75.00	16⅔%
Reaper	95.00	20%
Mower	60.00	15%

How much must a customer pay for each of the following orders, allowing a further discount of 3% for cash:

(1) 5 Common plows?
(2) 25 Hoes?
(3) 1 Surrey?
(4) 3 Truck-wagons?
(5) 2 Grain-drills?
(6) 2 Sulky-plows?
(7) 20 Hay-forks?
(8) 2 Runabouts?
(9) 1 Reaper?

Taxes

366. In towns, counties, and states, the expense of government and the cost of public enterprises are met by collecting from property owners a certain amount on each dollar of property valuation. Sums collected for these purposes are called **taxes.**

367. The **rate** of taxation is expressed by a rate per cent or by an amount on each dollar.

Thus, $\frac{1}{2}\%$, or 5 mills, means a tax of 5 mills on every dollar.

368. Taxes are usually not computed on the full value of the property. The value taken as a basis of taxation is called the **assessed value.**

Thus, if a house and lot worth $1,600 is valued for taxation at $1,200, it is said to be *assessed at* $\frac{3}{4}$ *of its value.*

WRITTEN EXERCISES

1. The following is the annual budget voted by a village board of trustees:

Contingent fund	$10,000
Street lighting	3,700
First district, highway fund	1,200
Second district, highway fund	1,200
Third district, highway fund	1,500
Fourth district, highway fund	$700
Fire department	3,500
Poor fund	1,700
Park fund	900
Bridge fund	4,000
Sidewalk fund	2,500
Library fund	700

Find the total appropriation.

2. What rate per cent of the total was the contingent fund? The poor fund?

3. Make ten other problems about these appropriations.

4. $\frac{3}{4}$ of the budget mentioned in Exercise 1 was raised by direct taxation, the rate being $1\frac{1}{2}\%$. Find the total assessment on which the tax was based.

REVIEW

ORAL EXERCISES

1. .05 of a number is what per cent of it?

2. 50% of a number is what part of it? 25%? 33⅓%?

3. How may we find how many per cent one number is of another?

4. 12 is ( )% of 18?

5. ( ) is 4% of 20?

6. ( ) is 33⅓% of 30?

7. 8 is ( )% of 40?

8. 25 is 200% of ( )?

9. 20 is ( )% of 30?

10. 3 is ( )% of 9?

11. ( ) is 10% of 80?

12. 16 is ( )% of 48?

13. 16 is ( )% of 80?

14. 75 is 75% of ( )?

15. 17 is ( )% of 34?

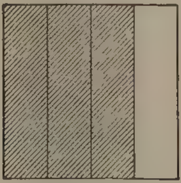
16. 100 lb. is how many per cent of a ton? 1,800 lb. is how many per cent.? 500 lb.?

17. 8 oz. is how many per cent of a pound? 4 oz.? 1 oz.?

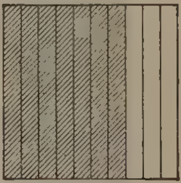
18. 18 in. is how many per cent of a yard? 9 in. is how many per cent? 27 in.?

19. 24 qt. is how many per cent of a bushel?

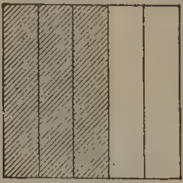
20. The squares represent the total population in 1900 of the states named, and the shaded areas represent the proportion of this population living in cities (*urban* population).



MASSACHUSETTS



NEW YORK



NEW JERSEY

How many per cent of the population of Massachusetts lived in cities? In the country?

21. Answer the same question for each of the other states.

WRITTEN EXERCISES

1. The United States silver dollar contains $371\frac{1}{4}$ gr. of pure silver; this is 90% of its weight. What is the weight of a silver dollar?

2. Bees wintered indoors require about $\frac{8}{15}$ as much food as when wintered out of doors. What per cent of the food required out of doors is saved by wintering indoors? If a colony requires 24 lb. of food when wintered indoors, how many pounds would it require when wintered out of doors?

3. A man's tax was $150 at $1\frac{1}{2}$%; the assessment was $\frac{2}{3}$ of the actual value of the property. What was its value?

4. A village resident owns a house valued at $4,500; it is assessed at $\frac{2}{3}$ of its value. What tax does its owner pay at the rate of $1\frac{1}{2}$%?

5. A property owner's tax is $30 on an assessment of $1,500. What is the rate of taxation?

6. According to the following table find within $\frac{1}{10}$ of 1% the rate of increase in the total number of votes cast for President of the United States in 1900 over that in 1896:

1896	13,923,102	1904	13,528,979
1900	13,959,653	1908	14,852,841

7. As in the last exercise, find to $\frac{1}{10}$ of 1% the rate of increase in 1908 over 1904.

8. Texas is the largest cotton-producing state in the United States; in 1903 the crop was 2,575,000 bales. How many per cent (within 1%) was this of 10,758,000 bales, the total production in the United States?

9. The urban population of Connecticut was 55% of its total population. Represent this relation graphically.

10. Represent graphically the fact that 40% of the total population of the United States lived in cities.

11. The table shows relative prices for a recent period of 14 years:

[The average price for this period is 100.]

YEAR.	Farm products.	Food.	Clothing.	Fuel and lighting.	House furnishing goods.	Lumber, building materials.	Metals and implements.	Drugs and chemicals.	Miscellaneous.
1.	110.0	112.4	113.5	104.7	111.1	111.8	119.2	110.2	110.3
2.	121.5	115.7	111.3	102.7	110.2	108.4	111.7	103.6	109.4
3.	111.7	103.6	109.0	101.1	106.5	102.8	106.0	102.9	106.2
4.	107.9	110.2	107.2	100.0	104.9	101.9	100.7	100.5	105.9
5.	95.9	99.8	96.1	92.4	100.1	96.3	90.7	89.8	99.8
6.	93.3	94.6	92.7	98.1	96.5	94.1	92.0	87.9	94.5
7.	78.3	83.8	91.3	104.3	94.0	93.4	93.7	92.6	91.4
8.	85.2	87.7	91.1	96.4	89.8	90.4	86.6	94.4	92.1
9.	96.1	94.4	93.4	95.4	92.0	95.8	86.4	106.6	92.4
10.	100.0	98.3	96.7	105.0	95.1	105.8	114.7	111.3	97.7
11.	109.5	104.2	106.8	120.9	106.1	115.7	120.5	115.7	109.8
12.	116.9	105.9	101.0	119.5	110.9	116.7	111.9	115.2	107.4
13.	120.5	111.3	102.0	134.3	112.2	118.8	117.2	114.2	114.1
14.	118.8	107.1	106.6	140.2	113.0	121.4	117.6	112.6	113.6

What was the rate per cent of decrease in the price of farm products from the first to the tenth year?

12. Answer the same question for the price of food. For the price of lumber. For the price of metals.

13. Make and solve 10 problems about relative prices, similar to Exercises 11 and 12.

14. What is the commission on 3,725 bales of cotton sold at $12.65 per bale, commission 5%?

15. A bill of books amounting to $500 was discounted $16\frac{2}{3}\%$ and 5%. What was the net amount of the bill?

16. A property valued at $80,500 was assessed at $\frac{3}{4}$ of its value. What was the tax at $\frac{5}{8}\%$?

17. When an owner's tax is $45.15 at the rate of $1\frac{1}{2}\%$, on what valuation is he taxed?

18. A fruit dealer stored 1000 bbl. of apples for 6 mo. at 5 cts. a barrel per month; the storage charge was 15% of the selling price. For how much did the 1000 bbl. sell?

19. A farmer stored 500 tons of cabbage for 3 mo. at 5 cts. a ton per month. The storage charge was 5% of the selling price. For how much did the 500 tons sell?

20. In a recent year the per capita expenditure for bread was \$17.92 and the per capita cost of meat \$9.68. The cost of meat was how many per cent of the cost of bread?

21. As in Exercise 20, the expenditure for clothing was \$19.18. The cost of bread was how many per cent of the cost of clothing?

22. During a recent year 70,900 persons took the Civil Service examinations in the United States, of whom 41% were appointed. How many received appointments?

23. There are 327,000 positions in the Executive Civil Service, 53$\frac{1}{3}$% of which are subject to competitive examinations. How many of these positions are subject to examinations?

24. The area of Lake Superior is 32,000 sq. mi., and that of Lake Michigan is 70.2% as large. What is the area of the latter?

25. A school district decides to build a schoolhouse costing \$10,000. The total property assessment of the district is \$2,000,000. What must be the rate of taxation (not counting the cost of collection) in order to raise the \$10,000?

26. There are in the district 115 property owners whose property is assessed as follows:

5 persons, each	\$95,000
10 persons, each	50,000
25 persons, each	10,000
50 persons, each	5,000
25 persons, each	1,000

Find the tax paid by one person in each class.

27. Find the total tax paid by the persons in each class.

28. The tax collector receives 1% on the amount collected and adds this to the amount collected from each owner. How much did he collect from each of the above owners, including his commission?

29. A house and lot valued at \$2,400 is assessed at $\frac{2}{3}$ of its value. How many dollars is this? What is the tax at $\frac{7}{8}$%?

CHAPTER XIII

INTEREST AND BANKING

INTEREST

369. A fee paid for the use of money is called **interest.**

370. The rate per cent paid for the use of money is called **the rate of interest.**

371. The money on which interest is paid is called the **principal.**

372. The sum of the principal and the interest for a given time is called the **amount.**

The Six-Per-Cent Method

373. Taking the year to be 12 months of 30 days each, the *six-per-cent method* is convenient for the following rates: 2%, 3%, 4%, and 6%.

INTEREST AT 6% ON $1.

Interest on $1 at 6% for 1 yr, is $.06. For 1 mo. is .005.
" " " " " " 2 mo. is .01. " 1 da. is .000$\frac{1}{6}$.

EXAMPLE: Find the interest on $350 at 6% for 2 yr. 4 mo. and 6 da.

Solution:

(1) Interest on $1 for 2 yr. at 6% is 2 × $.06 = $.12
(2) " " $1 " 4 mo. " 6% " 4 × .005 = .02
(3) " " $1 " 6 da. " 6% " 6 × .000$\frac{1}{6}$ = .001
(4) " " $1 " 2 yr. 4 mo. 6 da. at 6% = .141
(5) " " $350 " " " " " = 350 × $.141 = $49.35.

374. *In computing interest at 6%, take 6 cts. per dollar of the principal for each year, $\frac{1}{2}$ of a cent for each month, and $\frac{1}{6}$ of a mill for each day.*

WRITTEN EXERCISES

Find the interest on the following at 6%:

1. $500 for 2 yr. 8 mo. 12 da.
2. $360 for 4 yr. 9 mo. 10 da.
3. $450 for 2 yr. 11 mo. 5 da.
4. $700 for 6 yr. 1 mo. 18 da.
5. $1000 for 5 yr. 9 mo. 24 da.
6. $2,005 for 10 mo. 12 da.
7. $100.50 for 3 yr. 18 da.
8. $650.50 for 4 yr. 7 mo.
9. $755.75 for 5 yr. 6 mo. 20 da.
10. $900.80 for 3 yr. 5 mo. 15 da.
11. $45,000 for 1 yr. 5 mo.
12. $90.85 for 1 yr. 5 mo.
13. $185.60 for 3 yr. 5 mo. 15 da.
14. $400.75 for 5 yr. 6 mo. 20 da.
15. $475.90 for 4 yr. 7 mo.
16. $790.50 for 3 yr. 18 da.
17. $800.25 for 5 yr. 9 mo. 24 da.
18. $750.30 for 6 yr. 1 mo. 18 da.
19. $500.00 for 2 yr. 11 mo. 5 da.
20. $5,000 for 4 yr. 9 mo. 10 da.
21. $620 for 2 yr. 8 mo. 12 da.
22. $1,800 for 3 yr. 7 mo. 9 da.

375. Interest at 2% may be found by computing the interest on the whole sum for the whole time at 6% and taking $\frac{1}{3}$ of that result.

EXAMPLE: Find the interest on $350 for 2 yr. 4 mo. 6 da. at 2%.

Solution: (1) The interest at 6% is $49.35.
(2) $\frac{1}{3}$ of $49.35 = $16.45, the interest at 2%.

376. To find interest at 3%, we find the interest at 6% and take $\frac{1}{2}$ of it.

ORAL EXERCISES

1. How can the interest at 4% be found from that at 6%?

2. What must be added to the interest at 6% to find the interest at 7%?

3. What must be added to the interest at 6% to find the interest at 8%?

WRITTEN EXERCISES

Find the interest on the following:

1. \$300 for 4 yr. 9 mo. 10 da. @ 2%.
2. \$500 for 6 yr. 1 mo. 18 da. @ 8%.
3. \$451 for 5 yr. 9 mo. 24 da. @ 4%.
4. \$375 for 3 yr. 18 da. @ 7%.
5. \$400.50 for 4 yr. 7 mo. @ 3%.
6. \$525.50 for 5 yr. 6 mo. 20 da. @ 8%.
7. \$750.25 for 4 yr. 7 mo. 10 da. @ 2%.
8. \$800 for 2 yr. 8 mo. 12 da. @ 3%.

Direct Method

377. The interest for a given number of years and months can usually be best computed directly.

EXAMPLE: Find the interest on \$650 at 5% for 2 yr. 8 mo.

Solution: \$650 Principal
.05 Rate
\$32.50 Int. for 1 yr. 2 yr. 8 mo. = $2\frac{2}{3}$ yr.
$2\frac{2}{3}$
21.67
65.00
\$86.67 Int. for 2 yr. 8 mo.

378. *To find the interest, multiply the principal by the rate and the result by the time expressed in years.*

WRITTEN EXERCISES

Find the interest on the following:

1. \$500 for 2 yr. 10 mo. @ 5%.
2. \$650 for 3 yr. 6 mo. @ 4%.
3. \$760 for 4 yr. 2 mo. @ 7%.
4. \$800 for 5 yr. 4 mo. @ 6%.
5. \$700 for 3 yr. 9 mo. @ 3%.
6. \$900 for 2 yr. 6 mo. @ 6%.

Exact Interest

379. The Government and some banks use 365 days as a year and the exact number of days between dates. Interest calculated in this way is called **exact interest.**

EXAMPLE: Find the exact interest on $150 for 55 da. at 6%.

PLAN: 55 da. = $\frac{55}{365}$ yr. The interest therefore is $\frac{55}{365}$ of .06 × $150.

380. The following calendar shows the exact number of days between any day of any month and the same day of any other month within a year, leap years excluded:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January..	365	31	59	90	120	151	181	212	243	273	304	334
February.	334	365	28	59	89	120	150	181	112	242	273	303
March ...	306	337	365	31	61	92	122	153	184	214	245	275
April.....	275	306	334	365	30	61	91	122	153	183	214	244
May.....	245	276	304	335	365	31	61	92	123	153	184	214
June.....	214	245	273	304	335	365	30	61	92	122	153	183
July.....	184	215	243	274	304	335	365	31	62	92	123	153
August...	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	243	273	303	334	365	30	61	91
October ..	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	96	121	151	182	212	243	274	304	335	365

Thus, from February 17 to July 17 is 150 da. The 150 is found opposite February and under July. Then from February 17 to July 29 is 150 da. + 12 da. = 162 da.

WRITTEN EXERCISES

1. Find the exact interest on $6,000 from May 5 to August 16 at 6%.

Suggestion: Indicate the interest on $6,000 for 1 yr. at 6%; then for 1 da.; then for the exact number of days; finally, cancel and reduce.

Find the exact interest on:

2. $800 from April 15 to June 21 at 5%.

3. $15,000 from May 29 to June 24 at 4%.

4. $3,650 from April 2 to May 31 at 5%.

381. *Cancellation* is often used in computing interest for a given number of days.

Example: Find the interest on $240 at 6% for 93 days.

$$\textit{Solution:}\ \$.06\times\overset{2}{\cancel{240}}\times\frac{\overset{31}{\cancel{93}}}{\underset{3}{\cancel{360}}}=\$3.72.$$

WRITTEN EXERCISES

Find the interest on the following to nearest cent:

1. $500 for 63 da. @ 6%.

2. $350 for 93 da. @ 5%.

3. $400 for 120 da. @ 4½%.

4. $450 for 30 da. @ 4%.

5. $760 for 63 da. @ 3½%.

6. $325 for 93 da. @ 6%.

Interest Tables

382. Bankers and others frequently use an **interest table.**

A part of an interest table is shown opposite. The complete table consists of a similar page for each dollar from $1 to $99, for multiples of $100, and for multiples of $1000.

Thus, the interest on $87 at 3% for 33 da. is found in the column headed 3%, opposite 33 in the central or time column. Result, $.24.

Example: What is the interest on $87 for 2 yr. 3 mo. 18 da. at 6%?

Solution:

(1)	Interest for 2 yr. (from table)	$10.44
(2)	“ “ 3 mo. “ “	1.31
(3)	“ “ 18 da. “ “	.26
(4)	“ “ 2 yr. 3 mo. 18 da.	$12.01

ORAL EXERCISES

Find from the table the interest on $87 for:

1. 27 da. at 12%. **2.** 4 yr. at 4%. **3.** 11 mo. at 8%.

Solve:

4. A note for $87 runs 93 da. at 6%. What is the interest?

5. How can interest at 2½% be found from interest at 5%?

87 DOLLARS.

$87

3%	4%	5%	6%	TIME.	8%	7%	10%	12%
				DAYS' GRACE.				
.24	.32	.40	.48	33	.64	.56	.80	.96
.46	.61	.76	.91	63	1.22	1.07	1.52	1.83
.67	.90	1.12	1.35	93	1.80	1.57	2.25	2.70
				DAYS.				
.01	.01	.01	.01	1	02	.02	.02	.03
.01	.02	.02	.03	2	.04	.03	.05	.06
.02	.03	.04	.04	3	.06	05	.07	.09
.03	.04	.05	.06	4	.08	.07	.10	.12
.04	.05	.06	.07	5	.10	.08	.12	.15
.04	.06	.07	.09	6	.12	.10	.15	.17
.05	.07	.08	.10	7	.14	.12	.17	.20
.06	.08	.10	.12	8	.15	.14	.19	.23
.07	.09	.11	.13	9	.17	.15	.22	.26
.07	.10	.12	.15	10	.19	.17	.24	.29
.08	.11	.13	.16	11	.21	.19	.27	.32
.09	.12	.15	.17	12	.23	.20	.29	.35
.09	.13	.16	.19	13	.25	.22	.31	.38
.10	.14	.17	.20	14	.27	.24	.34	.41
.11	.15	.18	.22	15	.29	.25	.36	.44
.12	.15	.19	.23	16	.31	.27	.39	.46
.12	.16	.21	.25	17	.33	.29	.41	.49
.13	.17	.22	.26	18	.35	.30	.44	.52
.14	.18	.23	.28	19	.37	.32	.46	.55
.15	.19	.24	.29	20	.39	.34	.48	.58
.15	.20	.25	.30	21	.41	.36	.51	.61
.16	.21	.27	.32	22	.43	.37	.53	.64
.17	.22	.28	.33	23	.44	.39	.56	.67
.17	.23	.29	.35	24	.46	.41	.58	.70
.18	.24	.30	.36	25	.48	.42	.60	.73
.19	.25	31	.38	26	.50	.44	.63	.75
.20	.26	.33	.39	27	.52	.46	.65	.78
.20	.27	.34	.41	28	.54	.47	.68	.81
.21	.28	.35	.42	29	.56	.49	.70	.84
.22	.29	.36	44	30	.58	.51	.73	.87
				MONTHS.				
.44	.58	.73	.87	2	1.16	1.02	1.45	1.74
.65	87	1.09	1.31	3	1.74	1.52	2.18	2.61
.87	1.16	1.45	1.74	4	2.32	2.03	2.90	3.48
1.09	1.45	1.81	2.18	5	2.90	2.54	3.63	4.35
1.31	1.74	2.18	2.61	6	3.48	3.05	4.35	5.22
1.52	2.03	2.54	3.05	7	4.06	3.55	5.08	6.09
1.74	2.32	2.90	3.48	8	4.64	4.06	5.80	6.96
1.96	2.61	3.26	3.92	9	5.22	4.57	6.53	7.83
2.18	2.90	3.63	4.35	10	5.80	5.08	7.25	8.70
2.39	3.19	3.99	4.79	11	6.38	5.58	7.98	9.57
2.61	3.48	4.35	5.22	12	6.96	6.09	8.70	10.44
				YEARS				
5.22	6.96	8.70	10.44	2	13.92	12.18	17.40	20.88
7.83	10.44	13.05	15.66	3	20.88	18.27	26.10	31.32
10.44	13.92	17.40	20.88	4	27.84	24.36	34.80	41.76
13.05	17.40	21.75	26.10	5	34.80	30.45	43.50	52.20
3%	4%	5%	6%	TIME.	8%	7%	10%	12%

WRITTEN EXERCISES

What is the interest on $87 for:

1. 7 mo. 8 da. at 6%?

2. 1 yr. 2 mo. 28 da. at 4%?

3. 5 mo. 10 da. at 2½%?

4. 2 yr. 9 mo. at 4½%?

5. 5 mo. 13 da. at 7%?

6. 3 yr. 6 da. at 5%?

7. 5 yr. at 3½%?

8. 10 mo. 23 da. at 2%?

As we have only one page of the table, the principal must remain $87 in our problems. With the complete table, interest on any principal is found similarly. To find the interest on $7,849, the interest would be found (for the given time and rate) on the pages for $7,000, $800, $49, and these amounts would be added.

To Find the Time

383. To find the time when the principal, the rate, and the interest are given,

Divide the given interest by the interest on the principal for 1 yr. at the given rate.

EXAMPLE: The interest on $300 for a certain time at 6% was $30. What was the time?

Solution: (1) 6% of $300 = $18, the interest for 1 yr.
(2) $30 ÷ $18 = 1⅔, the number of years.
(3) 1⅔ yr. = 1 yr. 8 mo.

WRITTEN EXERCISES

Fill the blanks in the table:

	PRINCIPAL.	RATE.	TIME.	INTEREST.
1.	$500	6%	( )	$45.00
pd f **2.**	$300	5½%	( )	$67.50
3.	$250	4½%	( )	$33.75
4.	$400	3%	( )	$64.00
5.	$475	2½%	( )	$35.625
6.	$600	5%	( )	$100.00
7.	$900	7%	( )	$84.00

To Find the Rate

384. To find the rate when the principal, the interest, and the time are given,

Divide the interest by the interest on the principal for the given time at 1%.

EXAMPLE: The interest on $200 for 1 yr. 6 mo. is $15. What is the rate?

Solution: (1) 1% of $200 × 1½ = $3, the interest at 1%.
(2) $15 ÷ $3 = 5.
(3) Therefore, the rate of interest is 5%.

WRITTEN EXERCISES

Fill the blanks in the table:

	PRINCIPAL.	RATE.	TIME.	INTEREST.
1.	$500	( )	2 yr. 6 mo.	$67.50
2.	$300	( )	90 da.	$2.25
3.	$250	( )	60 da.	$2.50
4.	$400	( )	120 da.	$4.00
5.	$500	( )	1 yr. 3 mo.	$25.00
6.	$360	( )	3 yr. 8 mo.	$79.20
7.	$475	( )	2 yr. 9 mo.	$104.50
8.	$800	( )	4 yr. 6 mo.	$72.00

To Find the Principal

385. When the rate, time, and interest are given, the principal can be found by

Dividing the interest by the interest on $1 for the given time and rate.

EXAMPLE: What principal yields $25 in 2 yr. 6 mo. at 5%?

Solution: (1) 5% of $1 × 2½ = $.125, the interest on $1.
(2) $25 ÷ .125 = $200, the principal.

WRITTEN EXERCISES

Fill the blanks in the table:

	PRINCIPAL.	RATE.	TIME.	INTEREST.
1.	( )	6%	60 da.	$5.00
2.	( )	5%	90 da.	$7.50
3.	( )	4½%	120 da.	$9.00
4.	( )	3%	1½ yr.	$18.00
5.	( )	2%	2½ yr.	$22.50
6.	( )	3½%	2 yr. 9 mo.	$60.75
7.	( )	4%	3 yr. 8 mo.	$36.66⅔
8.	( )	5%	4 yr. 3 mo.	$31.875

PROMISSORY NOTES

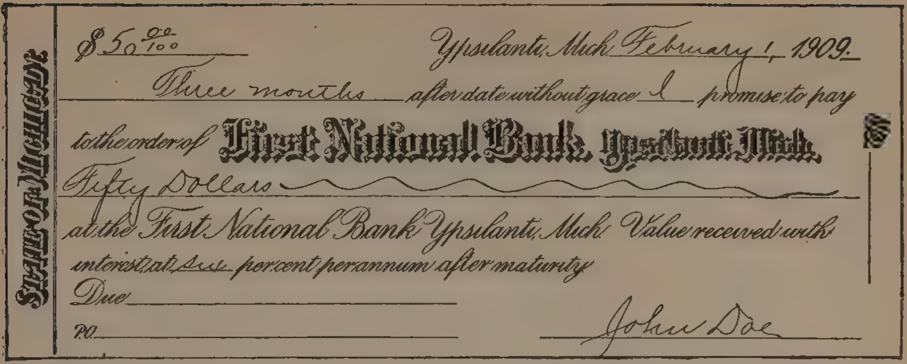
386. When money is borrowed, the borrower usually gives the lender a written acknowledgment of the indebtedness with a promise to pay a certain sum of money with interest at a certain time and place. Such a paper is called a **promissory note,** or simply a **note.**

387. The person who signs the note (agrees to pay) is the **maker,** and the person to whom it is to be paid is the **payee.**

388. The sum loaned is called the **face** of the note, and the face plus the interest is called the **amount.**

389. A note is said to **mature** on the day when it is legally due. When the time is specified in days, exact days are counted in finding maturity; when in months, calendar months are counted. Generally, if a note matures on a holiday or on Sunday, it is payable the day following.

390. Many states still allow **three days of grace** for the payment of notes. Thus, a 30-day note would be legally due in 33 days. When the law allows "days of grace," but does not compel their use, many notes are drawn to read "without grace."



State of Michigan

$50 00/100 Ypsilanti, Mich February 1, 1909.

Three months after date without grace I promise to pay

to the order of First National Bank, Ypsilanti, Mich.

Fifty Dollars

at the First National Bank Ypsilanti, Mich. Value received with

interest at six per cent per annum after maturity

Due

P.O.

John Doe

Who is the maker of the note above? The payee? What is its face? Time? Rate of interest?

WRITTEN EXERCISES

Write notes to cover the following conditions, all to be dated the day of writing:

	MAKER.	PAYEE.	FACE.	TIME.	RATE OF INTEREST PER ANNUM.
1.	John Smith	Henry Jones	$125	3 mo.	5%
2.	Richard Roe	John Doe	$87	2 mo.	4%
3.	Yourself	Your teacher	$580	6 mo.	6%

391. If the payee's name only is written in the note, it cannot be sold; but if it reads "*Pay to the order of* ——," or "*to bearer,*" it can be sold, and is called a **negotiable note.**

392. When a note is transferred, the payee named must first write his signature across the back. This is called **indorsing** the note. Every indorser of a note is responsible for its payment, unless he writes also the words "*without recourse.*"

The responsibility of the indorser ceases upon the maturity of the note unless he is promptly notified of its non-payment, but the maker of the note is responsible for its payment until it is paid.

If the payee simply writes his name across the back, he is said to indorse it **in blank.** This makes it payable to the bearer. But if he designates the payee, he is said to indorse it **in full.**

ORAL EXERCISES

1. What is meant by the face of a note?

2. When does a 6-months' note dated April 1 mature without grace? With grace? Are days of grace allowed in your state?

3. A note is drawn by Richard Roe to pay John Doe. Who is the payee?

4. What is meant by a note being negotiable? What must John Doe do to the note mentioned in Exercise 3 in order to make it negotiable?

5. How may a person indorse a note and not become responsible for its payment?

6. What is meant by indorsing a note "in blank"? "In full"?

WRITTEN EXERCISES

1. Write an indorsement in full for a note to your order.

2. Write an indorsement for a note to your order so as not to become responsible for the payment of the note.

Use the rate of interest common in your vicinity to find the amounts of the following notes:

3. Face $175, date Jan. 10, time 1 yr. 6 mo.

4. Face $360, date Apr. 15, time 2 yr. 3 mo. 15 da.

5. Face $680, date Feb. 10, time 1 yr. 6 mo. 21 da.

Solve:

6. Mr. Brown bought a house for $1,500. He paid $700 in cash and $800 by note at 5%, payable semiannually. What was the amount of interest due at each payment?

7. Mr. Lee bought a threshing machine for $300. He paid $150 in cash and gave his note for the rest with interest at 6%, payable annually. What was the amount of interest due at each payment?

8. A dealer sold an automobile for $1000, receiving $400 cash and a note for the rest due in 3 yr. with interest at 6%, payable semiannually. How much interest was paid on the note altogether?

9. A farmer borrowed $600 on three equal notes, one due in 1 yr., the second in 2 yr., and the third in 3 yr., all with interest at 5%, payable annually. How much was paid at the end of each year, if all payments were made when due? What was the total amount of interest paid?

10. Mr. Perkins bought a house and lot for $6,000; he paid $900 cash and gave three notes of equal amounts for the balance, due respectively in 4, 8, and 12 mo. with interest at 4½%. What sum was due at each payment?

11. Mr. Williams purchased an apartment building for $12,000. He paid $4,000 cash and gave five notes of equal amounts for the rest, payable respectively 6, 12, 18, 24, and 30 mo. after date, with interest at 5%, payable semiannually. What was each semiannual payment?

12. A firm accepted the following settlement of an account amounting to $2,050: A note for $1000 due in 3 mo., one for $500 due in 6 mo.; and one for $550 due in 1 yr., all bearing 6%. What was the amount of these notes? Would the firm have received more or less by accepting a note for $2,050 for 6 mo. at 6%?

Partial Payments

393. Notes often contain a provision permitting the borrower to pay a part of the principal before the note matures. Such advance payments are usually made on "interest days," the days on which interest is due.

394. When a partial payment of a note is made, the payment is indorsed on the back of the note. This indorsement serves as a receipt.

395. Preparatory.

1. A note for $500 dated Jan. 2, 1909, due in 5 yr. with interest at 6% payable annually, provides the privilege of prepayment in multiples of $50 on interest days. When is the first interest due? How much is it? If the maker pays $100 of the face besides the interest, how much does he still owe?

2. What is the interest Jan. 2, 1911? If $100 of the face is paid on this date, how much does the maker still owe?

WRITTEN EXERCISES

1. A promissory note for $2,500 due in 4 yr. with interest at 4% payable annually includes the privilege of prepayment. If the maker prepays $500 and interest each interest day, what is the last payment of principal and interest?

2. On July 1, 1908, John Doe signed a note for $400 due in 3 yr., interest at 5%, payable semiannually with the privilege of prepayment on interest days. He paid $150 and interest at each opportunity. What was the interest Jan. 1, 1909? What amount did he still owe? Find the amount still due after each further payment until the obligation was settled.

3. A note for $500, dated Jan. 2, 1907, with interest at 6%, payable annually for 3 yr. with privilege of prepaying $100, or more, on interest days, had the following indorsements:

Received, Jan. 2, 1908, $130. John Doe.
" " 2, 1909, 224. John Doe.

Each indorsement included the interest paid on that date. How much was due Jan. 2, 1910?

4. Upon a note for $350 dated July 1, 1908, with interest at 5%, payable semiannually, the following payments, including interest, were indorsed:

Received, Jan. 1, 1909, $58.75. Richard Roe.
" July 1, 1909, 82.50. Richard Roe.
" Jan. 1, 1910, 105.63. Richard Roe.

What was due July 1, 1910?

In the following notes the interest is payable annually, and prepayment of any sum is permitted on interest dates. Find the amount due at the date of settlement:

	FACE.	RATE.	DATE OF NOTE.	INDORSEMENTS.		DATE OF SETTLEMENT.
5.	$525	4%	June 15, 1905	June 15, 1906,	$125	June 15, 1909
				June 15, 1907,	250	
				June 15, 1908,	125	
6.	250	6	April 1, 1907	April 1, 1908,	120	April 1, 1910
				April 1, 1909,	110	
7.	1000	5½	Aug. 20, 1906	Aug. 20, 1907,	425	Aug. 20, 1909
				Aug. 20, 1908,	425	
8.	500	6	May 1, 1905	May 1, 1906,	180	May 1, 1910
				May 1, 1907,	121	
				May 1, 1908,	115	
				May 1, 1909,	109	

INSTALMENT PAYMENTS

WRITTEN EXERCISES

1. Mr. Peters buys a house for $6,500, on which he pays $500 cash, and the remainder in monthly payments of $25, with interest at 6%.

That is, at the end of each month he pays the interest for one month on the sum still due, and besides pays an *instalment* of $25 on that sum. For example, at the end of the first month he pays $25 and the interest on the $6,000 for one month, leaving $5,975 still due. How much is his first monthly payment? His second? His third? His fourth?

How much has the sum due been reduced in one year? What is still due at the end of the first year?

2. What was his payment at the end of the first month of the second year? The second month?

3. What was his payment at the end of the first month of the third year? The second month? How much has he paid off in ten years? What was his payment at the end of the first month of the eleventh year? The second month?

4. How long would it take him to pay for the house completely? What was his last payment?

5. Make and solve six problems about the accompanying advertisement.

6. Miss Martin bought a piano for $400 on the instalment plan, paying $40 at the time of purchase, and the remainder in monthly instalments of $10 each, plus the interest on the unpaid balance at 6%. What was her first monthly payment? Her second?

FOR SALE—A BARGAIN

$7,450

Only $750 cash and $50 per month; 4912 Washington Place; interest 4 1-2 per cent; 2-story stone front 8-room residence; quarter-sawed oak finish, steam heat; tile bathroom. Open for inspection.

7. What was her payment at the end of the first month of the second year? How long did it take to pay for the piano?

BANKING

396. Many banks receive deposits and issue check books to the depositor. The check book contains blanks which when filled out are orders upon the bank to pay money and charge the same to the depositor's account.

WRITTEN EXERCISES

1. A man deposited $580.60 and drew by check $432.90. How much had he left?

2. A depositor had $365 to his credit. He deposited a note for $85 and drew by check $105.50. How much had he then in the bank?

3. A depositor placed in the bank: $236.15 and $906.12; he drew by check $26.84, $83.33, $100, $11.45, and $250. How much had he left on deposit?

4. On Jan. 1 a man's bank balance was $835.16; during January he drew by check $110.50, $15.60, $25.30, $75.50. What was his balance February 1?

397. In depositing money at a bank, it is customary to file a memorandum, or **deposit slip.**

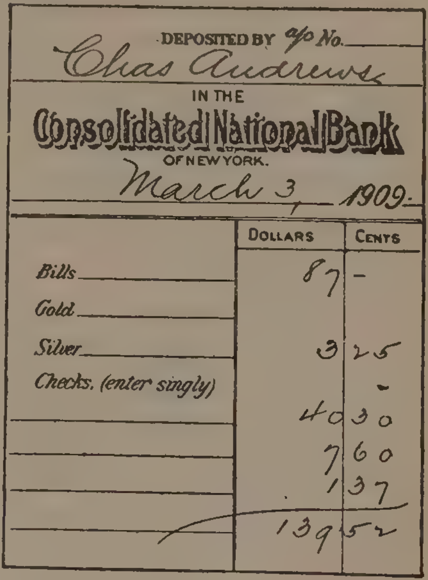
WRITTEN EXERCISES

1. What was the total amount deposited, according to the deposit slip shown?

2. What was the amount deposited by means of checks? By means of bills and silver?

3. A man entered on a deposit slip gold, $25; silver, $123.75; and checks, $108.50, $17.50, and $216.25. How much did he deposit in all?

Find the total amount of deposits made by a merchant on each of the following dates:



DEPOSITED BY a/c No.
Chas Andrews
IN THE
Consolidated National Bank
OF NEW YORK.
March 3, 1909

	DOLLARS	CENTS
Bills	87	-
Gold		
Silver	3	25
Checks, (enter singly)		
	40	30
	7	60
	1	37
	139	52

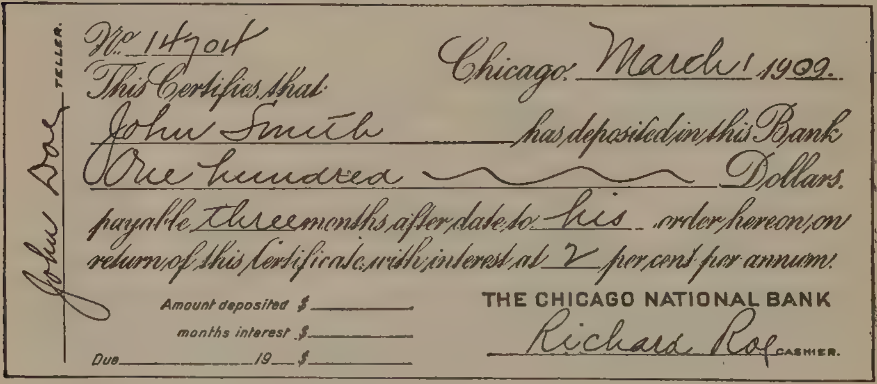
	4.	5.	6.	7.
ITEMS.	NOV. 6.	NOV. 13.	NOV. 20.	NOV. 27.
Gold...........	$ 50	$ 50	$ 20	$ 10
Silver..........	230	189	300	40
Bills...........	185	264.50	160	300
Checks.........	380.59	740.82	590.94	1,265.17

Certificates of Deposit

398. No interest is paid by banks on money subject to check, unless the daily balance is regularly quite large. Banks will usually pay interest on money not subject to check. In this case a **certificate of deposit** is issued, stating conditions of deposit. If the money is repayable at a fixed time named in the certificate, it is a **time certificate of deposit.** If the money is repayable at any time the holder of the certificate may desire, it is a **demand certificate of deposit.** The amount for which the certificate is drawn is called its **face.**

If the deposit is made for a definite time, the interest is sometimes computed in advance on the sum deposited, and included in the face of the time certificate. The certificate then bears no further interest.

Usually no interest is paid on a demand certificate unless the money is allowed to remain at least a specified time.



No. 14704 Chicago, March 1 1909.

This Certifies that John Smith has deposited in this Bank One hundred Dollars.

payable three months after date to his order hereon, on return of this Certificate, with interest at 2 per cent per annum.

Amount deposited $

months interest $

Due 19 $

John Doe TELLER.

THE CHICAGO NATIONAL BANK

Richard Roe CASHIER.

WRITTEN EXERCISES

1. Is the above a time or a demand certificate of deposit? Find the numbers to fill the blanks in the lower left corner of the certificate.

2. Write a demand certificate of deposit for $50 on the Chicago National Bank.

The following demand certificates of deposit bear interest at 2% per annum provided the deposit is allowed to remain at least 60 da. Find the amount of interest received in each case:

	FACE.	DATE OF DEPOSIT.	DATE OF PAYMENT.
3.	$125	Sept. 6, 1909	July 6, 1910
4.	50	Sept. 16, 1909	Oct. 16, 1910
5.	200	Jan. 2, 1909	May 2, 1909
6.	175	July 5, 1910	Sept. 16, 1910

7. Mr. Walker deposited $150 in the Second National Bank of St. Louis for 4 mo., with interest at the rate of 2½% per annum. What was the face of his certificate, drawn to include the interest?

Compute the faces of the following certificates of deposit, including interest:

	AMOUNT DEPOSITED.	TIME.	RATE OF INTEREST.	FACE.
8.	$225	6 mo.	3%	( )
9.	120	8 "	3¼%	( )
10.	450	3 "	2½%	( )
11.	670	2 "	2¼%	( )
12.	1,500	2 "	2¼%	( )
13.	2,500	3 "	2½%	( )
14.	2,750	10 "	4%	( )

15. Write a demand certificate of deposit with face $225, bearing 2% interest, and payable to yourself.

Bank Discount

399. If the holder of a promissory note desires to use the money promised before the time of payment, his bank will usually advance him the money, provided he can give satisfactory guarantee that it will be paid when due. This is called **discounting** the note, and the compensation of the bank for this service is called **bank discount.**

Discount, like interest, is computed on the face of the note at a certain rate per cent per annum for the time from the day on which the note is discounted until it is due.

In some localities both days are counted, and the method of exact interest is used.

400. The face of the note less the discount constitutes the sum paid to the holder and is called the **proceeds.**

Thus, if a note for \$100 having 60 da. to run is discounted at 6%, the discount is \$.99, and the proceeds \$99.01.

EXAMPLE: Find the discount and proceeds of a note for \$3,650, without interest, dated Aug. 5, 1908, time 60 da., and discounted Aug. 29 at 6%.

Solution: (1) Aug. 5 + 60 da. = Oct. 4. Aug. 29 to Oct. 4 = 36 da.
(2) $\frac{36}{360}$ of .06 × \$3,650 = \$21.90, discount.
(3) \$3,650 − \$21.90 = \$3,628.10, proceeds.

WRITTEN EXERCISES

In solving the problems below, use 360 da. as 1 yr., and do not include the day of discount.

Find the proceeds:

	DATE OF NOTE.	FACE.	TIME.	DATE OF DISCOUNT.	RATE OF DISCOUNT.
1.	June 3, 1909	\$1,000	60 da.	June 25	6%
2.	July 16, 1909	3,500	90 "	Aug. 2	5%
3.	May 25, 1910	8,000	60 "	June 28	6%

401. If a note bears interest, the discount is usually computed upon the amount (the face plus the interest) to the date of maturity. The method varies.

EXAMPLE: Find the discount at 6% on a note of \$500 with interest at 6% for 3 mo., discounted 1 mo. after date.

Solution: (1) \$500 + $\frac{1}{4}$ × .06 × \$500 = \$507.50, the amount.
(2) $\frac{1}{6}$ × .06 × \$507.50 = \$5.08, the discount.
(3) \$507.50 − \$5.08 = \$502.42, the proceeds.

WRITTEN EXERCISES

1. If the notes in Exercises 1, 2, 3 on page 344 bear interest at 5%, find the proceeds.

Find the discounts and proceeds of the following notes:

	DATE OF NOTE.	FACE.	TIME.	DATE OF DISC.	RATE OF	
					INT.	DISC.
2.	Jan. 1, 1908	$250	30 da.	Jan. 15, 1908	6%	5%
3.	Feb. 15, 1908	350	60 "	Mch. 15, 1908	5½	5
4.	Dec. 20, 1907	400	90 "	Feb. 1, 1908	4½	4
5.	Jan. 1, 1909	425	120 "	Mch. 10, 1909	6	5
6.	June 1, 1908	600	1 yr.	Jan. 1, 1909	4	3½
7.	April 1, 1909	500	1½ "	Apr. 1, 1910	4½	4
8.	May 15, 1908	375	8 mo.	July 15, 1908	5	5
9.	Sept. 10, 1909	240	10 "	Jan. 15, 1910	5	5
10.	Nov. 1, 1907	225	6 "	Feb. 1, 1908	6	5½
11.	Mar. 25, 1908	875	4 "	May 25, 1908	5½	5

COMPOUND INTEREST

402. Interest that is computed on the original principal only is called **simple interest.**

403. It is the practice of savings banks to compute interest on a deposit at regular intervals and to add this to the depositor's credit, thus paying interest upon interest already earned. The total interest paid is called **compound interest.**

When the interest is added to the principal semiannually, it is said to be *compounded semiannually;* when added annually, it is said to be *compounded annually.*

Compound interest is almost invariably computed by the use of tables that usually give the result of compounding annually the interest on 1 dollar at various rates. The interest is found by deducting the principal from the given amount.

A small portion of an interest table limited to periods of 1 yr. to 5 yr. is given on the following page.

AMOUNT OF $1 AT COMPOUND INTEREST, COMPOUNDED ANNUALLY:

YEARS.	RATE: 2%	2½%	3%	4%	5%
1	1.02000	1.02500	1.03000	1.04000	1.05000
2	1.04040	1.05063	1.06090	1.08160	1.10250
3	1.06121	1.07689	1.09273	1.12486	1.15763
4	1.08243	1.10381	1.12551	1.16986	1.21551
5	1.10408	1.13141	1.15927	1.21665	1.27628

Thus, $1 at 3% compounded for 4 yr. amounts to $1.12551. This is found in the column under 3% and in the line opposite 4 at the left. The compound interest on $100 for 4 yr. at 3% would be 100 × $1.12551, and similarly for any other principal.

Interest otherwise compounded can easily be found from the tables. Thus, interest at 4% per annum compounded semiannually is the same as interest at 2% compounded annually for twice the number of years.

WRITTEN EXERCISES

Find from the table the amounts of the following, compounded annually:

	PRINCIPAL.	TIME.	RATE.
1.	$1,000	5 yr.	3%
2.	500	3 "	4
3.	650	4 "	2
4.	425	3 "	2½
5.	775	4 "	2

6. A man deposited $2,000 in a savings bank, July 1, at 3% per annum, interest credited semiannually. What interest was credited the following Jan. 1? What was the new principal?

7. What was the interest and new principal the next July 1? How much money would the man have in the bank at the end of 3 yr.?

RULES FOR PARTIAL PAYMENTS

1. There are various rules for determining the amount due at final settlement of an interest-bearing note having no fixed time for the payment of the interest, and on which the debtor is allowed to make partial payments at such times and of such amounts as may suit his convenience. Of these rules, the most important are the **Merchant's Rule** and the **United States Rule.**

2. The **Merchant's Rule** is as follows:

1. *Find the amount of the face of the note with interest from its date to the date of final settlement.*

2. *Find the amount of each partial payment with interest from its date to the date of settlement.*

3. *The sum due at final settlement is the amount of the face of the note less the sum of the amounts of the partial payments.*

EXAMPLE: A note for $500 with interest at 6% is dated Jan. 12, and bears indorsement of the following payments in the same year: April 27, $120; June 16, $132.60; Aug. 21, $217. What was due at final settlement, Dec. 3 of the same year?

Solution:

Amount of $500 for 10 mo. 21 da.		$526.75
Amount of $120 for 7 mo. 6 da.	$124.32	
Amount of $132.60 for 5 mo. 17 da.	136.29	
Amount of $217 for 3 mo. 12 da.	220.69	481.30
Due at final settlement		$45.45

WRITTEN EXERCISES

1. A note for $600 dated Feb. 3, with interest at 6%, bears indorsement of the following payments in the same year: Mar. 18, $123; April 27, $42.20; June 6, $136.75. What was due on Nov. 10 of that year?

2. A note for $390 with interest at 5% is dated March 18, and the following payments have been made on it in the same year: June 10, $260; Aug. 17, $135. What was due on Dec. 20 of that year?

3. A man purchased furniture costing $635. He paid $115 cash, and gave his note, dated Jan. 2, at 6%, for the balance. During that year he made the following payments on the note: June 14, $178; Aug. 12, $92.40; Nov. 1, $127.20. What was due on Dec. 30 of that year?

4. A note for $286 is dated Aug. 12, and bears interest at 5½%. In the following year payments were made on it as follows: Jan. 5, $70; April 18, $120; Sept. 15, $30. What was due at the final settlement, Dec. 30 of that year?

5. On May 1 a peddler bought horse, wagon, and goods for $327, giving his note at 6% interest. He paid $29 on June 12, $53 on Aug. 19, and $32 on Oct. 12; the next summer he paid $46 on May 27, $82 on Sept. 3, and the balance on Nov. 1. How much did he then pay?

6. A note for $450, with interest at 5½%, dated Jan. 10, was settled a year after date. What was due at final settlement, if $120 was paid on May 1, and $140 on Sept. 15?

3. The **United States Rule** is the one commonly applied by courts in disputed cases, and should be used whenever another method is not designated. This rule is as follows:

1. *Compute the interest from the date of the note to that of the first partial payment.*

2. *If the interest equals or is less than the payment, add the interest to the principal and deduct the payment.*

3. *Regard the remainder as a new principal and proceed as before.*

4. *If the interest exceeds the payment, add this payment to the one next following and treat the sum as a simple payment made at the date of the later payment.*

Example: A note for $280 was dated Feb. 1, and bore interest at 6%. Payments were made on it in the same year as follows: March 1, $15; June 1, $3; July 1, $20; Sept. 1, $2.49; Dec. 1, $86.27. The note was settled a year after its date. What was the final payment?

Solution:

	Original principal	$280.00
Mar. 1.	Interest for 1 mo	1.40
		281.40
	Payment	15.00
	Second principal	266.40
June 1.	The interest for 3 mo. ($4.00) is more than the payment.	
July 1.	Interest for 4 mo	5.33
		271.73
	Payment ($3 + $20)	23.00
	Third principal	248.73
Sept. 1.	Interest for 2 mo	2.49
		251.22
	Payment	2.49
	Fourth principal	248.73
Dec. 1.	Interest for 3 mo	3.73
		252.46
	Payment	86.27
	Fifth principal	166.19
Feb. 1.	Interest for 2 mo	1.65
	Final payment	167.84

WRITTEN EXERCISES

1. A note for $340 is dated Dec. 1, and bears interest at 6%. During the next year payments are made on it as follows: Feb. 1, $3.40; March 1, $2; April 1, $82.50; July 1, $63.29. The note was settled 11 mo. after date. What was the final payment?

2. On June 1, a farmer bought implements to the amount of $133 from the village merchant, and gave his note at 6% for the amount. He sold the merchant butter and eggs, the value to be applied on the note as follows: June 13, $9.63; July 3, $27.18; July 31, $61.75. What was due at the final settlement on Sept. 19?

3. On May 10, a cattle dealer sold a farmer 5 cows at $80 each, taking his note at 5% interest for the amount, and agreeing that the farmer might from time to time at his convenience pay on the note his profits from the sale of milk. The following payments were made: June 2, $12.50; June 29, $18.36; July 20, $31.27; Aug. 25, $28.73; Sept. 30, $103.20. What was due at the final settlement on Nov. 15?

4. On April 15, a dealer sold to a man a merry-go-round outfit costing $650, for $100 cash and his note at 6% interest for the balance, with privilege of partial payments at his convenience. He made payments as follows: June 1, $8.50; June 23, $41.62; July 5, $123; Aug. 18, $372.15; and final settlement, Oct. 1. How much did he pay at that time?

5. A note for $900 at 5½% interest, dated July 1, 1908, bears indorsement of the following payments: Oct. 1, 1908, $200; Jan. 20, 1909, $100; July 6, 1909, $172.50. What is due at final settlement, Dec. 1, 1909?

6. A note for $1200 at 6% interest, dated Jan. 20, 1909, bears indorsement of the following payments: July 3, 1909, $125; Dec. 15, 1909, $20; June 23, 1910, $380; Aug. 10, 1910, $15. What is due at final settlement, Feb. 17 1911?

REVIEW

ORAL EXERCISES

1. What is meant by a promissory note? By the maker of a note? The payee?

2. How would you word a promissory note for $200 dated to-day and payable in 3 yr. with interest at 5%?

3. What is meant by days of grace? Are days of grace legal in your state?

4. What is meant by negotiable? What is indorsing a note? Illustrate an indorsement in blank. In full.

5. How is exact interest computed? What do business men commonly use in computing interest?

6. What is a certificate of deposit? A demand certificate? A time certificate?

7. How is the interest computed on deposits made in a savings bank?

8. What is bank discount? Illustrate your answer.

9. What is meant by proceeds? Illustrate your answer.

10. How does a partial payment note differ from an ordinary promissory note? How is the interest computed on such a note?

11. What is the interest at 5% on a promissory note for $500 for 6 mo.?

12. What is the amount of the promissory note of Exercise 11?

13. When does a 6 mo. note mature, dated Jan. 2, 1909, if drawn in a state where grace is allowed?

14. How may one indorse a note and not become responsible for its payment?

15. What is the bank discount at 6% on a note for $200 due in 3 mo.?

WRITTEN EXERCISES

Find the interest at 6% on the following notes:

1. $675.50 for 3 yr. 7 mo.

2. $1,007.75 for 4 yr. 5 mo.

3. $380.25 for 2 yr. 10 mo. 15 da.

4. $160 for 5 yr. 4 mo. 10 da.

Find the interest on:

5. $560 for 2 yr. 6 mo. 18 da. at 2%.

6. $840.90 for 3 yr. 11 mo. 10 da. at 3%.

7. $690 for 5 yr. 2 mo. 20 da. at 3½%.

8. $1,200 for 4 yr. 3 mo. 27 da. at 4½%.

Find the exact interest at 5% on:

9. $350 from May 1, 1909, to Feb. 15, 1910.

10. $275.75 from Dec. 14, 1910, to May 1, 1911.

11. $400 put out at interest at 3½% earned $21 in a certain time. How many years was it lent?

12. $350 was lent for 2 yr. 3 mo. and earned $27.56. What was the rate of interest?

13. A certain sum of money earned $45.84 at 5½% in 1 yr. 8 mo. What was the principal lent?

14. A man bought a house and lot for $8,000 and paid $1,500 in cash; he gave a note for the rest for 4 yr. 6 mo. at 5½%. What was the total interest due on the note?

15. A bank has a capital of $150,000 and a surplus (undivided profits) equal to 101.32% of this. Find the surplus.

16. Find the surplus in each case as in Exercise 15:

	(a)	(b)	(c)	(d)	(e)
Capital......	$250,000	$500,000	$300,000	$750,000	$100,000
Surplus.....	210.73%	134.82%	120.54%	119.73%	120.22%

17. At 4½% how long would it take $1,800 to earn $100 interest?

18. Make and solve five problems about the annexed advertisement, using $5\frac{1}{2}\%$ as the rate of interest.

BE YOUR OWN LANDLORD

6405 JACKSON AVE.

For Sale: New detached 2-story brick and stone residence, containing parlor, dining-room, kitchen, 4 bedrooms and bath; an ideal home. Price $5,500. ONLY $500 CASH and monthly payments of $40 and interest required to buy this house.

19. Find the interest upon the following certificates of deposit at 2%, allowing 30 da. from date of deposit before the certificate draws interest:

	FACE.	DATE OF DEPOSIT.	DATE OF PAYMENT.
(*a*)	$235.	May 2, 1912	March 15, 1913
(*b*)	46.50	Apr. 1, 1911	Dec. 20, 1911
(*c*)	186.25	June 15, 1910	July 10, 1911 (time = $\frac{355}{360}$ yr.)

20. Find the proceeds of the following notes:

	DATE OF NOTE.	FACE.	TIME.	DATE OF DISCOUNT.	RATE OF DISCOUNT.
(*a*)	June 20, 1909	$250	90 da.	July 25	$5\frac{1}{2}\%$
(*b*)	July 15, 1910	375	60 da.	Aug. 10	6
(*c*)	Feb. 1, 1911	1,950	90 da.	Apr. 25	5

21. On a note for $675, bearing 6% interest, dated Aug. 1, 1908, and due in 5 yr., the following payments were made:

Jan. 1, 1909, $200.
Jan. 1, 1910, $150.
June 1, 1911, $100.
July 1, 1911, $75.

Using the United States Rule, find the amount due at maturity.

22. What is the compound interest on $100 for 4 yr. at 3%, compounded annually?

23. What is the amount of $500 for 3 yr. at 4%, compounded annually?

24. What is the compound interest on $250 for 2 yr. at 2%, compounded semiannually?

25. At simple interest at 7%, how many years will it take $150 to earn $39.375?

26. A man bought a house for $10,000, paying $2,000 cash and $1000 annually with interest at $5\frac{1}{2}\%$. How much did he pay for the house in principal and interest?

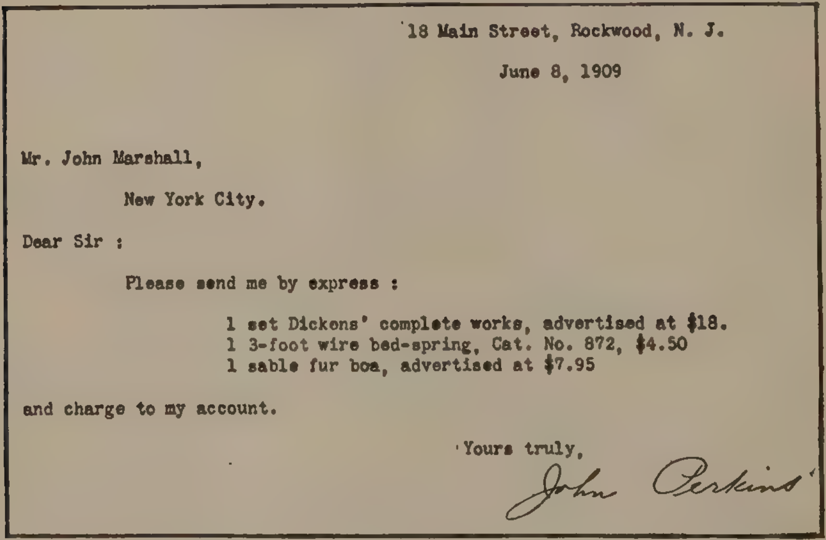
CHAPTER XIV

BUSINESS APPLICATIONS

ORDERING GOODS

WRITTEN EXERCISES

1. Make out a bill for the goods mentioned in this order:



18 Main Street, Rockwood, N. J.
June 8, 1909

Mr. John Marshall,
New York City.

Dear Sir :

Please send me by express :

1 set Dickens' complete works, advertised at $18.
1 3-foot wire bed-spring, Cat. No. 872, $4.50
1 sable fur boa, advertised at $7.95

and charge to my account.

Yours truly,
John Perkins

Write an order and a bill for each of the following, the purchases being made of Wiley Smith & Co., Chicago:

2. Roland Ames, Springfield, Missouri:

2 lawn mowers,	catalogue number	6900, each	$2.33.
2 doz. camp chairs,	" "	7053, "	.70.
½ doz. folding cots,	" "	7770, "	1.65.

3. A. R. Childs, Newark, New Jersey:

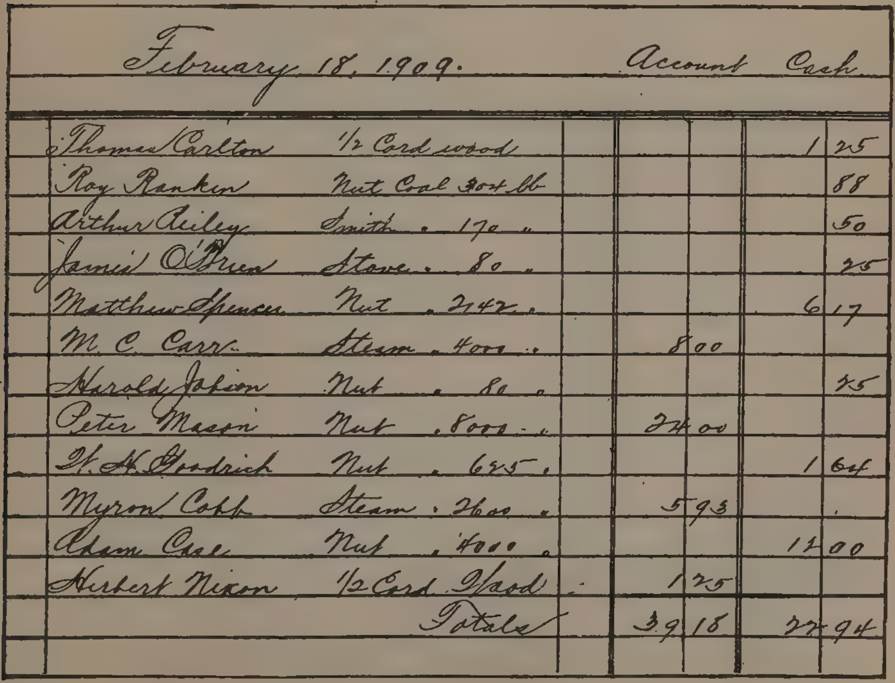
Two 5-pound cans Java coffee, at 31½ cts. a pound.
5 bbl. flour, catalogue number A 723, at $5.50 a barrel.
3 doz. cans sardines, catalogue No. A 271, at $4.25 a dozen.

RECORD OF SALES

404. A common method of recording sales is that of noting in a book, called a **day book,** the name of the customer, a description of the goods bought, and whether the price is charged to the customer's account or paid in cash.

WRITTEN EXERCISES

1. The following is a coal dealer's record of one day's sales. Verify the sums in each column, and find the total amount of the sales.



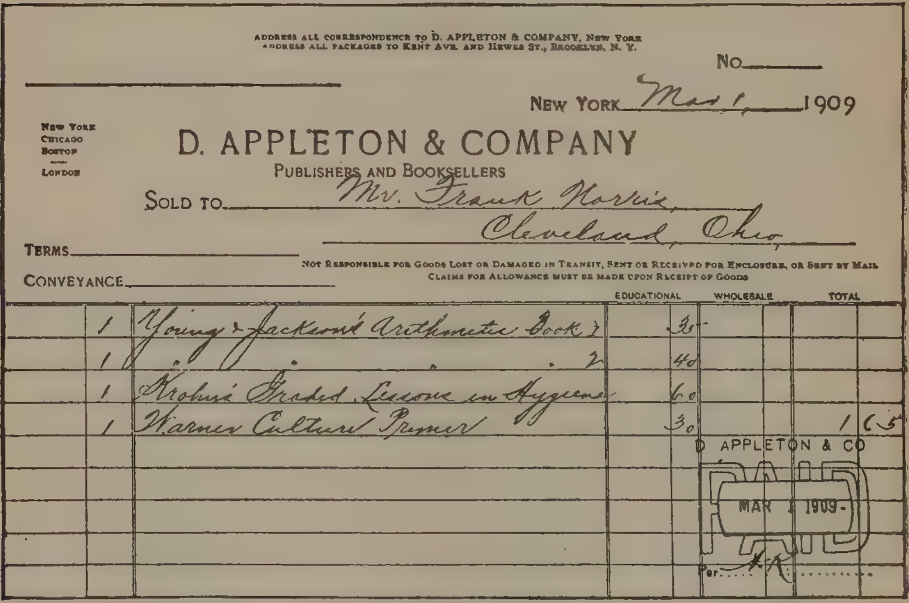
February 18, 1909.		Account		Cash	
Thomas Carlton	1/2 Cord wood			1	25
Roy Rankin	Nut Coal 304 lb				88
Arthur Riley	Smith " 170 "				50
James O'Brien	Stove " 80 "				25
Matthew Spencer	Nut " 2142 "			6	17
M. C. Carr	Steam " 4000 "	8	00		
Harold Jobson	Nut " 80 "				25
Peter Mason	Nut " 8000 "	24	00		
W. H. Goodrich	Nut " 625 "			1	64
Myron Cobb	Steam " 2600 "	5	93		
Adam Cole	Nut " 4000 "			12	00
Herbert Nixon	1/2 Cord Wood	1	25		
	Totals	39	18	22	94

2. On a certain day a dealer's cash items were: $2.50, $5.75, $11.16, $4.93, $.30, $1.15, $.33, $4.60; the account items were: $4.78, $13.60, $5.12, $3.17. Find the total.

3. Make other records and find the totals.

BILLS AND RECEIPTS

405. When a bill is paid it is customary to give a written acknowledgment of payment, called a **receipt.** The receipt may be either written or stamped on the bill itself, as in the bill shown below, or it may be drawn up separately.



ADDRESS ALL CORRESPONDENCE TO D. APPLETON & COMPANY, NEW YORK
ADDRESS ALL PACKAGES TO KENT AVE. AND HEWES ST., BROOKLYN, N. Y.

No.______

NEW YORK Mar 1, 1909

NEW YORK
CHICAGO
BOSTON
LONDON

D. APPLETON & COMPANY
PUBLISHERS AND BOOKSELLERS

SOLD TO Mr. Frank Norris,
Cleveland, Ohio,

TERMS______

CONVEYANCE______

NOT RESPONSIBLE FOR GOODS LOST OR DAMAGED IN TRANSIT, SENT OR RECEIVED FOR ENCLOSURE, OR SENT BY MAIL
CLAIMS FOR ALLOWANCE MUST BE MADE UPON RECEIPT OF GOODS

		EDUCATIONAL	WHOLESALE	TOTAL
1	Young & Jackson's Arithmetic Book 1	35		
1	" " " " 2	40		
1	Krohn's Graded Lessons in Hygiene	60		
1	Warner Culture Primer	30		1 65

D APPLETON & CO
PAID
MAR 1 1909
Per A.K.

WRITTEN EXERCISES

Make out receipted bills for the following items, with yourself as purchaser, and as seller some firm that deals in the commodities named:

1. 3 sacks of flour at $1.69, one 16-pound ham at 12½ cts. a pound, 9 lb. of tea at 39 cts. a pound.

2. 1 dining table, $27; 8 chairs at $2.98 each.

3. 6 violin strings at 19 cts. each; 1 cake rosin, 8 cts.; 3 copies of sheet music at 29 cts. each, 5 at 15 cts., and 1 at 60 cts.

Write receipts showing that you have made the following payments:

4. A monthly instalment of $10 for the current month on a piano that you have bought from a dealer known to you.

5. To the publisher of a local paper or magazine for one year's subscription in advance. (Use an actual name and rate.)

Make a receipted bill for each of the following:

6. 12 Geographies at $1 each, 125 Spellers at 22 cts. each, 300 Primers at 25 cts. each, 50 Arithmetics at 35 cts. each. Bought by A. Dunn from Baker, Taylor and Co.

7. E. D. Clark bought of R. H. Macy and Co.

40 yd. of silk at	$ .90 a yd.
120 dress patterns at	5.25 each
2 doz. blankets at	3.75 per pair
13 mattresses at	4.65 each

8. F. Klein purchased of Tiffany and Co.

5 gold watches at	$75.50 each
12 sets of silver knives at	48.00 a set
22 sets of silver spoons at	22.50 a set
5 silver trays at	27.50 each

9–13. Make five bills and receipt them, using the following prices:

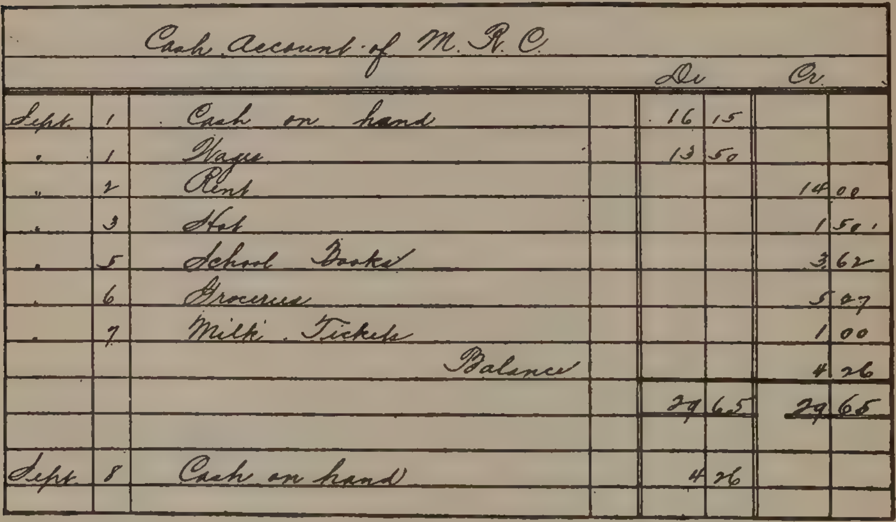
Men's hats	$3.50
“ shoes	4.00
“ hose	.75
Silk vests	2.25
Collars	1.50 per dozen.
Ties	.50 each

Calico	$.06	a yd.
Muslin	.08	“
Bleached muslin	.12	“
Broadcloth	1.75	“
Lansdowne	1.25	“
Cheviot	.75	“

CASH ACCOUNTS

406. An account of money received and paid out is called a **cash account.**

Dr. stands for debtor and *Cr.* for creditor. "Cash" may be thought of as a debtor to the person keeping the account for all money received and as creditor for all money paid out.



Cash Account of M. R. C.

				Dr.		Cr.	
Sept.	1	Cash on hand		16	15		
"	1	Wages		13	50		
"	2	Rent				14	00
"	3	Hat				1	50
"	5	School Books				3	62
"	6	Groceries				5	07
"	7	Milk Tickets				1	00
		Balance				4	26
				29	65	29	65
Sept.	8	Cash on hand		4	26		

WRITTEN EXERCISES

Make out and balance the following accounts:

1. Cash on hand Jan. 1, $500. Expenditures: Jan. 5, $80.30; Jan. 10, $30.75; Jan. 15, $4.25; Jan. 21, $106; Jan. 30, $40.80.

2. Cash on hand Monday, $90.50. Receipts: $10 a day for the week of 6 days. Expenditures: Monday, $7.50; Tuesday, $12.25; Thursday, $18.40; Saturday, $21.43.

3. Cash on hand July 1, $74.90. Receipts: July 6, $60.70; July 15, $50; July 21, $18.90; July 31, $50. Expenditures: July 2, $25.50; July 7, $6.50; July 16, $40.59.

4. Cash on hand Oct. 1, $185.50. Expenditures: Oct. 3, $16.50; Oct. 11, $89.70; Oct. 15, $5.53; Oct. 25, $17.75; Oct. 28, $15.90; Oct. 30, $12.

5. Jan. 1, '09, on hand $75. Expenditures: Jan. 2, books $3.50; Jan. 10, board $7.50; Jan. 15, rent $25; Jan. 25, clothes $22.50. Jan. 30, salary received $250.

Balance the following accounts:

6.

EXPENDITURES.		CASH.	
May 1,	$10.	On hand	$125.50
May 3	15.75		
May 5	8.30		
May 10	12.60	Int.	20.00
May 15	1.25		
May 25	50.00	Salary	175.00

7.

EXPENDITURES.		CASH.	
July 1,	$125.50	On hand	$1000
July 5	75.60	Profits	500
July 10	200.30		
July 16	190.05	Int.	300
July 25	400.25	Dividends	650
July 30	90.83	Profits	2010

REMITTANCES

Registered Mail

407. Money may be safely forwarded by mail in a letter that is **registered** at a post-office. The fee for this is 10 cents. A receipt showing the delivery of the letter is returned to the sender by the post-office.

Postal Money Orders

408. Payment to a person at a distance may be made also by sending him a **postal money order** purchased at a post-office. This is an order directing the postmaster at the place where the other person lives to pay him the money. The person presenting the order for payment must be known to the postmaster or be identified.

The fees charged for money orders are:

For orders for sums not exceeding	$2.50	3 cts.
“ “ over $2.50 and not exceeding	5.00	5 “
“ “ “ 5.00 “ “	10.00	8 “
“ “ “ 10.00 “ “	20.00	10 “
“ “ “ 20.00 “ “	30.00	12 “
“ “ “ 30.00 “ “	40.00	15 “
“ “ “ 40.00 “ “	50.00	18 “
“ “ “ 50.00 “ “	60.00	20 “
“ “ “ 60.00 “ “	75.00	25 “
“ “ “ 75.00 “ “	100.00	30 “

The money is not actually sent from one post-office to the other. The transfer is merely a matter of book-keeping, since the Government has received the money at one office and paid it out at another.

ORAL EXERCISES

1. State the fee for the following post-office orders:

$85	$3.75	$10.25	$40	$55.60	$75	$63.25
95	7.20	15.60	45	65.80	30	83.20

Telegraphic Money Orders

409. The most rapid method of remitting money is by telegraph. The sum to be remitted is paid to the agent of a telegraph or express company, who telegraphs the agent at any other place to pay out an equal sum to the person named.

A common charge is 1% of the sum sent in addition to double the usual rate for a 10-word message. The following are the rates for such messages from Chicago to the places named:

San Francisco	75 cts.	New Orleans	50 cts.
Detroit	25 “	Galveston	60 “
New York	40 “	Kansas City	30 “

ORAL EXERCISES

1. What is the cost of sending $150 by telegraphic order from Chicago to San Francisco, at the charge stated above?

2. Make and solve five similar problems.

Cabled Money Orders

410. The cost of cable messages is often reduced by using one word to represent several. A list of such words is called a **code.** Both the sender and the receiver must have the code. The following will illustrate:

WORD.	MEANING.
Fichant	One hundred dollars.
Feststadt	Fifty dollars.
Ficheron	One thousand dollars.
Cicatrix	American Express Company's office.
Fenderent	Have cabled \$— through —— at ——.

ORAL EXERCISES

1. Mr. Marks, traveling in London, receives the following message from Mr. Case in New York City:

Marks, Ten Oxford Street, London.
Fenderent Ficheron Cicatrix London. Case.

Translate the above message. What did it cost at 25 cts. a word, including the address and the signature? What was the cost of cabling the money at 1½% of the sum sent?

2. Make and solve other problems on the cost of cabled messages.

Express Payments

411. Money may be *shipped by express*, and large payments in gold are so shipped. But small amounts are usually paid by an **express money order.**

The plan and rates are similar to those of the postal order.

ORAL EXERCISES

1. Using the rates given on page 356 state the fee for each of the following express orders:

\$9.75 \$75.00 \$11.60 \$99.50 \$40.15 \$.65

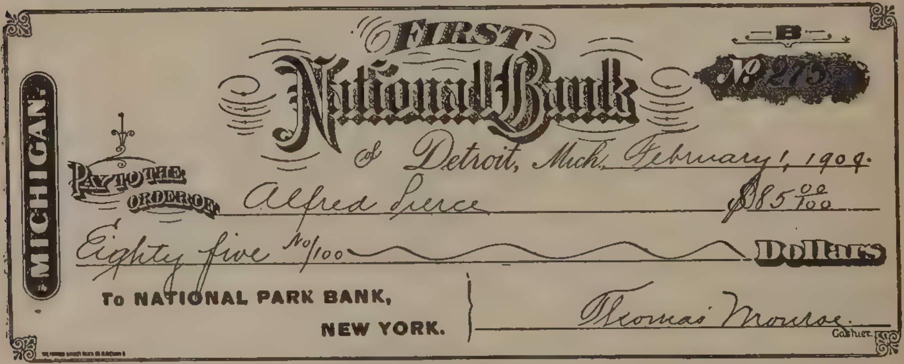
Checks

412. A check, drawn upon a bank in which the sender has a deposit, also may be sent. The person receiving the check may deposit it in a bank in his locality. That bank will collect the amount of the check from the bank on which it was drawn.

Many banks charge a small fee for collecting a check; consequently business houses object to this method of remittance and it is customary to use some other form.

Bank Drafts

413. Money is frequently sent by mail by means of **bank drafts.** A bank draft is merely a check drawn by one bank upon another, and may have the form here shown.



B
No. 275

FIRST National Bank of Detroit, Mich. February 1, 1909.

MICHIGAN.

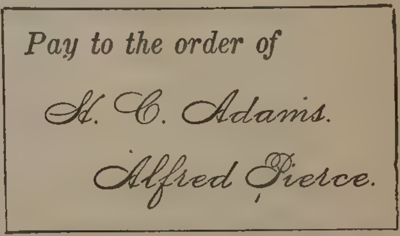
PAY TO THE ORDER OF Alfred Pierce $85 00/100

Eighty five no/100 Dollars

To NATIONAL PARK BANK,
NEW YORK.

Thomas Monroe
Cashier.

The bank will charge Mr. Pierce $85 and possibly a small fee in addition for this draft. Mr. Pierce will indorse this draft in full and mail it to Mr. Adams, to whom he wishes the money paid. The National Park Bank at New York will pay Mr. Adams $85 on presentation of the draft. The indorsement is as here shown. If Mr. Adams desires, he may deposit this draft with some other bank for collection.



Pay to the order of

H. C. Adams.

Alfred Pierce.

WRITTEN EXERCISES

1. Write a draft drawn by the cashier of the Mechanics' Bank of Rochester for yourself, payable at the National Park Bank of New York City.

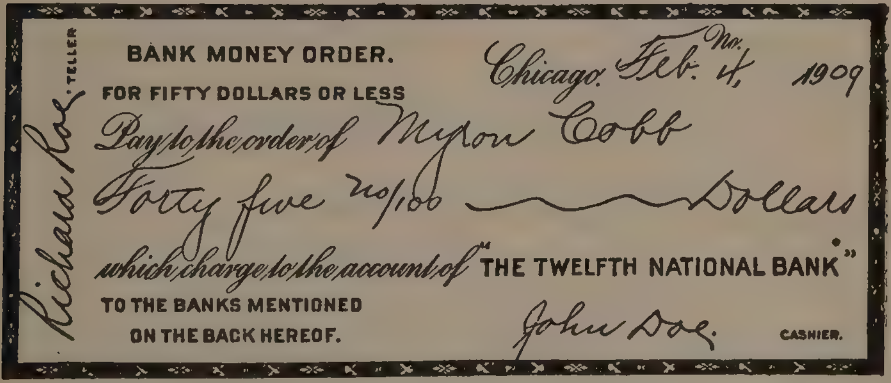
2. Indorse this draft in favor of John Shipman.

3. The sender may have the draft drawn payable to the creditor, in which case no indorsement is necessary. Write such a draft.

4. The sender may have a draft drawn payable to himself and indorse it in favor of any other person. Write such a draft.

Bank Money Orders

414. Banks often issue money orders. The following is a specimen:



BANK MONEY ORDER.
FOR FIFTY DOLLARS OR LESS
No.
Chicago. Feb. 4, 1909
Pay to the order of Myron Cobb
Forty five no/100 —— Dollars
which charge to the account of "THE TWELFTH NATIONAL BANK"
TO THE BANKS MENTIONED
ON THE BACK HEREOF.
John Doe. CASHIER.
Richard Roe. TELLER

On the back of such orders is printed a list of the banks that have agreed to pay these orders when presented. But usually any bank will pay them on satisfactory identification of the holder. The orders are marked "Fifty dollars or less," "One hundred dollars or less," "One thousand dollars or less," etc., according to the amount of the order.

415. The amount charged by a bank for a draft or a money order is called the **premium.**

ORAL EXERCISES

1. What is the premium at $\frac{1}{4}\%$ on a bank money order for $1000? What is the total cost of such an order?

2. What is the total cost of an order for $2,500 at $\frac{1}{8}\%$ premium?

3. Name two other kinds of money orders.

4. Name five ways of remitting money.

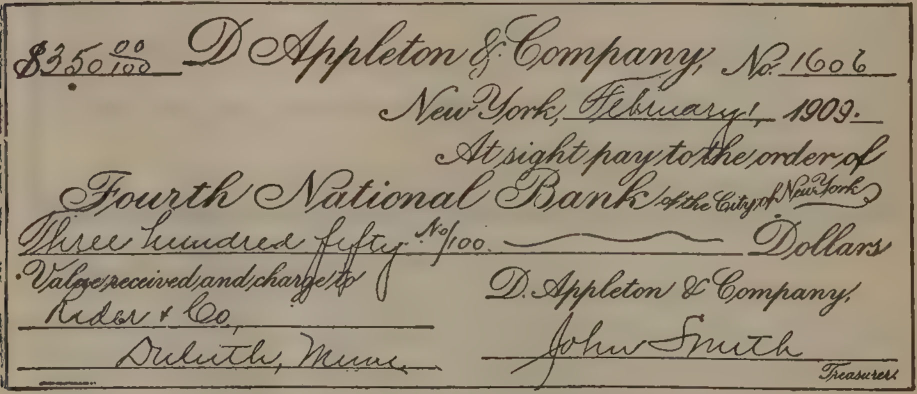
WRITTEN EXERCISES

1. Find the total cost of bank money orders for the following sums, premium $\frac{1}{4}\%$:

$1,500	$3,500	$10,650	$1,225
1,625	86.50	195.50	40.30

Commercial Drafts

The firm, Rider & Company of Duluth, owe D. Appleton & Company of New York $350 for books. D. Appleton & Company may collect the account by means of the following draft:



$350 00/100 D. Appleton & Company, No. 1606

New York, February 1, 1909.

At sight pay to the order of

Fourth National Bank of the City of New York

Three hundred fifty No/100 Dollars

Value received and charge to

Rider & Co.,

Duluth, Minn.

D. Appleton & Company,

John Smith

Treasurer

This is sent by the Fourth National Bank of New York to a bank of Duluth. This bank sends the draft by messenger to Rider & Co., who may accept the draft and pay the amount. If they refuse to pay it, the draft is returned to its maker.

416. Such a draft is called a **commercial draft;** and, since it was payable at sight, a **sight draft.** Instead of the words "at sight," the drawer may state any future time of payment. In that case, the draft is called a **time draft.** When the draft is presented, the debtor, if he intends to pay it, writes "Accepted" with date and signature across its face. An accepted time draft may be discounted like a note.

ORAL EXERCISES

1. John Doe draws a sight draft in favor of the Park National Bank of New York and charges the same to Richard Roe of Detroit, Michigan. In what bank does Mr. Roe deposit the draft for collection?

2. What kind of draft is the one mentioned in Exercise 1?

3. How would the Park National Bank proceed to collect this note?

4. What would become of the note if Richard Roe refused to accept it?

5. If it were a time draft for 60 da. and Mr. Roe accepted it, what would he do with the draft?

WRITTEN EXERCISES

1. The bank making the collection may charge a fee. What would be the cost of collecting a draft for $589.75 at the rate of ½%?

2. A time draft for $325 had been accepted; the holder discounted it 30 da. before it was due, at 6%. What were the proceeds?

3. What is the cost of collecting a draft for $4,500 at ¼%? What are the proceeds of the draft?

4. A time draft for $1,500 had been accepted; the holder discounted it 2 mo. before it was due, at 6%. What were the proceeds?

INSURANCE

Fire Insurance

417. When a company insures a house for a certain sum—say $2,000—against loss by fire, it agrees to make good the owner's loss to the extent of $2,000, if the house is accidentally burned during the period specified.

418. In return, the owner pays the company in advance a stipulated amount called a **premium.**

419. The contract between the company and the owner is called a **policy.**

420. The amount for which the property is insured is called the **face** of the policy.

WRITTEN EXERCISES

1. A house is insured against fire for $3,000; the rate is $.65 per $100 per annum. What is the annual premium? Two annual premiums in advance usually pay for 3 years' insurance. What is the amount paid in advance for three years' insurance on the house?

2. Mrs. Brown had her jewels and wardrobe insured against fire for $3,000; the rate was $.45 per $100 per annum. What was the annual premium? Three annual premiums in advance usually pay for five years' insurance. What did Mrs. Brown pay for five years' insurance?

3. A cargo was insured for $12,360 at $2\frac{1}{2}$%. What was the annual premium?

4. A house valued at $9,500 was insured for $\frac{3}{4}$ of its value at 3%. What was the annual premium?

5. The annual premium on a policy for insuring a block at 3% was $465. What was the face of the policy?

6. A stock of goods valued at $2,500 was insured, and the annual premium was $62.50. What was the rate of insurance?

7. A house was insured by a policy on which the premium was $85.32 and the rate was 6%. What was the face of the policy?

Life Insurance

421. The essential feature of life insurance is, that the insurance company pays a fixed sum to the estate of a person named (**beneficiary**), whenever the insured person dies. In many cases the condition is added that the sum shall be paid to the insured himself, if living, at the end of a stipulated time; for example, 20 years. The first may be called a **life policy,** the second, a 20-year **endowment policy.**

422. In return, the insured person pays (usually annually in advance) a fixed sum called a **premium.**

The amount of the annual premium depends upon the age of the insured at the time when the payments are begun, and the character of the policy. It is usually stated at a rate per $1000 of the insurance. There are many different kinds of policies, with different conditions and privileges. The following specimen table of rates may be used in the problems:

Age when policy is issued.	*21*	*22*	*23*	*24*	*25*
Annual premium, life.....	15.01	15.35	15.71	16.03	16.46
“ “ 20 yr...	42.44	42.50	42.55	42.61	42.66
Age when policy is issued.	*26*	*27*	*28*	*29*	*30*
Annual premium, life.....	16.87	17.31	17.76	18.24	18.74
“ “ 20 yr...	42.73	42.80	42.88	42.97	43.07
Age when policy is issued.	*31*	*32*	*33*	*34*	*35*
Annual premium, life.....	19.27	19.83	20.42	21.04	21.70
“ “ 20 yr...	43.17	43.29	43.41	43.67	43.73

WRITTEN EXERCISES

1. Mr. Gray at the age of 21 insured his life for $1000 on the 20-year plan. At the expiration of the twenty years he was still living and received $1000. How much more did he receive than he had paid in?

2. How much does a man pay, beginning at 35 years of age, for $1000 of insurance on the life plan, if he lives to be 75 years of age?

3. Make and solve five problems about life insurance.

BROKERAGE

423. An agent who arranges for purchases or sales without actually receiving and delivering the goods is called a **broker.**

Thus, a broker in Chicago representing a wheat owner in Minneapolis may sell a carload of wheat in the Chicago market to another broker who represents a miller in Cincinnati. The wheat would be shipped from Minneapolis direct to Cincinnati after the transaction.

424. The fee paid the broker for buying or selling is called **brokerage.** It is usually a fixed sum per bushel, barrel, etc.

425. Some common rates of brokerage are:

For grain, ⅛ ct. a bushel, the least charge being $2.00.
For cotton, 5 cts. a bale, the least charge being $5.00.
For pork, 2½ cts. a barrel, the least charge being $6.25.
For coffee, 4 cts. a bag of 130 lb.

ORAL EXERCISES

Find the brokerage at the above rates on each of the following sales:

1. 10,000 bu. wheat.
2. 100 bbl. pork.
3. 210,000 bu. wheat.
4. 1000 bu. corn.
5. 2,000 bales cotton.
6. 500 bags coffee.
7. 4,000 bbl. pork.
8. 5,000 bu. oats.

Solve:

9. 10 problems of your own, similar to those preceding.

10. A trader sold "10 wheat" (10,000 bu.) through a broker at 65 cts. a bushel. How much did he realize from the sale?

11. A firm bought 1000 bales of cotton on the Exchange at $50 a bale. How much did it cost, including brokerage?

12. A broker bought for a customer 20,000 bu. of wheat at 65 cts. a bushel and sold it at 78 cts. How much did the customer gain, allowing the usual brokerage for buying and selling?

WRITTEN EXERCISES

1. Express the brokerage in oral exercises 10 and 11 above as a rate per cent.

2. In the market quotations, grain is listed per bushel, coffee is listed per pound, metal per ton, and cotton per bale of 500 lb. The following table contains the highest and lowest quotations of various staple commodities during a recent year:

	HIGHEST.	LOWEST.
Wheat	$1.15\frac{1}{4}$	.80
Corn	$.81\frac{1}{2}$	.52
Oats	$.62\frac{1}{2}$	.39
Coffee	$.07\frac{1}{2}$	$.06\frac{5}{16}$
Iron	26.00	19.75
Tin	$44.47\frac{1}{2}$	$30.87\frac{1}{2}$
Copper	25.55	12.25
Lead	6.25	4.60
Cotton	13.55	10.55

Find the brokerage at $\frac{1}{8}$ ct. per bushel on 5,000 bu. of wheat and the total cost when bought at the highest price as quoted.

3. According to the table, how much was invested in wheat at the lowest price by a broker who received $250?

4. Similarly, make and solve from this table five problems.

STOCKS AND BONDS

Stock Companies

426. When several persons wish to conduct a business enterprise together, they may organize a **company.**

427. The money invested in the business is called **capital.**

428. If the organization of the company is authorized by the state (in accordance with certain laws) the company is said to be **incorporated.** When incorporated, the company, **corporation,** can legally transact business like an individual.

429. The capital of a corporation is divided into equal amounts called **shares.** In this case the capital is called **capital stock,** or simply **stock.** Whatever amount a share represents is called the **face value,** or **par value,** of the share.

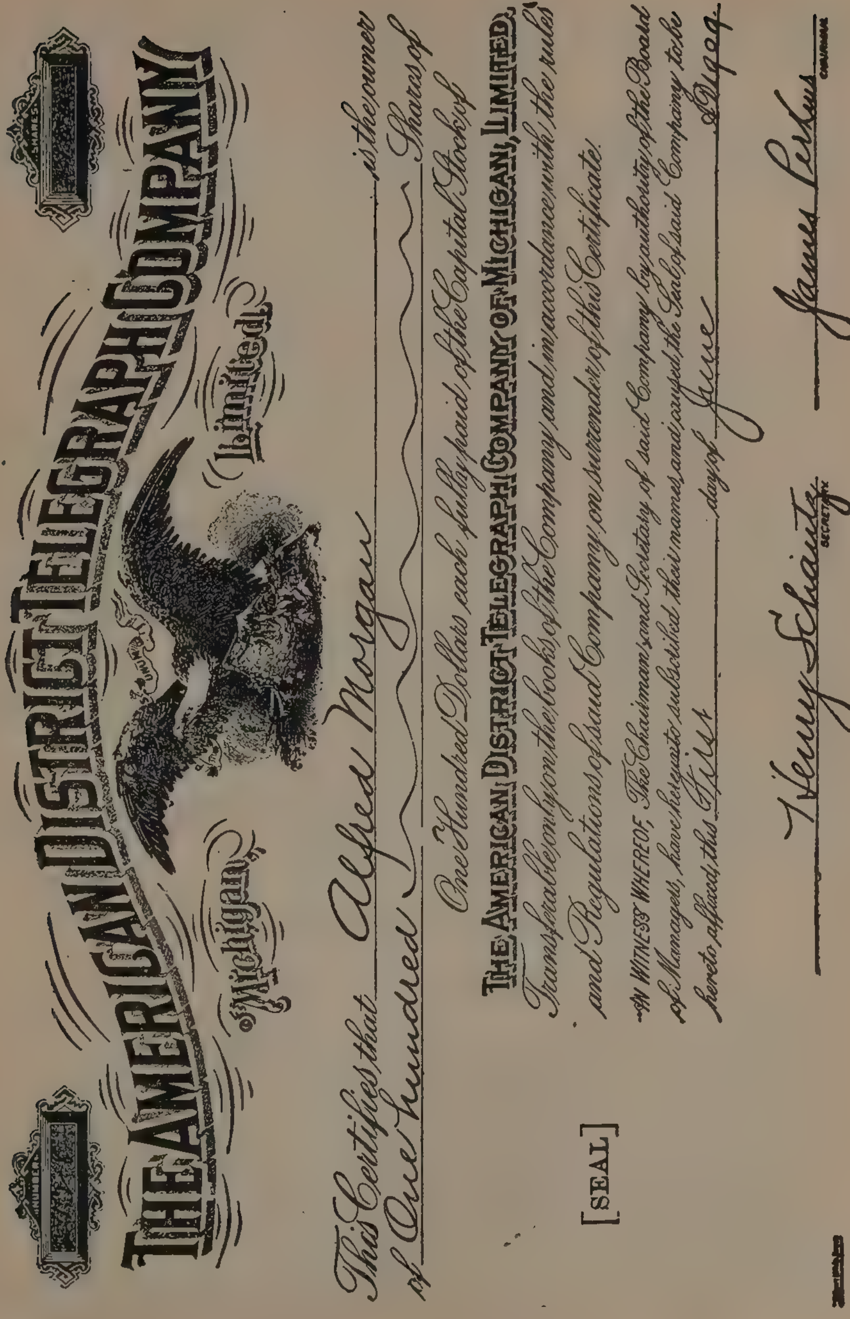
Capital is invested in a stock company by the purchase of one or more of the shares of stock.

430. The owners of stock are called **stockholders,** and certificates are issued for the shares of stock.

The illustration on the opposite page is a certificate of stock showing that Alfred Morgan owns 100 shares of the American District Telegraph Company.

431. The business is carried on under the management of officials elected by the stockholders. At regular intervals, usually twice or four times a year, the profits are divided among the stockholders, except what may be reserved for the needs of the business. The sum paid is called a **dividend,** and is usually expressed as a rate per cent of the face value of the share.

432. A stockholder may sell his stock to any purchaser for its face value, or for more or less. If it sells for its face value, it is said to be **at par;** otherwise, **above** or **below par.**



NUMBER SHARES

THE AMERICAN DISTRICT TELEGRAPH COMPANY

of Michigan (Limited)

This Certifies that Alfred Morgan is the owner of One hundred Shares of One Hundred Dollars each fully paid of the Capital Stock of THE AMERICAN DISTRICT TELEGRAPH COMPANY OF MICHIGAN, LIMITED, Transferable only on the books of the Company and in accordance with the rules and Regulations of said Company on surrender of this Certificate.

IN WITNESS WHEREOF, The Chairman and Secretary of said Company by authority of the Board of Managers, have hereunto subscribed their names and caused the Seal of said Company to be hereto affixed, this First day of June A.D. 1909.

[SEAL]

Henry Schiautz SECRETARY

James Perkins CHAIRMAN

When the price of stock is quoted 90, 85, 45, 37½, or 284¾, it means that a share whose par value is $100 can be bought for $90, $85, and so on.

There are "stock exchanges" in the large cities to facilitate the purchase and exchange of stocks. The stock broker charges a commission of ⅛ of 1% on the *par value* for either buying or selling.

ORAL EXERCISES

In the exercises following, the par value of a share is to be regarded as $100, unless otherwise specified.

1. What must one *add* to each quotation to find the cost of a share, including brokerage? What must one *subtract* to find what the sale of a share would net the seller?

2. Mr. Richmond owned 12 shares of New York Railway stock; he received an annual dividend of $6 a share. What was his yearly income from these shares?

3. Mr. Marsh received a semiannual dividend of 4% on 10 shares of railroad stock. What was his annual income?

4. Mr. A owns 10 shares of stock on which a dividend of 3% has been declared. How much does he receive?

5. What is the annual income from 160 shares of stock that pays a quarterly dividend of 1¾%?

WRITTEN EXERCISES

What is the cost of the following purchases of stock:

1. 17 shares of Baltimore and Ohio Railway stock at 105?

2. 150 shares of Illinois Central Railway stock at 128?

3. 2,000 shares of St. Louis and S. W. stock at 128?

4. 1500 shares of Niagara Power Co. stock at 84¼?

5. 5,000 shares of Texas Pacific Railway stock at 41½?

6. What did the owner of the stock in each of the above exercises realize from its sale, after having paid the broker?

433. To distinguish the rate of dividend from the rate of earnings per dollar invested, the latter is called the *rate of income on the investment.*

EXAMPLE: A man bought 1 share of railroad stock at $90, including brokerage; the stock paid an annual dividend of 4%. What was the man's rate of income on his investment?

The man received $4 per $90 invested, or $4\frac{4}{9}$ cts. per dollar, or $4\frac{4}{9}$% income on the investment.

WRITTEN EXERCISES

1. If Diamond Match stock is bought for 140, including brokerage, and pays an annual dividend of 10%, find the rate of income on the investment.

2. What is the rate of income on an investment in 10 shares of stock, paying 4% annually, and bought at 80, including brokerage?

3. A company whose stock is listed at $49\frac{7}{8}$ declares an annual dividend of $4.75 a share. Finding first the cost per share, including brokerage, find the rate of income.

434. Sometimes the capital stock of a corporation is divided into two kinds, called **preferred stock** and **common stock.** A dividend must be paid on the preferred stock, before any dividend whatever may be paid on the common stock.

ORAL EXERCISES

1. A carpet company organized with a capital of $1,000,000. How many 100-dollar shares of stock were there?

2. The organizers subscribed for $\frac{1}{2}$ of the stock. How many shares were sold to other investors?

3. 2,000 shares were preferred stock. How many shares of common stock were there?

4. What sum was required annually to pay the dividends on the preferred stock at 5%?

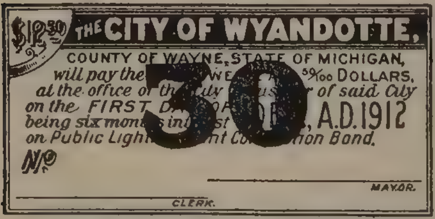
Bonds

435. When the Government (national, state, county, or city) or a corporation borrows large sums of money for a long time, it acknowledges the receipt of the money and promises to repay it in a statement called a **bond.** A bond is usually issued for each $1000 borrowed. The opposite page shows a bond whose price is $500.

436. The amount of the bond is called its **par value.**

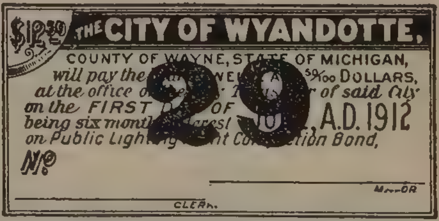
Thus, the face value of the above bond is $500.

437. Bonds bear interest at a specified rate. This is usually paid upon presentation of certificates called **coupons** attached to the bonds. The coupons are promissory notes, which, if not paid when due, bear interest at the legal rate.



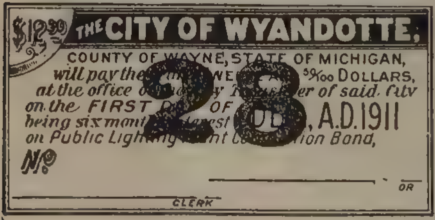
30

$12.50 THE CITY OF WYANDOTTE,
COUNTY OF WAYNE, STATE OF MICHIGAN,
will pay the [illegible] 50/100 DOLLARS,
at the office of the City [illegible] of said City
on the FIRST D[illegible] OF [illegible], A.D. 1912
being six months interest
on Public Ligh[illegible]nt Co[illegible]tion Bond.
No.
MAYOR.
CLERK.



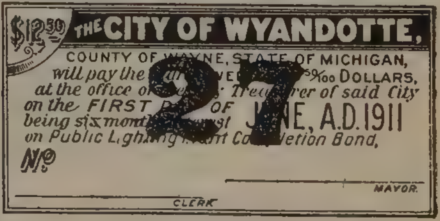
29

$12.50 THE CITY OF WYANDOTTE,
COUNTY OF WAYNE, STATE OF MICHIGAN,
will pay the [illegible] 50/100 DOLLARS,
at the office o[illegible] of said City
on the FIRST [illegible] OF [illegible], A.D. 1912
being six month[illegible]
on Public Ligh[illegible]nt Co[illegible]tion Bond,
No.
MAYOR
CLERK.



28

$12.50 THE CITY OF WYANDOTTE,
COUNTY OF WAYNE, STATE OF MICHIGAN,
will pay the [illegible] 50/100 DOLLARS,
at the office o[illegible] Treasurer of said City
on the FIRST [illegible] OF [illegible], A.D. 1911
being six mont[illegible]
on Public Ligh[illegible]nt Co[illegible]ion Bond,
No.
OR
CLERK



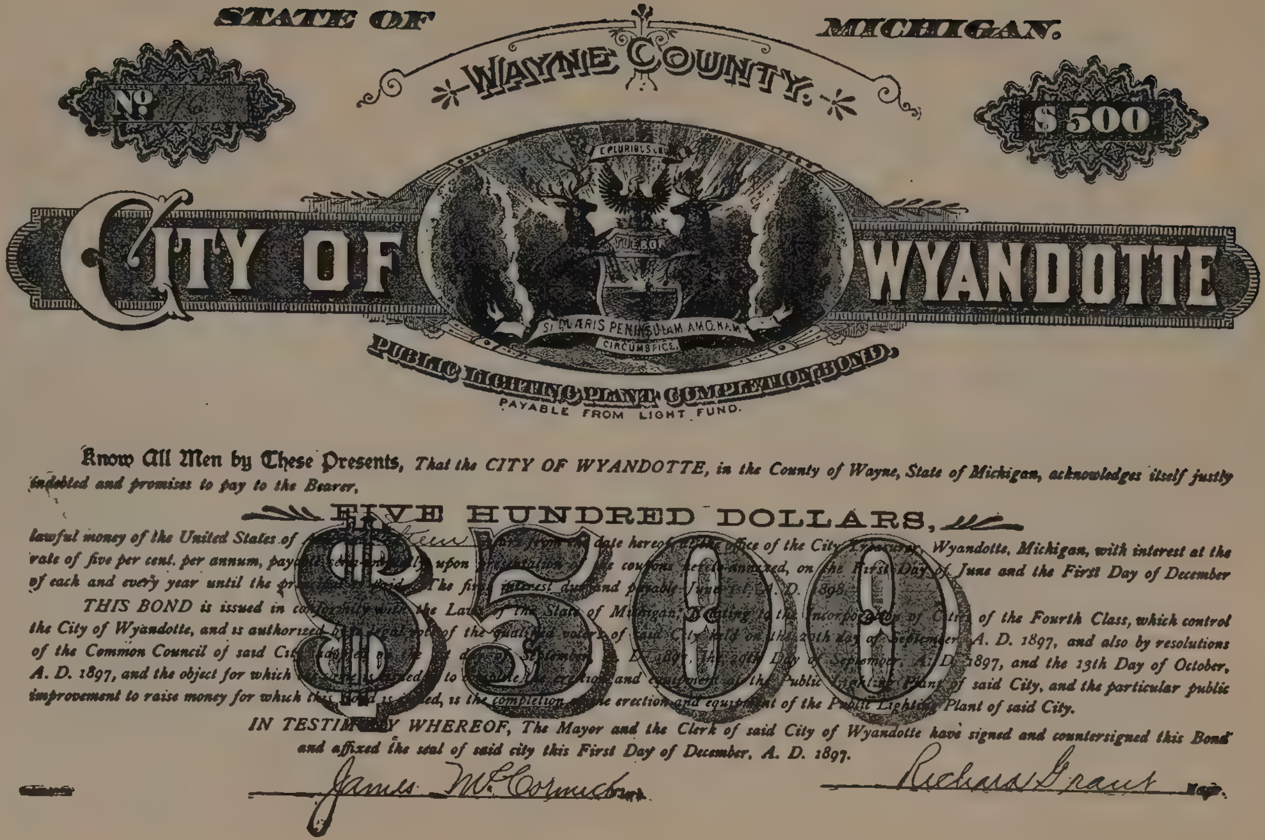
27

$12.50 THE CITY OF WYANDOTTE,
COUNTY OF WAYNE, STATE OF MICHIGAN,
will pay the [illegible] 50/100 DOLLARS,
at the office o[illegible] Treasurer of said City
on the FIRST [illegible] OF JUNE, A.D. 1911
being six mont[illegible]
on Public Lighting Plant Construction Bond,
No.
MAYOR.
CLERK

ORAL EXERCISES

1. What is the face of the bond shown on page 371? What is the annual interest at 5%?

2. The coupons above are the last 4 of the 30 coupons of this bond. Each represents the interest for 6 months. What was the whole amount of interest on the bond?



STATE OF MICHIGAN.

WAYNE COUNTY.

No.

$500

CITY OF WYANDOTTE

E PLURIBUS UNUM

TUEBOR

SI QUÆRIS PENINSULAM AMŒNAM CIRCUMSPICE.

PUBLIC LIGHTING PLANT COMPLETION BOND.

PAYABLE FROM LIGHT FUND.

Know All Men by These Presents, *That the CITY OF WYANDOTTE, in the County of Wayne, State of Michigan, acknowledges itself justly indebted and promises to pay to the Bearer,*

FIVE HUNDRED DOLLARS,

lawful money of the United States of America, Ten years from the date hereof, at the office of the City Treasurer, Wyandotte, Michigan, with interest at the rate of five per cent. per annum, payable semi-annually upon presentation of the coupons hereto annexed, on the First Day of June and the First Day of December of each and every year until the principal is paid. The first interest due and payable June 1st, A. D. 1898.

THIS BOND is issued in conformity with the Laws of the State of Michigan, relating to the incorporation of Cities of the Fourth Class, which control the City of Wyandotte, and is authorized by a legal vote of the qualified voters of said City held on the 20th day of September, A. D. 1897, and also by resolutions of the Common Council of said City adopted on the [illegible] day of September, A. D. 1897, the 29th Day of September, A. D. 1897, and the 13th Day of October, A. D. 1897, and the object for which this bond is issued is to complete the erection and equipment of the Public Lighting Plant of said City, and the particular public improvement to raise money for which this bond is issued, is the completion of the erection and equipment of the Public Lighting Plant of said City.

IN TESTIMONY WHEREOF, The Mayor and the Clerk of said City of Wyandotte have signed and countersigned this Bond and affixed the seal of said city this First Day of December, A. D. 1897.

James McCormick Clerk.   Richard Grant Mayor.

$500

438. Bonds differ from stock in that the bonds are secured by the property of the company and bear a permanent rate of interest.

The interest must be paid on the bonds before any dividend can be paid on the stock. A bondholder is a creditor; a stockholder is a part owner. Bonds, like stocks, are bought and sold in the market.

When bonds are purchased at prices below par and held until maturity, the rate of income, or yield, differs from the rate of interest. But this phase of the subject is too difficult to present here.

439. The broker's commission for buying or selling bonds is $\frac{1}{8}$ of 1% of the *par value.*

Amounts bought and sold are stated in par value, and the price is quoted at a rate per $100 of par value. The rate of interest is usually added.

Thus, U. S. 4's 142 means that United States bonds bearing 4% interest on par value are sold at $142 for each $100 of par value.

EXAMPLE: What is the cost of $3,000 (par value) Erie Canal bonds at 101?

Solution: (1) Each $100 of par value costs $101.
(2) $3,000 of par value cost 30 × $101, or $3,030.
(3) Brokerage, $\frac{1}{8}$ of 1% of par value, $3.75.
(4) Total cost = $3,033.75.

WRITTEN EXERCISES

1. What is the cost of $15,000 city bonds at 105?

2. Find the proceeds from each of the following sales:

$5,000 Central Union Gas Co. of N. Y. 1st mortgage 5% bonds.	$108\frac{1}{2}$
$10,000 Union Gas Light Co. of Kings Co. 1st mortgage 6% bonds......................	$106\frac{3}{8}$
$20,000 Northern Pacific Gt. Northern R. R. (C., B. & Q. coll.) 4% bonds.............	$94\frac{1}{8}$
$6,000 Southern R. R. 1st mortgage 5% bonds..............	$115\frac{3}{8}$

3. Find the amount of annual interest from each purchase above.

4. Find the brokerage on each of the above sales.

REVIEW

ORAL EXERCISES

1. Mr. Carelton of Detroit owes Rice and Co. of St. Louis $175 for some goods. How may he discharge the indebtedness? Will he be likely to use an express or a postal order or a bank draft?

2. What is a bank draft? What is a commercial draft?

3. What is a sight draft? A time draft? How is a commercial draft collected?

4. Smith & Co. of New York draw upon Jones & Co. of Johnstown for $250, 30 days after sight. What may Jones & Co. write on the face of the draft to show that they will pay it?

5. What is the usual rate of brokerage for buying and selling grain on the Exchanges? At this rate find the brokerage on the following purchases:

32,000 bu. of wheat;
20,000 bu. of oats;
10,000 bu. of barley;
7,000 bu. of corn.

6. A firm bought 5,000 bales of cotton at $50 a bale, brokerage 5 cts. a bale. What did it cost?

7. What is meant by the stock of a corporation? What is the usual par value of stock? Name a price that is above par.

8. What is a dividend? On what value of the stock is it declared?

9. Where are stocks usually bought and sold, and through whom? What is the fee for buying or selling called?

10. What is meant by rate of income on the investment? How is it computed?

11. What is a bond? What is usually attached to a bond to represent the interest due?

12. Where are bonds usually bought and sold? What must one add to the quoted price of a bond to find its cost?

WRITTEN EXERCISES

1. Write an order to Sargent and Smith, 18 Grand St., New York, for a tool chest which they advertise in "The Youth's Companion" for $8.95, purchaser to pay carriage. State in what form you send the money and give directions as to sending the tool chest to you. (When ready for shipment it weighs 65 lb. If convenient, ascertain which would be the cheaper way of having it come to you, by freight or by express, remembering that you must also pay cartage from the freight depot to your home, if sent by freight.)

2. Make and receipt a bill for 5 tons of hard coal at $6.75, plus 25 cts. per ton for carrying it to the bin; 1 load of kindling wood, $1.25.

3. Mr. Johnson bought 100 shares of Pennsylvania Railroad stock at 130. How much did he pay his broker? How much did the seller receive from his broker?

4. If New York Central stock pays a dividend of 5%, what is the income from 238 shares?

5. What is the income from 250 shares of stock paying a dividend of $4\frac{1}{2}$%?

6. A man purchased some stock at $110 a share, including brokerage, which paid a dividend of $5\frac{1}{2}$%. What was the rate of income on the investment?

7. A premium of $157.50 was paid on a policy whose face was $4,500. What was the rate of insurance?

8. What is the cost of 500 Metropolitan bonds (par $100) quoted at $109\frac{7}{8}$?

9. A man bought 10 bonds (par $100) quoted at $149\frac{7}{8}$ and paying an annual interest at the rate of $4\frac{1}{2}$%. What was the amount of his annual interest?

CHAPTER XV

POWERS AND ROOTS

POWERS

440. The product of two equal factors is called the **second power,** or **square,** of the repeated factor.

Thus, 4, the product of 2 and 2, is the second power, or square, of 2.

441. The product of three equal factors is called the **third power,** or **cube,** of the repeated factor.

Thus, 8, the product of 2 and 2 and 2, is the third power, or cube, of 2.

442. The product of four equal factors is called the **fourth power** of the repeated factor, and so on.

Thus, 16, the product of 2, 2, 2, and 2, is the fourth power of 2. Similarly, 32 is the fifth power of 2, 64 is the sixth power, and so on.

ORAL EXERCISES

1. 25 is what power of 5?

2. 125 is what power of 5?

3. 27 is what power of 3?

4. 81 is what power of 3?

5. What is the area of a square 2 in. on a side?

6. What is the area of a square 5 yd. on a side?

7. What is the volume of a cube 3 ft. on an edge?

443. Powers of numbers can be denoted by exponents.

Thus,
The second power of 2 (or 4) = 2^2.
The third power of 2 (or 8) = 2^3.
The fourth power of 3 (or 81) = 3^4.

444. *A power of any number is found by multiplication.*

WRITTEN EXERCISES

Find the following squares:

1. 15^2.
2. $(\frac{9}{15})^2$.
3. 11^2.
4. 25^2.
5. 8^2.
6. $(\frac{3}{4})^2$.
7. 2.5^2.
8. $.1^2$.

Find the volume of a cube whose edge is:

9. 4 in.
10. 5 ft.
11. 1.2 in.
12. 3 ft.
13. .6 in.
14. 9 ft.
15. 4 yd.
16. 7 ft.

SQUARE ROOTS

445. *Any number is the square root of its square.*

Thus, 4 is the square root of 4^2, or 16.
1.5 is the square root of 1.5^2, or 2.25.
$\frac{3}{4}$ is the square root of $(\frac{3}{4})^2$, or $\frac{9}{16}$.

446. A root of a number may be indicated by placing over the number the symbol $\sqrt{}$, called the **radical sign.**

Thus, the square root of 25 is denoted by $\sqrt{25}$. The cube root of 8 by $\sqrt[3]{8}$; the fourth root of 16 by $\sqrt[4]{16}$, and so on.

To Find the Square Root by Factoring

447. *When it is possible to find the square root by factoring, factor the number into its prime factors and arrange them in two equal sets. Either set is the square root of the number.*

EXAMPLE: Find $\sqrt{225}$.

$225 = 3 \times 5 \times 3 \times 5 = 15 \times 15 = 15^2$. Therefore, $\sqrt{225} = 15$.

WRITTEN EXERCISES

Find by factoring the square roots of:

1. 144.
2. 400.
3. 256.
4. 961.
5. 289.
6. 9,801.
7. 625.
8. 441.
9. 196.
10. 2,601.
11. 324.
12. 3,600.
13. 576.
14. 169.
15. 900.

To Find the Square Root when the Number Cannot Easily Be Factored

448. A knowledge of the way in which a square is formed will assist in finding square roots.

EXAMPLE: What is the square of 14?

$14^2 = 14 \times 14 = 196.$

But $14 = 10 + 4$, and we may multiply $10 + 4$ by $10 + 4$, as follows:

$$\begin{array}{l} (10 + 4) \times 10 = 10^2 + 10 \times 4 \\ (10 + 4) \times \ \ 4 = \qquad\quad\ 10 \times 4 + 4^2 \\ \hline \qquad\qquad\quad 10^2 + 2 \text{ times } 10 \times 4 + 4^2 \end{array}$$

Then, $(10 + 4) \times (10 + 4) = 100 + 80 + 16 = 196.$

The work above shows that the square of a number composed of tens and units is the square of the units plus twice the product of the tens and the units plus the square of the tens.

$$\begin{aligned} \text{Thus, } 15^2 &= 10^2 + 2(10 \times 5) + 5^2 \\ &= 100 + 100 + 25 = 225. \end{aligned}$$

449. We can determine in advance the number of figures in the square root of the square of an integer.

1. Read: $1^2 = 1.$ $5^2 = 25.$ $9^2 = 81.$
$2^2 = 4.$ $7^2 = 49.$ $10^2 = 100.$

A one-figure integer has at most how many figures in its square? What is the least number it can have?

2. Read: $10^2 = 100.$ $25^2 = 625.$ $99^2 = 9{,}801.$
$15^2 = 225.$ $50^2 = 2{,}500.$ $100^2 = 10{,}000.$

A two-figure integer has at most how many figures in its square? What is the least number it can have?

3. Similarly, the square of an integer of 3 figures is composed of 5 or 6 figures, and so on.

Thus, separate the square into periods of 2 figures each, beginning at the decimal point, and the number of places is the number of figures in the root. The place at the left may not be complete. For example, 1 ′69 ′00 has 3 periods; hence the square root contains 3 places.

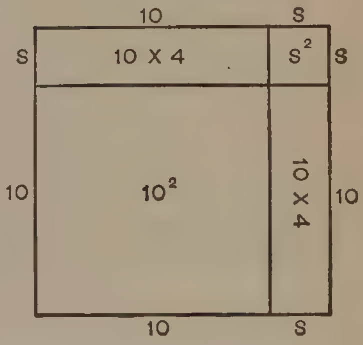
450. Example: Find the square root of 196.

Our knowledge of how squares are formed will help to find the square root. Consider 196 to be the area of a square of which we wish to find the side.

By marking off the periods in 1 96 we see that the number contains the square of 10.

Root 14
Number 1′96
1 00
96
96

Subtracting this, there is left 96, which shows that the side of the square sought is more than 10. If we call S the part to be added to the side already found, we must make the additions shown in the figure in order to form a new square.



Thus, 96 contains $2 \times (10 \times S) + S^2$. We shall find S approximately by dividing 96 by 2×10. The result is 4. Using this for S, $2 \times 10 \times S + S^2$ becomes $2 \times 10 \times 4 + 16 = 96$.

Subtracting 96, the remainder is zero; hence we have found the side, 14, of a square whose area is 196. That is, we have found the square root of 196 to be 14.

451. *To find the square root of any number:*

1. *Point off the number into periods of two figures each, beginning at the right (at the decimal point in a number containing a decimal).*

2. *By inspection find the largest integer whose square is not greater than the left period.*

(In Example A it is 9.)

(A)
Root 9 2
Number 84′64
81
18 364
182 364

3. *Use this integer as the first digit of the root. Subtract its square from the left period.*

(In Example A this square is 81.)

4. *Bring down the next period.*

(In Example A this makes 364.)

5. *Multiply the part of the root already found by 2. This number is called the* **trial divisor.** (18 in Example A.)

6. *Divide the remainder (omitting the right digit) by the trial divisor and use the digit found as the next digit of the root.*

(In Example A, 36 ÷ 18 = 2.)

7. *Annex this digit to the trial divisor. This forms the* **complete divisor.**

(182 in A.)

8. *Multiply the complete divisor by the digit of the root just found and subtract.*

Note.—It may happen that the product to be subtracted is larger than the number from which it is to be subtracted. This indicates that the trial divisor gave too large a digit. Try the next smaller digit.

9. *Repeat steps 4 to 8 until all the periods have been brought down.*

(B)

Root	3 0. 6 9
Number	**9′41′.87′61**
	9
6	41
60	4187
606	3636
612	55161
6129	55161

(C)

Root	1 4. 1 4 +
Number	**2′00**
	1
2	100
24	96
28	400
281	281
282	11900
2824	11296
	604

If a zero occurs in the root, as in Example B, when 4 is not divisible by 6, bring down the next period and proceed as before.

If the last remainder is zero, as in Example B, the process is ended, the given number is a perfect square, and its root has been found exactly. If the last remainder is not zero, as in (C), the process may be continued as far as desired by supplying zeros.

Test: The square of the root should equal the given number.

452. *To extract the square root of a fraction, either reduce the fraction to a decimal and proceed, or extract the square root of both terms.*

WRITTEN EXERCISES

Extract the square root of:

1. 729.
2. 72.25.
3. 1.69.
4. $\frac{16}{49}$.
5. 1,225.
6. 6,889.
7. 361.
8. 841.
9. $\frac{16}{25}$.
10. 5,625.
11. 4.41.
12. 625.
13. $\frac{121}{625}$.
14. 4,225.
15. 22,500.
16. 784.
17. 20.25.
18. $\frac{49}{144}$.
19. 7,225.
20. 2.25.
21. 2,025.
22. $\frac{49}{2025}$.
23. 1,444.
24. 2.56.
25. 1,681.
26. 3,025.
27. $\frac{169}{289}$.
28. 11,025.
29. 156.25.
30. 3,136.
31. .2025.
32. 8,836.
33. 96.04.
34. 1,936.
35. 6.25.

Solve:

36. How long is each side of a square court of 1,225 sq. yd.?

37. What is the perimeter of a field 16 rd. wide and 144 rd. long? What would be the perimeter if the field were square and had the same area?

38. What is the distance in rods around a square tract of land containing 1,681 sq. rd.?

39. A square field of corn contains 252,004 hills of corn. How many rows in the field?

40. How long is each side of a square floor of 7,225 sq. ft.?

Find the following roots to 3 places of decimals:

41. $\sqrt{2}$.
42. $\sqrt{11}$.
43. $\sqrt{13}$.
44. $\sqrt{15}$.
45. $\sqrt{3}$.
46. $\sqrt{2.5}$.
47. $\sqrt{.3}$.
48. $\sqrt{.2}$.
49. $\sqrt{5}$.
50. $\sqrt{.02}$.
51. $\sqrt{.05}$.
52. $\sqrt{1.5}$.
53. $\sqrt{7}$.
54. $\sqrt{1.05}$.
55. $\sqrt{75}$.
56. $\sqrt{.75}$.

Solve:

57. What is the length in rods of a side of a square field containing 1 acre? Find the result to two decimal places.

CHAPTER XVI

MENSURATION

THE RIGHT TRIANGLE

453. A triangle one of whose angles is a right angle is called a **right triangle.**

The side opposite the right angle of a right triangle is called the **hypotenuse.**

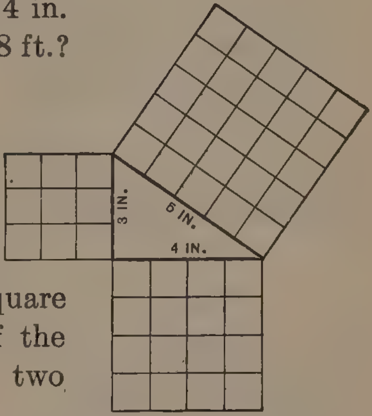
ORAL EXERCISES

1. What is the area of a square 4 in. on a side? 5 in.? 3 in.? 9 in.? 8 ft.? 10 yd.?

2. What is the length of each side of the triangle in the figure?

3. What is the area of the square drawn on the longest side?

4. Compare the area of the square on the longest side with the sum of the areas of the squares on the other two sides.



454. *In any right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.*

If h represents the length of the hypotenuse, and a and b those of the sides, then $h^2 = a^2 + b^2$.

This relation is called the *Pythagorean Theorem,* after the Greek philosopher and mathematician, Pythagoras (about 500 B.C.), who is supposed to have discovered it.

WRITTEN EXERCISES

Compute the hypotenuse of a right triangle having the following sides forming the right angle:

1. 9 in., 12 in. **2.** 5 in., 12 in. **3.** 12 in., 16 in.

4. Compare the square on the hypotenuse with the sum of the squares on the other two sides.

455. PREPARATORY.

The hypotenuse of a right triangle is 10 in. Find the area of the square having this line for a side. The length of another side is 6 in. What is the area of the square on this side? What is the area of the square on the third side of the triangle? How long is the third side of the triangle?

EXAMPLE: Find the length of the hypotenuse of a right triangle whose other two sides are 8 ft. and 15 ft. respectively.

Solution: $h = \sqrt{8^2 + 15^2} = \sqrt{289} = 17.$

The hypotenuse is 17 ft.

WRITTEN EXERCISES

Find the unknown sides of the following right triangles:

	1.	2.	3.	4.	5.
Hypotenuse:	5	x	13	10	x
Side:	4	3	x	6	6
Side:	x	4	12	x	8

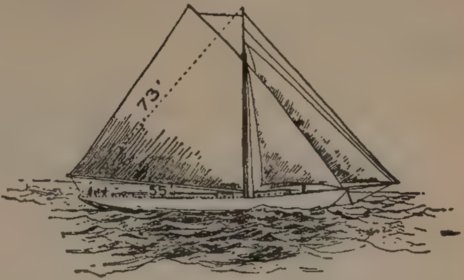
Solve:

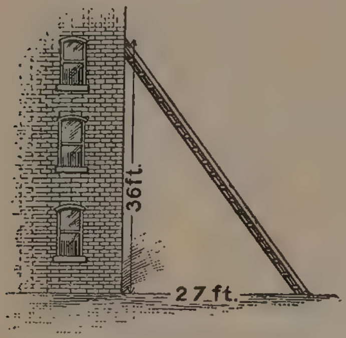
6. The width of the end of a house was 24 ft., and the height of the ridge of the roof above the level of the eaves was 12 ft. Find the length of the slope of the roof from the ridge to the eaves.

7. A garden has the form of a right triangle. Its longest side is 149 ft., and one side is 51 ft. What will it cost to surround it with a fence at $.70 a running foot?

8. A garden in the form of a right triangle has a hypotenuse 50 ft. long, and one side 30 ft. long. At $.70 a running foot, what is the cost of a fence enclosing the garden?

9. A rope 73 ft. long is fastened to the top of a boat's mast, and when drawn taut, it touches a point on the deck 55 ft. from the foot of the mast. Find the height of the mast.





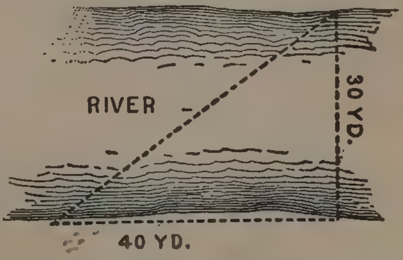
10. A ladder is placed against a building, as shown in the figure. How long is the ladder?

11. If a ladder 41 ft. long rests against a wall with its foot 9 ft. from the wall, how high is the top of the ladder from the ground?

12. A mason measures from one corner along the adjacent sides of a wall 30 ft. and 40 ft. He then connects the ends of these lines to see if the triangle thus formed is a right triangle. If it is, what is the length of this third line?

13. A rectangular tract of land was 3 mi. on one side and 4 mi. on the adjacent side. What was the distance diagonally from one corner of the tract to the other?

14. A swimmer in crossing a stream 30 yd. wide landed 40 yd. farther down stream. How many yards was it from his starting point to the landing place?



15. A trunk is 3 ft. long and 2 ft. wide. What is the longest umbrella that can be laid flat on the bottom of the trunk?

CIRCLES

456. Preparatory.

The distance through a round body, as a pencil or a drinking cup, is called its **diameter,** and may be measured by use of triangles and a scale as shown in Figure 1.

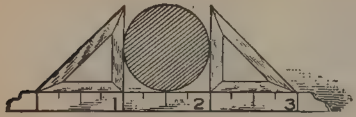


Fig. 1.

1. Measure the diameters of several round objects and record the results.

2. Wrap a piece of paper about each object until the ends overlap, as in Figure 2. Prick the ends with a pin. Release the paper and measure between the pinholes, as in Figure 3. This is the distance around the object and is called its **circumference.**

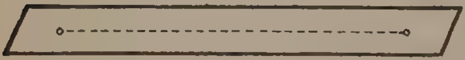


Fig. 3.

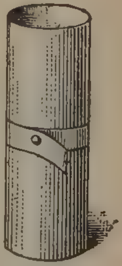


Fig. 2.

3. Find the ratio of the circumference of each to its diameter.

457. A **circle** is a closed curve every point of which is at the same distance from a fixed point called the **center.** A straight line joining the center and any point of the circle is called a **radius.**

458. The ratio of the circumference of a circle to its diameter is represented by the Greek letter π (pronounced pī). The value of π can be found approximately only; 3.14 is correct to the nearest hundredth, and should be used in the problems following.

459. If d denotes the diameter of a circle, πd denotes its circumference. *If r denotes the radius, $d = 2r$, and $\pi d = 2\pi r$, or circumference $= 2\pi r$.*

For example, the circumference of a circle 8 in. in diameter is approximately 3.14 × 8 in., or 25.12 in.

The circle is often defined as a surface, but the definition on page 388 is preferable because it is the one used in geography, advanced mathematics, and common parlance. The area enclosed by a circle is called the area of the circle.

WRITTEN EXERCISES

1. Find the circumference of a circle of diameter 6 in. Of diameter 1 ft. Of radius 6 in. Of radius 3 yd.

2. If the circumference of a circle is given, by what must it be divided to produce the diameter?

3. Find the diameter of a circle of circumference 3.14 ft.

4. Find the length of the radius of a circle whose circumference is 6.28 in. Of one whose circumference is 9.42 yd.

5. How many times will a bicycle wheel 28 in. in diameter revolve in going 1 mi.?

460. PREPARATORY.

Cut out of paper a circle of radius 3 in. and cut along the lines marked in Figure 1. Open out the circle as shown in Figure 2. The parts are called *sectors*.

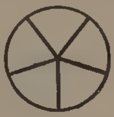


Fig. 1.

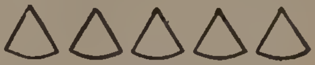


Fig. 2.

461. A sector of a circle resembles a triangle, the curve being the base, and a radius the altitude.

462. It is shown in geometry that the area of a sector is found by taking one-half of the base times the altitude.

463. *Denoting the base of a sector by* b *and its altitude by* r*, the area is* $\frac{br}{2}$.

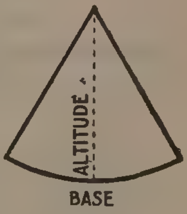


Fig. 3.

As all the sectors of a circle have the same altitude, the area of all of them (and, therefore, the area of the circle) is evidently the sum of their bases times half of the common altitude. The sum of all the bases is the circumference, since the circumference was cut up to make them. Therefore, *the area of a circle* $= \textit{circumference} \times \frac{1}{2}\,\textit{radius} = 2\pi r \times \frac{r}{2} = \pi r^2$.

464. *The area of a circle is π times the square of the radius.*

WRITTEN EXERCISES

1. Find the length of a circle of radius 4 in., using 3.14 for π. Find its area.

2. Find the area of circles of radii: 6 in.; 1 ft.; 2 in.; 2 ft.; 10 ft.; 10 in.

3. The area of a circular garden was 12.56 sq. rd. What was the diameter of the garden?

4. What was the cost of a fence around it at 30 cts. per foot?

5. The area enclosed by a circular walk is 28.26 sq. yd. What is the radius of the circular area?

6. Find the area included within a semicircle and a diameter, the diameter being 5 ft.

7. The rectangular court shown in the picture is broken by a circle in the center, by semicircles at the ends, and by a trapezoid at the side. From the dimensions given in feet, find the area of the court not occupied by these spaces.

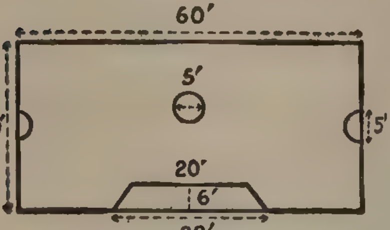


Fig. 1.

8. If a figure is drawn as indicated by Figure 2, what is the area of the square?

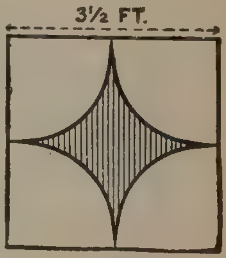


Fig. 2.

9. What part of a circle occupies each corner? What is the area of these parts?

10. Draw a figure like that shown for Exercise 8, using a square of side 6 in. What is the area of the shaded part?

11. If the circular tiles shown in Figure 1 are 1 in. in diameter, what is the area of that part of the rectangle not covered by the tiles?

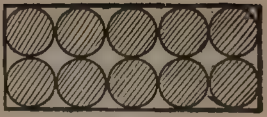


Fig. 1.

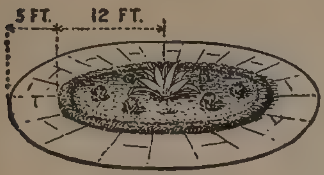


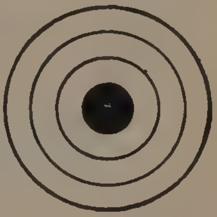
Fig. 2.

12. A walk of uniform width surrounds a circular grass plat, as shown in Figure 2. Find the area of the plat. Of the plat and the walk together. Of the walk.

13. What is the cost of a floor for a round tower 40 ft. in diameter at 32 cts. per square foot?

14. A round tree trunk has a circumference of 14.13 ft. What is its diameter? What is the area of a cross section?

15. A target is 8 ft. in diameter, the bull's-eye is 2 ft. in diameter, and each of the rings formed by the other circles is 1 ft. wide. What part of the whole target is the bull's-eye? What part of the whole is each ring?



16. The rotunda of the capitol in Washington is 98 ft. in diameter. Find the cost of a floor for it at 49 cts. per square foot.

17. What is the cost of carpeting a semicircular platform 20 ft. in diameter at $1.20 a square yard, allowing for waste an amount equal to 5% of the surface of the platform?

18. Find the cost of clearing a circular skating space 500 ft. in diameter, if the snow is 7 in. deep on the ice, and the cost of removal is 5 cts. a cubic yard.

19. A cow is tethered in the center of a field 100 ft. square with a rope 50 ft. long. Over how many square feet can she graze? Over what fractional part of the field?

20. The earth is about 8,000 mi. in diameter and makes one rotation on its axis in 24 hr. How far does a point on the equator move in a day? In an hour? In a minute?

HEXAGONS

465. The **regular hexagon** occurs frequently in nature and is much used in architecture and designs.

It is constructed by drawing a circle and drawing successive chords equal to the radius. (See Figure 1.)

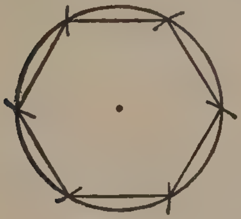


Fig. 1.

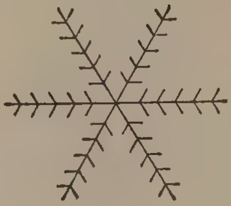


Fig. 2.

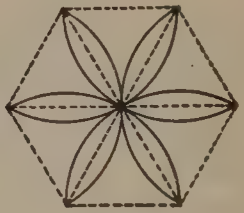


Fig. 3.

WRITTEN EXERCISES

1. Construct a regular hexagon, using dotted lines for the sides.

2. From the center of the figure draw full lines to the corners. Along each such line draw short parallel lines as shown in Figure 2. The resulting figure is a common pattern among snow crystals.

3. Draw a regular hexagon in dotted lines. By using each vertex as a center, draw the curves shown in Figure 3. Complete the figure and erase the dotted lines.

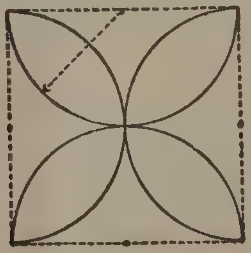


Fig. 4.

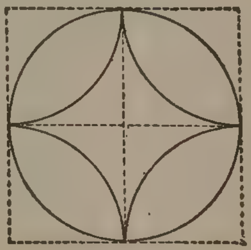
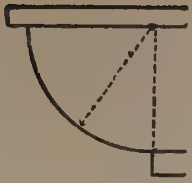
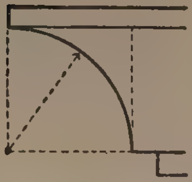
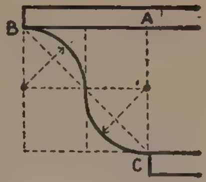


Fig. 5.

4. Show how to draw the designs of Figures 4 and 5.



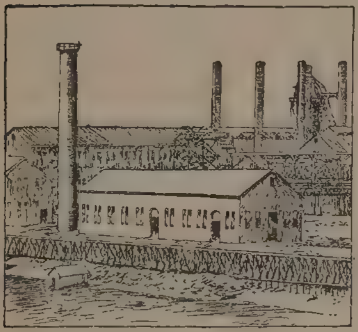




5. Explain from the figures how to construct a design for each of these moldings.

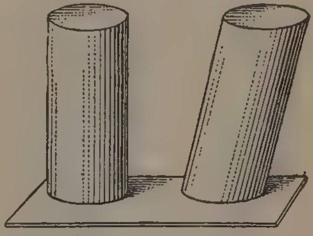
CYLINDERS

466. Curved surfaces, like those of sections of stovepipe, factory smokestacks, and funnels of ships, are called **cylindrical surfaces.**



467. If the ends of a cylindrical surface are in parallel planes the whole is called a **cylinder.** The ends are called the **bases.** If the base is a circle, the cylinder is called **circular.**

468. Right and Oblique Cylinders. If the curved surface is perpendicular to the plane of the base, the cylinder is called a **right cylinder.** Cylinders that are not right cylinders are **oblique cylinders.**



We shall treat only right circular cylinders, and shall call them merely cylinders.

469. Preparatory.

1. Wrap a piece of paper around a cylinder. Cut it so that the edges meet. Trim the ends even with the cylinder. Unroll the paper. What kind of figure is the paper?

2. Measure the length and breadth of the paper and find the area. What is the area of the curved surface of the cylinder? How does the length of the paper compare with the distance around the cylinder?

470. The surface of an object exclusive of its ends is usually called its **lateral surface.**

471. *The lateral surface of a right cylinder is the product of its altitude and the circumference of its base.*

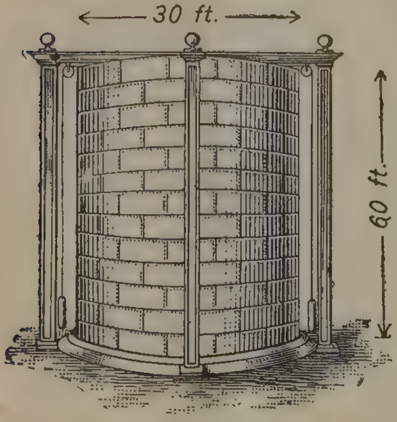
WRITTEN EXERCISES

Find the numbers to fill the blanks:

	DIAMETER.	ALTITUDE.	LATERAL AREA.
1.	4 in.	10 in.	( )
2.	2 in.	( )	31.4 sq. in.
3.	( )	5 in.	62.8 sq. in.

Solve:

4. Find the area of the curved surface of the cylindrical gas tank shown in the picture.



5. What will it cost to paint the surface mentioned in Exercise 4 at $.50 a square yard? Test.

6. How many square feet are there in the top and curved surface of a gas tank 36 ft. in diameter and 45 ft. high?

7. What will it cost to paint the top and curved surface of the tank of Exercise 6 at $.45 a square yard?

472. *The volume of a cylinder is the product of its altitude and the area of its base.*

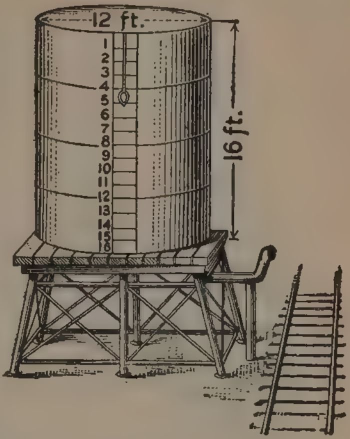
WRITTEN EXERCISES

1. How many cubic feet in a cylindrical tank 1 ft. high, whose base covers 40 sq. ft.? In a tank 2 ft. high having the same base? In one 5 ft. high?

2. Find the area of the base of the water tank shown in the picture. From the height given, find the volume in cubic feet.

3. Taking 1 cu. ft. as 7½ gal. how many gallons does this tank hold?

4. When the gauge reads 5, it means that the water is 5 ft. deep in the tank. How many gallons are there in it then?



Find the numbers to fill the blanks for these cylinders:

DIAMETER.	ALTITUDE.	VOLUME.
5. 3 in.	8 in.	( )
6. ( )	5 in.	62.8 cu. in.
7. 1 ft.	( )	12.56 cu. ft.

8. Find the number of square inches of iron in a stovepipe 3 ft. long and 6 in. in diameter.

9. What will it cost to paint the lateral surface of a standpipe 52 ft. in circumference and 68 ft. high, with open top, at 35 cts. per square yard?

10. How many cubic yards of water will the pipe hold?

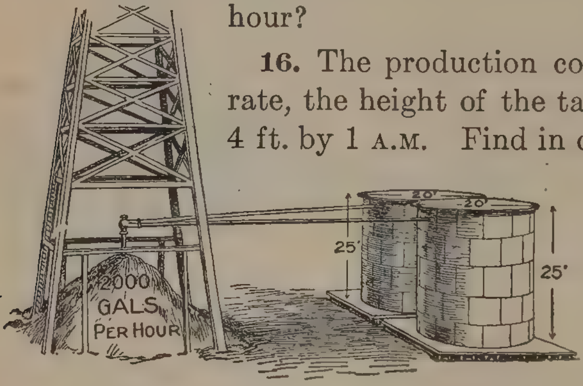
11. The pit for a turntable is 24 ft. in diameter and 4 ft. deep. Find the cost of excavation at 70 cts. per cubic yard.

12. A sprinkling wagon has a cylindrical tank 12 ft. long and 4 ft. in diameter. How many gallons does it contain? (Use 7½ gal. = 1 cu. ft.)

13. How many gallons will the standpipe in Exercise 9 hold?

14. A drain pipe has an internal diameter of 30 in. It is half full of water which flows at the rate of 2 mi. per hour. How many cubic feet of water pass any point per minute?

15. At 7 A.M. a gas tank 220 ft. in circumference is 10 ft. high. At 5 P.M. it has been raised to the height of 40 ft. by the manufactured gas, none having been consumed. How many cubic feet of gas have been manufactured per hour?



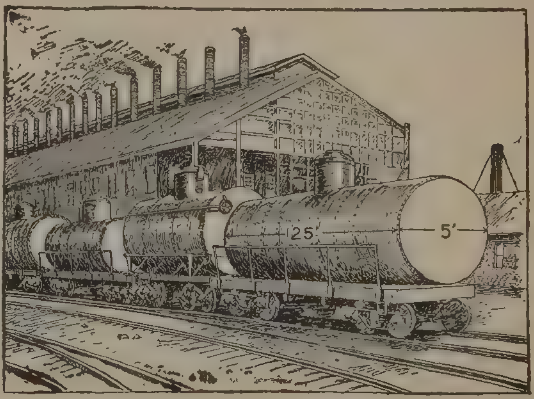
16. The production continuing at the same rate, the height of the tank had diminished to 4 ft. by 1 A.M. Find in cubic feet the average hourly consumption from 5 P.M. to 1 A.M.

17. How many gallons of petroleum does a well flowing at the rate mentioned in the picture produce in a day? In a week? In a year?

18. What is the capacity of the tanks?

19. A pipe-line for carrying oil from Pittsburg to New York City delivers 300 gal. a minute. How many gallons does it deliver in a day? In 365 days? How long would it take to empty 100 tanks like those above?

20. The refined oil is shipped in tank cars. How many gallons does this car carry? What is the value of such a carload of kerosene at 9 cts. a gallon? Of gasoline at 13 cts. a gallon?



21. A wagon for watering streets had a cylindrical tank 12 ft. long and $3\frac{1}{2}$ ft. in diameter. How many gallons did it hold?

PYRAMIDS

473. If pieces of paper be cut like Figures 1 and 2 and folded so that the edges of the triangles come together, the forms made are **pyramids.**

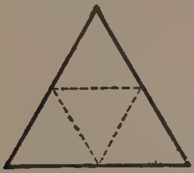


Fig. 1.

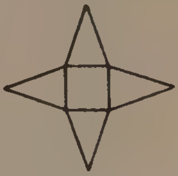


Fig. 2.

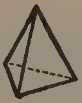


Fig. 3.

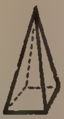


Fig. 4.

474. The base, or bottom, of a pyramid may have any number of sides. Pyramids are called *triangular, square,* or the like, according to the shape of the base. What kind of figures are the other faces of a pyramid?

475. Preparatory.

1. A pupil wrapped a piece of paper around a square pyramid and trimmed it off even with the base. He also wrapped a piece of paper around a rectangular block with base and altitude equal to those of the pyramid, leaving one end open and trimmed off even with the solid. After pasting the margins so as to hold the forms firmly in shape, he filled the paper pyramid with sand and then poured the sand into the paper prism. By doing this three times, he filled the prism.

2. If convenient, perform an experiment similar to this.

3. The volume of the prism was found to be three times as great as that of the pyramid. Their bases and altitudes were equal. Hence, the volume of a square pyramid is $\frac{1}{3}$ of the product of its base and altitude.

476. *The volume of any pyramid, no matter what its base, is* $\frac{1}{3}$ *of the product of the altitude and the area of the base.*

WRITTEN EXERCISES

1. Find the volume of a triangular pyramid, the area of whose base is 36 sq. in. and whose altitude is 8 in.

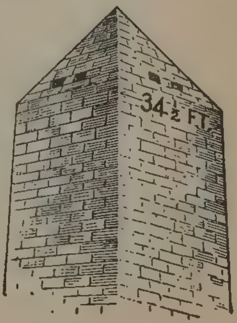
2. The pyramid of Cheops, shown in the picture, stands on a square base 746 ft. on a side. What is the area of the base? The altitude of this pyramid is 480 ft. Find its volume.

3. What kind of figures are the lateral sides of a pyramid? How is the area of a triangle computed? Find the lateral area (surface, excluding the base) of the pyramid of Cheops (altitude of side 608 ft.).

4. Find the numbers to fill the blanks in the following·

ALTITUDE OF PYRAMID.	BASE (Rectangular).	VOLUME.
10 ft.	4 ft. by 5 ft.	( )
16 yd.	( ) sq. yd.	1,728 cu. yd.
( )	10 in. by 9.5 in.	1,187.5 cu. in.
5½ in.	16 sq. in.	( )
75 ft.	51 sq. ft.	( )

5. A pyramid is 50 ft. square and 75 ft. high. Find its volume.



6. The Washington monument is crowned with a square pyramid as shown in the picture. The top of the pyramid is 25 ft. above its base. Find its volume.

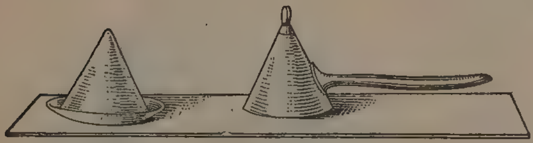
7. Find the altitude of a pyramid of which the volume is 84 cu. ft. and the base is a square 6 ft. on a side.

CONES

477. PREPARATORY.

1. Draw a circle of 3 in. radius. Cut along the circle and along a diameter.

2. Roll one of the pieces of paper into a form like the ice-cream mold and place it upright on a sheet of paper. Such a figure is called a **cone.**



3. Mark around the base of the cone. What is the figure thus made?

4. Measure the diameter of this circle. Compute its circumference.

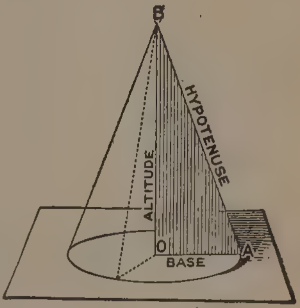
5. Find the area of the sector which formed the cone.

6. Cut from cardboard a right-angled triangle having one perpendicular side longer than the other.

7. Set the triangle upright on a piece of paper, as shown in the picture, and rotate it about the vertical line O B.

8. What figure does the point A trace on the paper?

9. As the triangle rotates about B O, what kind of figure does the hypotenuse generate?



478. The length of the hypotenuse is called the **slant height** of the cone. The point of the cone is called its **vertex.**

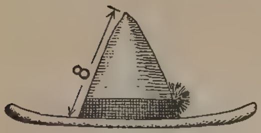
479. *The lateral area of a cone is $\frac{1}{2}$ the product of the perimeter of its base and the slant height.*

EXAMPLE: Find the lateral area of a cone having a circumference of 12 in. and a slant height of 3 in.

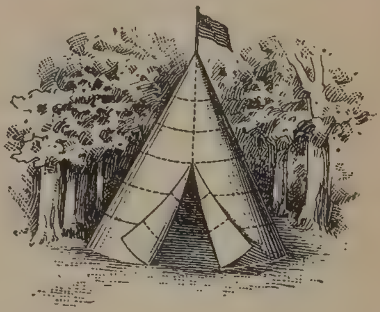
Solution: $\frac{1}{2} \times 3 \times 12$ sq. in. = 18 sq. in.

WRITTEN EXERCISES

1. How many square inches of material are there in the crown of this hat, if the band is 18 in. long?



2. Roy and Harold made a tent for camping. The radius of the base was 6 ft., and the slant height 16 ft. How many square feet were there in its surface?



3. How many square yards in the surface of this tent? What would be the cost of duck for the tent at 12½ cts. a sq. yd., allowing for waste an amount equal to $\frac{1}{10}$ of the surface of the tent?

4. What is the area of the surface of a cone made from a sector that is $\frac{3}{4}$ of a circle of radius 6 in.? Also one made from a sector that is $\frac{2}{3}$ of the same circle?

480. Preparatory.

1. Jennie wrapped a piece of paper about a cone (Fig. 1) and about a cylinder (Fig. 2) having base and altitude equal to those of the cone. What was the shape of the piece of paper in each case?

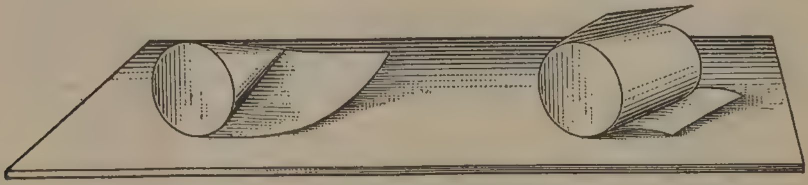


Fig. 1. Fig. 2.

She rolled the sector into a cone and the rectangle into a cylinder and found that three conefuls of sand just filled the cylinder.

2. The volume of the cylinder was how many times that of the cone? The volume of the cone was what part of that of the cylinder?

3. How did their bases and altitudes compare? How is the volume of a cylinder computed?

4. How may the volume of a cone be computed?

481. *The volume of a cone is $\frac{1}{3}$ of the product of the altitude and the area of the base.*

WRITTEN EXERCISES

Find the numbers to fill the blanks:

	1.	**2.**	**3.**	**4.**
Altitude	3 in.	6 ft.	( )	3 yd.
Base	12 sq. in.	40 sq. ft.	12 sq. ft.	( )
Volume	( )	( )	144 cu. ft.	96 cu. yd.

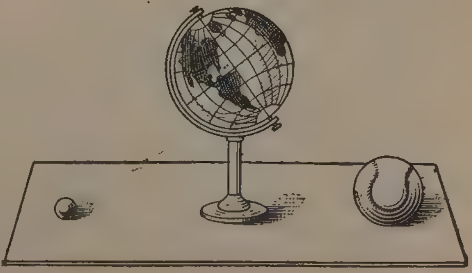
SPHERES

482. If a circle be turned about a diameter as an axis, the surface described is called a **sphere.** The center of the circle is the **center** of the sphere. The space inclosed is called the **volume** of the sphere.

483. The straight line connecting any point of the sphere with the center is called a **radius** of the sphere.

484. A straight line passing through the center of a sphere and terminated by the surface is a **diameter** of the sphere.

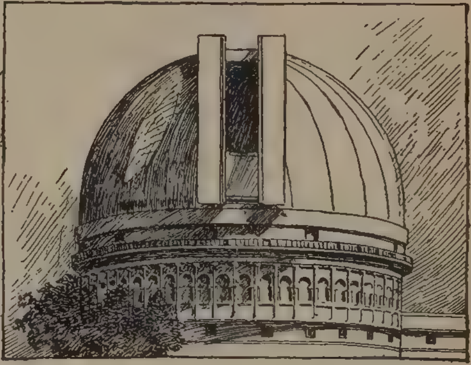
Marbles, balls, and globes are examples of spherical forms.



485. When a flat surface or plane cuts a sphere, the section made is a circle. When the cutting plane passes through the center of the sphere, the circle is called a **great circle**; otherwise, it is called a **small circle.**

Such circles may be seen by immersing some round object, as an orange or a croquet ball, in water to various depths, also by actually cutting spherical bodies, for example, balls of clay.

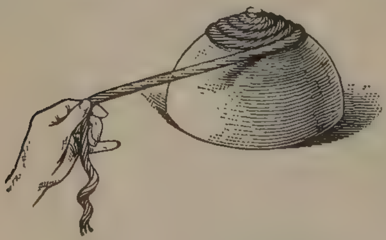
486. The two portions into which a sphere is divided by any cutting plane are called **segments.** When the cutting plane passes through the center, the segments are equal and are called **hemispheres.**

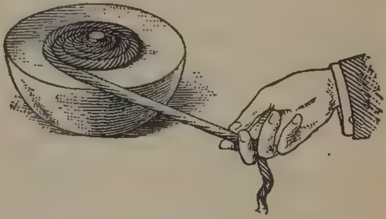


What is the shape of the dome of this observatory?

487. Preparatory.

1. By winding a hemisphere closely with a string and then winding this string on the base of the hemisphere, it is found





that twice as much string is required to cover the convex surface as to cover the base.

Call the area of the base 1. How does the area of the hemispherical surface compare with this area? How does the area of the sphere compare with it?

2. How does the radius of a sphere compare with that of one of its great circles? What represents the area of a circle of radius r? Then what does $4\pi r^2$ express?

488. *The area of a spherical surface is 4π times the square of its radius.*

WRITTEN EXERCISES

1. Using the letter π, state the area of a sphere of radius 3 in.; 6 in.; 4 ft.; 1 yd.

Thus, for radius 3 in. the area is 36π.

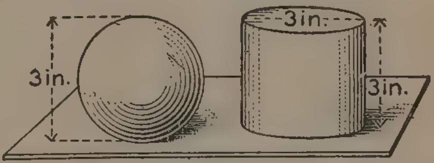
2. Using 3.14 as the value of π, compute the area of each sphere of Exercise 1.

3. The radius of the base of the hemispherical dome of an observatory is 45 ft. Find the perimeter of the base.

4. What is the cost of painting the outer surface of the dome at 2 cts. a square foot, regarding it as a hemisphere?

489. Preparatory.

1. Louise modeled from clay a cylinder and a sphere with the dimensions given. The clay cylinder weighed 30 oz. and the sphere 20 oz. The weight of the sphere was what part of that of the cylinder?



2. Find the volume of a cylinder whose base has a diameter $2r$ and whose altitude is $2r$.

3. According to Exercise 2 what part of this volume is the volume of a sphere of diameter $2r$? What represents the volume of the sphere?

490. *The volume of any sphere of radius r is $\frac{1}{3}$ of the radius r times the surface $4\pi r^2$.*

WRITTEN EXERCISES

1. What is the value of $\frac{4}{3}\pi r^3$, taking $r = 8$ and $\pi = 3.1416$?

2. What is the volume of a sphere whose radius is 7 in.?

3. Find the volume of a sphere of radius: 3 ft.; 1 yd.; 2 in.; 4 in.; 5 ft.

Fill the blanks:

For any sphere, let d = diameter, a = area,
r = radius, v = volume.

	4.	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**
$r =$	( )	2	( )	( )	$\frac{1}{3}$	( )	$1\frac{2}{10}$	( )
$d =$	( )	( )	( )	8	( )	$\frac{4}{7}$	( )	$\frac{6}{10}$
$a =$	4π	( )	36π	( )	( )	( )	( )	( )
$v =$	( )	( )	( )	( )	( )	( )	( )	( )

Solve:

In the following problems use $\pi = \frac{22}{7}$, and simplify by canceling whenever possible.

12. Regarding the earth as a sphere of radius 4,000 mi., find its volume.

13. The gilding of a ball on a church spire cost \$126.72 at \$1.12 per square foot. Find the diameter of the ball.

14. The inner surface of the dome of the capitol at Washington is covered by a fresco painting. Regarding the surface as a hemisphere 165 ft. in diameter, how many square feet are there in the painting?

15. A balloon has the form of a sphere 10 yd. in diameter. How many square yards of material are needed to make the balloon?

16. What is the cost of inflating the balloon with gas costing \$1.40 per 1000 cu. ft.?

17. A kettle is in the form of a hemisphere 28 in. in diameter. How many cubic inches of water does it hold? How many gallons?

18. On a church spire there is a ball 2 ft. in diameter. What will it cost to gild the ball at \$1.20 per square foot?

19. What represents the volume of a hemisphere of radius r? What represents the volume of $\frac{1}{8}$ of a sphere of radius r?

20. Express the volume of a sphere whose diameter is d. What is the volume when $d = 15$ in.?

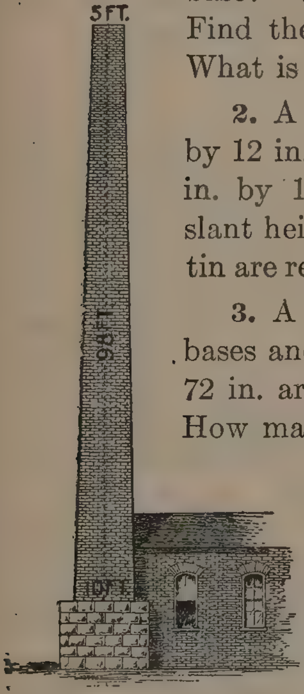
FRUSTUMS

491. When a pyramid is cut parallel to the base, a smaller pyramid is cut off. What is left is called a **frustum of a pyramid.**

Frustums of pyramids are common in such forms as are shown below.

WRITTEN EXERCISES

1. The chimney in the figure is a frustum of a square pyramid with the dimensions marked. What is the area of each base? What kind of figures are the lateral faces? Find the area of one of them. Of all of them. What is the total surface of the frustum?

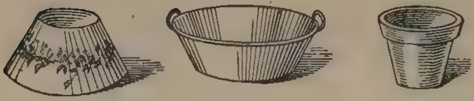

5 FT.
98 FT.
10 FT.

2. A tin pan measures 8 in. by 12 in. on the bottom and 10 in. by 15 in. on the top; the slant height is 6 in. How many square inches of tin are required to make it?

3. A waste basket has regular hexagons as bases and measures 54 in. around the bottom and 72 in. around the top; the slant height is 17 in. How many yards of silk 24 in. wide are required to line its inner sides, excluding the bottom, and allowing ½ yd. for waste?

4. A manufacturer wishes to purchase cardboard to make 10,000 lampshades; the shades are frustums of regular hexagonal pyramids and measure 15 in. around the top and 36 in. around the bottom with a slant height of 6 in; he buys cardboard 19 in. by 24 in. by the ream of 480 sheets and finds that 10% of each sheet is wasted in cutting. How many reams must he buy?

492. By cutting a cone parallel to the base, a smaller cone is cut off. What is left is called a **frustum of a cone.**



The objects here pictured are frustums of cones.

If C = convex surface,
S = slant height,
A = altitude,
R = radius of lower base,
r = radius of upper base,
V = volume,

it can be proved that:

1. $C = (R + r)\pi S.$

2. $V = \dfrac{\pi A\,(R^2 + r^2 + Rr)}{3}.$

These formulas should be used in solving the following exercises. Use $\frac{22}{7}$ for π and simplify by canceling as much as possible.

WRITTEN EXERCISES

1. The bottom of a dish pan is 10 in. in diameter, the top is 14 in. in diameter, and the slant height of the pan 9 in. How many square inches of tin are required to make it?

2. A tub is 26 in. in diameter at the bottom, 40 in. at the top, and 24 in. high, inside measure. How many gallons does it hold?

3. The slant height of the same tub is 25 in., inside measure. How many square feet of lumber are there in it, disregarding the thickness of the lumber?

4. A lighthouse tower is in the shape of a frustum of a cone 20 ft. in diameter at the base, $10\frac{1}{2}$ ft. at the top, and 45 ft. high. The hollow portion within is a cylinder 8 ft. in diameter. How many cubic feet of masonry in the tower?

5. The slant height of the same tower is 45 ft. 3 in. What will it cost to paint the walls outside and inside at $3\frac{1}{8}$ cts. a square foot?

6. The tank of a street sprinkler is in the form of a frustum of a cone with inside diameters 8 ft. at the bottom and 6 ft. at the top, and inside height 5 ft. Find its capacity in cubic feet.

REVIEW

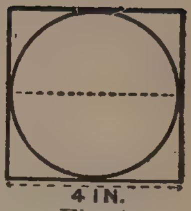


Fig. 1.

1. If a circle is drawn in a square, as shown in Figure 1, what is the area of the circle?

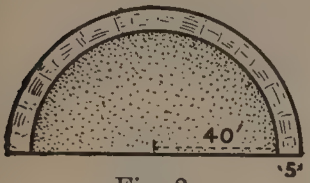


Fig. 2.

2. What is the area of the part of the square not included in the circle?

3. Figure 2 shows a walk about a semicircular garden. Find the length of the outside of the walk. Of the inside.

4. Find the area of the garden and the walk. Of the garden alone. Of the walk.

Find the missing numbers in this table of circles:

	DIAMETER.	RADIUS.	CIRCUMFERENCE.	AREA.
5.	17 in.	( )	( )	( )
6.	( )	3.5 ft.	( )	( )
7.	( )	( )	( )	3.14 sq. ft.
8.	( )	( )	18.84 ft.	( )

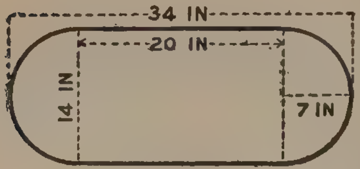
Find the missing numbers relating to cylinders:

	DIAMETER.	ALTITUDE.	LATERAL AREA.	TOTAL AREA.	VOLUME.
9.	10 in.	12 in.	( )	( )	( )
10.	6 in.	( )	150.72 sq. in.	( )	( )
11.	( )	8 ft.	( )	( )	226.08 cu. ft.
12.	( )	10 in.	62.8 sq. in.	( )	31.4 cu. in.

Solve:

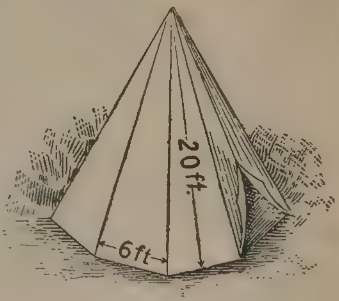
13. An assembly hall is in the form of a rectangle with one semicircular end; the rectangle is 20 ft. by 40 ft. What is the cost of boards for the floor at $18.20 a thousand feet?

14. A wash boiler is 16 in. deep and has the base indicated in the figure. What is its capacity?



15. What is the radius of a circle whose area is 3.14 sq. ft.?

16. A steam gauge indicates the pressure on a square inch. When the steam gauge reads 45 lb. it means that the steam pressure is 45 lb. on every square inch of inner surface of the boiler. At this rate what is the entire pressure in tons on a cylindrical boiler 2 ft. in diameter and 8 ft. long?



17. An 8-sided tent has the dimensions indicated. How many square yards of canvas are required for it?

18. If the perimeter of the base of a rectangular solid is 16 and its altitude is 5, what is its lateral surface?

19. If the lateral surface is 120 and the altitude 6, what is the perimeter of the solid?

20. If the perimeter of the base of a right pyramid is 20 in. and the slant height is 18 in., what is the lateral surface?

21. What is the perimeter of the base, if the lateral surface is 200 sq. ft. and the slant height 20 ft.?

Fill out the blanks for cylinders:

	22.	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**
Lateral area:	4	( )	180	( )	$\frac{15}{4}$	$2\frac{1}{2}$	$12m^2$
Altitude:	7	9	( )	$\frac{1}{2}$	$\frac{3}{4}$	( )	( )
Perimeter of base:	( )	20	15	$\frac{4}{5}$	( )	$1\frac{2}{3}$	$3m$

Fill out the blanks for pyramids:

	29.	**30.**	**31.**	**32.**	**33.**	**34.**	**35.**
Slant height:	28	( )	16	75	( )	3.5	( )
Lateral surface:	( )	400	320	375	625	( )	$25d^2$
Perimeter of base:	30	40	( )	( )	15	8.2	$5d$

GENERAL REVIEW

1. Write in words:
2,100,625; 120,865,407; 6,732.086; 15,905.003005.

2. Write, using exponents for powers of 10:
9,000,000· 19,000,000,000.

Add and test:

3.	**4.**	**5.**
463281	7632845	96.514
732008	4297637	109.009
496207	2330099	136.705
553484	4280691	297.894
236729	2336417	633.946
808909	1234567	525.893
111222	5432098	600.009
367428	6789026	200.946
444333	9998885	111.222
222555	6767676	333.444

Subtract and test:

6.	**7.**	**8.**
893645	208763	106.00897
598764	145679	24.36899

Multiply:

9. 63485 × 363. **10.** 46390 × 879. **11.** 49.0076 × 129.7.

Divide and test:

12. 634389 ÷ 623. **13.** 548623 ÷ 1287. **14.** 18.6032 ÷ 1.763.

Add:

15. $\frac{3}{4}$; $\frac{8}{9}$; $\frac{7}{8}$. **17.** $\frac{1}{12}$; $\frac{9}{10}$; $\frac{7}{15}$. **19.** $\frac{4}{5}$; $\frac{5}{7}$; $\frac{8}{15}$.

16. $2\frac{1}{2}$; $3\frac{2}{8}$; $5\frac{4}{5}$. **18.** $12\frac{1}{2}$; $9\frac{3}{4}$; $1\frac{1}{8}$. **20.** $6\frac{1}{3}$; $33\frac{2}{5}$; $2\frac{8}{9}$.

Subtract:

21. $\frac{9}{34} - \frac{3}{17}$. **23.** $\frac{40}{9} - \frac{17}{18}$. **25.** $\frac{23}{25} - \frac{4}{5}$.

22. $9\frac{1}{2} - 4\frac{7}{8}$. **24.** $12\frac{3}{8} - 9\frac{7}{16}$. **26.** $40\frac{3}{5} - 29\frac{3}{4}$.

Multiply:

27. $\frac{5}{8} \times \frac{10}{12}$.

28. $1\frac{1}{2} \times 4\frac{3}{4}$.

29. $\frac{6}{7} \times \frac{13}{14}$.

30. $5\frac{3}{5} \times 6\frac{2}{3}$.

31. $\frac{1}{2} \times \frac{3}{4} \times \frac{8}{9}$.

32. $12\frac{1}{2} \times 33\frac{1}{3}$.

Divide:

33. $\frac{3}{4} \div \frac{7}{16}$.

34. $4\frac{1}{2} \div 3\frac{3}{4}$.

35. $\frac{4}{5} \div \frac{9}{10}$.

36. $5\frac{1}{6} \div 1\frac{11}{12}$.

37. $\frac{3}{11} \div \frac{9}{22}$.

38. $8\frac{9}{10} \div 4\frac{3}{5}$.

Add:

39. 17 da. 15 hr.
183 20

40. 125 mi. 205 ft.
368 735

41. 40 bu. 3 pk.
87 2

Subtract:

42. 53 gal. 1 qt.
19 3

43. 12 yd. 2 ft.
7 2

44. 127 cu. yd. 8 cu. ft.
99 19

Multiply:

45. 17 A. 40 sq. rd.
5

46. 45 gr. 11 doz.
15

47. 6 mi. 5,000 ft.
9

Divide:

48. 9)23 hr. 45 min.

49. 15)226 yd. 1 ft.

50. 121)400 ft. 9 in.

51. Find the amount of the following bill:

Sept. 6	1 enameled dish pan	$.75
	1 “ basin	.20
	1 8-quart pan	.14
	2 oz. putty	.08
“ 8	1 saw	.90
	1 wrench	.45
	2 oz. 2-inch brads	.12
	2 doz. wardrobe hooks	.20
	1 file	.10
	1 pair 8-inch pliers	.60
	1 brace	.75
	1 plane	1.25
	1 ¼-inch bit	.25
Sept. 8	1 ½-inch bit	$.45
	1 pt. turpentine	.10
“ 10	oil and turpentine	.10
“ 12	1 steamer	.25
	1 screw-driver	.18
	1 flour sieve	.20
“ 15	2 asbestos mats	.10
“ 17	1 letter plate	.50
	3 papers tacks	.15
“ 21	1 chopping knife	.25
“ 25	4 Welsbach gas lamps	4.00
“ 27	1 box tapers	.10
	1 gas lighter	.25

52. Mr. Marsh's grocery bill for November contained items costing the following amounts:

Nov. 1–2	*4–8*	*9–14*	*15–20*	*21–25*	*26–30*
$0.15	$0.53	$0.15	$0.10	$0.48	$0.18
.05	.75	.38	.75	.56	.30
.15	.25	.22	.30	.12	.12
.44	.22	.15	.26	.15	.20
.15	.10	.10	.15	.08	.28
.08	.35	.14	.13	.10	.20
.15	.10	.25	.18	.20	.10
.24	.10	.30	.24	.05	.10
.30	.15	.24	.06	.10	.09
.25	.22	.35	.09	.35	.15

Find the total of the bill.

53. How many quarts in 4 bu. 3 pk. 7 qt.? In 10 bu. 1 pk. 5 qt.?

54. How many square inches in 1 sq. yd.? 5 sq. ft.? 100 sq. ft.?

55. How many yards, feet, and inches in 1,620 in.? In 2,000 in.?

56. How many days, hours, and minutes in 8,256 min.? In 12,000 min.?

Express decimally to the nearest thousandth:

57. $\frac{5}{8}$ ft. **59.** $\frac{7}{12}$ yd. **61.** $\frac{11}{64}$ gal. **63.** $\frac{23}{200}$ mi.

58. $\frac{14}{125}$ bu. **60.** $\frac{9}{32}$ hr. **62.** $1\frac{4}{9}$ da. **64.** $9\frac{13}{16}$ cu. yd.

65. Express decimally as days: 3 da. 19 hr. 40 min.

66. Express decimally as miles: 5 mi. 80 yd. 2 ft.

67. Express 7.65 gal. as gallons, quarts, and pints.

68. A chimney 115 yd. high is built at the rate of 2 yd. 1 ft. 8 in. per day. How many days are required to build it?

69. Find the cost of 6 sacks of flour at 99 cts. a sack. Of 12 shirts at 99 cts. each. Of 7 dozen handkerchiefs at $1.99 a dozen.

70. What is the area of a floor 16 ft. by 25 ft.? What did it cost at 25 cts. a square foot?

71. 100 oranges sold for $1.50. Find the rate per dozen.

72. 25 cans of corn sold for $3.75. Find the rate per dozen.

Find the cost of:

73. 27 lb. sugar at 5¼ cts. a lb.

74. 38 lb. steak at 19 cts. a lb.

75. 176 lb. coffee at 33⅓ cts. a lb.

76. 15 lb. 4 oz. tea at 80 cts. a lb.

77. 197 lb. crackers at 12½ cts. a lb.

78. What is the cost of ¼ T. of range coal at $7.25 a ton, and 100 lb. of cannel coal at $8 a ton?

79. A butcher sold a lamb as follows: leg, 22.2 lb. at 14 cts.; loin, 17.5 lb. at 11 cts.; rib, 14.5 lb. at 10 cts.; chuck, 19.8 lb. at 4 cts. What did he receive altogether?

80. What is the cost of 12 chairs at $1.12½ each?

81. What is the cost of 8 books at $.87½ each?

82. How many flags at $15 each can be bought for $285?

83. How many wagons at $87½ each can be bought for $2,450?

84. In a certain year the ten leading butter states of the United States produced the following amounts:

STATE.	MILLION POUNDS.	STATE.	MILLION POUNDS.
Iowa	139	Illinois	87
New York	115	Minnesota	82
Pennsylvania	111	Michigan	68
Wisconsin	107	Kansas	60
Ohio	88	Indiana	55

Represent these amounts graphically to the nearest 10 million, using parallel lines.

85. In a recent year 99,311,678 lb. of crude rubber were imported into the United States; the average price was \$4.47 per pound. What was it worth?

86. Spain yields 32,800 T. of cork annually, valued at \$183 a ton. What is the yield worth?

87. The area of Douglas Park in Chicago is 179.79 A.; it is a rectangle about $\frac{1}{3}$ mi. wide. How long is it in miles? In feet?

88. A man set out geraniums 8 in. apart along a bed 5 yd. 2 ft. 4 in. long. If the end plants stand 4 in. from the edges of the bed, how many plants were required?

89. What is the cost of a hardwood floor for a room 15 ft. by 18 ft. at 90 cts. a square yard?

90. A rug 9 ft. wide covers 12 sq. yd. of floor. How long is the rug?

91. A manufacturer wishes to make 25 dozen tables 44 in. wide and 10 ft. long, with square ends. How many square feet of lumber are required to make the tops?

92. In 1900 there were manufactured in the United States 1,825,769 watch movements. If these were placed side by side on a straight line, find the length of the line, estimating that, on an average, one watch would require 2 in.

First make a rough estimate in miles.

93. The fence around a rectangular corner lot is 279 ft. long. Find the frontage of the lot on both streets together.

94. A boy measured the side of the house with a stick which he thought was a yard long, and reported the house to be 42 ft. long. He afterward found that his stick lacked 2 in. of being a yard long. What was the length of the house?

95. A room 21 ft. by 18 ft. is to be covered with carpet 27 in. wide, the breadths running the long way of the room. The cost of sewing is 3 cts. per yard of seam, the cost of the carpet is \$1.12$\frac{1}{2}$ a yard, and the cost of laying, $\frac{1}{2}$ ct. a yard. Find the cost of the carpet put down.

96. A farmer put 671 bu. of apples into barrels holding 2 bu. 3 pk. each. How many barrels were required?

97. 4 is what per cent of 32? Of 24? Of 12? Of 100?

98. 5 is 10% of what number? 6 is 6% of what number?

99. A 10% cut in the wages of the Rhode Island cotton-mill operators reduced their weekly earnings $20,000. How much had they been receiving a week?

100. Wages of 75,000 operatives earning $10 a week were cut 8%. What was the reduction of their total weekly earnings?

101. Ice-cold water increases 4% of its volume if heated to the boiling point. What is the volume of boiling water which measured 5 gal. 3 qt. 1 pt. when ice-cold?

102. If oak wood is burned, the ashes weigh about 3% of the original weight of the wood. How many pounds of wood are burned in producing 5 lb. of ashes?

103. In a recent year 39,673 applications were made for patents at the Patent Office in Washington; 26,418 were granted. What per cent of the number of applications was this?

104. 1,802 of these applications were made by foreigners. How many per cent of the total were made by Americans?

105. An ocean steamer requires provisions as follows:

20 T. of flour and 125% as many tons of potatoes.
2,000 qt. of milk and cream, and 75% as much ice cream.
35,000 lb. fresh meat; 8½% as much fish, and 14% as much salt meat.

How many pounds of fish are required? Of salt meat? How many quarts of ice cream? How many tons of potatoes?

106. The total coal production of the United States in 1903 was 359,421,311 T., an increase of 19% over 1902. How many tons were produced in 1902?

107. The value of the coal produced during a recent year was $504,190,733, an increase of 38% over the previous year. What was the value of the product of the previous year?

108. 1,000,000 bales of cotton often change owners on the New York Exchange in one day during the winter season. What is the value of such a sale at 12 cts. a pound, allowing 500 lb. to the bale?

109. A cotton gin cleans 1000 lb. of cotton in the time it would take a man to clean 5 lb. The cotton gin does how many times the work of one man?

110. Find the total number of bushels of wheat received in the markets of these cities on the date of the statement. On the same date a year before.

111. If the wheat was sold at an average price of $.75 a bushel, what was its value in each case?

The receipts of wheat were as follows:

	To-day. Bushels.	A year ago. Bushels.
Chicago..........	15,000	136,208
Milwaukee........	6,160	880
Minneapolis.......	71,200	132,050
Duluth...........	6,953	17,260
St. Louis.........	23,000	28,000
Kansas City......	18,900	36,000

112. What was the brokerage for selling each amount at $\frac{1}{8}\%$?

113. Mr. Roe sold 5 shares of a carpet company's stock at $110 a share. How much was this above the par value of the stock? How much did he gain on 5 shares bought at par?

114. An investor in the stock of a cement company sold ten 100-dollar shares at $90 each. How much below par was this? If he bought them at par how much did he lose?

115. A broker sold 150 shares of copper stocks at $125 a share and charged $\frac{1}{8}\%$ brokerage. How much did the seller receive?

116. What did 75 shares of railroad stock at $110 per share cost a purchaser, with brokerage, $\frac{1}{8}\%$?

117. Mr. White's broker sold 25 shares of railroad stock for him at $116 a share, $\frac{1}{8}\%$ brokerage. What did Mr. White receive?

118. A man bought 50 shares of Pullman Palace Car stock at 220. If an annual dividend of 8% is paid, how much does he receive a year? What per cent of the amount invested is this?

119. In Galveston, Texas, 11,000,000 cu. yd. of earth were used to build an embankment, costing 18½ cts. per cu. yd. To pay for the improvement the city issued $1000 bonds. How many were required?

120. A collector was to receive 2% of the first $5,000 of his receipts and 1½% of the remainder of his receipts. He turned in $8,934.20. How much did he collect?

121. A commercial traveler receives a commission of 5% on his sales; on Tuesday his sales amounted to 3 times as much as on Monday; his commission for the two days was $24. How much were his sales for each day?

122. Four workmen receive together $546 for a job on which they worked 30, 24, 20, and 17 days, respectively. How much should each one receive?

123. Taking the population of the United States to be 76,000,000, how many bushels of wheat are required to feed the people, allowing 4⅗ bu. to each inhabitant?

124. The wheat crop of the world in a recent year was 2,226,745,000 bu.; this was only 95% of the demand. How many bushels did the crop lack of supplying the demand?

125. In another year the crop was 2,879,924,000 bu., which was 113% of that year's demand. How many bushels were there in excess of the demand?

126. 47,000,000 A. of wheat were sown in this country in one year. How many bushels of seed were required, allowing 1½ bu. to the acre?

127. 50,000 men are engaged in the Dakotas during a wheat harvest. At $65 a month, how many dollars does it take to pay them for July and August?

128. In a recent year Minnesota sowed 5,000,000 A. of wheat and harvested 78,000,000 bu. What was the yield per acre?

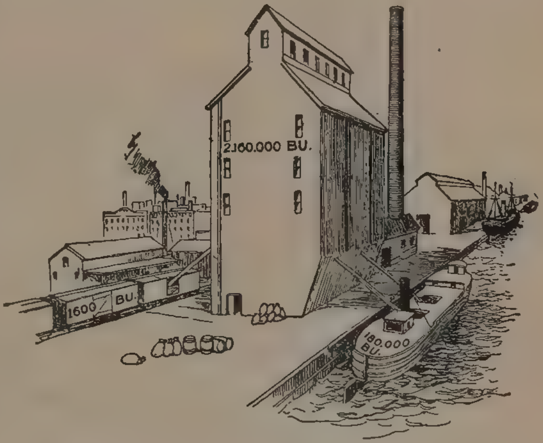
129. Pennsylvania consumed $\frac{9}{13}$ of her wheat crop of 26,000,000 bu. How many bushels were shipped out of the state?

130. A cargo of wheat on the Great Lakes contained 360 carloads of 700 bu. each. How many bushels was this?

131. How many boatloads of 180,000 bu. each does the elevator in the picture hold? How many carloads?

132. How many times would the annual wheat yield of Minnesota (about 80,000,000 bu.) fill the elevator shown?

133. Recently, Minnesota's wheat crop (see Exercise 132) was $1\frac{1}{3}$ times as large as that of Great Britain, twice that of Australasia, and $6\frac{2}{3}$ times that of Egypt. How many bushels were grown by each?


2,160,000 BU.
1600 BU.
180,000 BU.

134. A New England cotton mill uses 15 bales of 500 lb. each a day. How many tons does it use in 300 days?

135. This mill employs 600 hands and has a weekly pay roll of $2,000. What is the average weekly wage for each employee?

136. A cotton plantation of 600 A. yields 250 lb. of cotton an acre. How many 500-pound bales are produced? What is the value of the cotton at 12 cts. a pound?

137. For each pound of cotton of Exercise 136, $2\frac{1}{2}$ lb. of seed were obtained. How many pounds of cotton seed did the plantation yield?

138. The total wheat crop of the United States in a certain year was 670,000,000 bu., of which 40% was consumed at home. How many bushels were consumed at home? How many were exported?

139. Recently, America produced 769,000,000 bu., or about $\frac{1}{4}$ of the world's production. What was the approximate production of wheat in that year? Europe produced $\frac{1}{2}$ of this amount. How many bushels was this?

140. There were in the same year 550,000,000 bread eaters in the world, consuming $4\frac{1}{2}$ bu. per capita. How many bushels were consumed? Was the wheat crop sufficient to meet the demand?

141. The lower Mississippi basin and Atlantic slope is the greatest cotton producing region in the world; its area is $\frac{1}{6}$ of that of the United States. Taking the latter to be 3,025,600 sq. mi., how many square miles are there in the cotton region?

142. A cotton plantation contained 675 acres. Allowing 250 lb. to the acre and 500 lb. to the bale, how many bales are produced by the plantation?

143. The planter sells his cotton for 10 cts. a pound. Find the value of one bale. Find the value of one season's product from this plantation.

144. 28 million acres of cotton were grown in the United States in 1903; the total crop was 10,758,000 bales. What was the average yield in bales per acre? In pounds per acre?

145. What was the total value of this product at 12 cts. a pound?

146. Nearly all of the raw cotton grown in the United States is shipped to the northern states or to Europe, for manufacture. 6,482,849 bales were exported to Europe in 1903. According to Exercise 144 how many bales were retained at home? What per cent (within 1%)?

147. Refined oil is exported in 5-gallon cans. A certain factory for making cans has a daily capacity of 70,000 cans. How many gallons of oil can be shipped in the cans that it makes in one year (300 days)?

148. In 1880 two men could solder 1000 cans in a day; by use of the latest machinery, three men can now solder 24,000 cans in a day. Taking the wages to be the same, the cost of soldering a can now is what part of the cost in 1880?

149. In a recent year the Standard Oil Company imported 60,000 tons of tin, which cost $10,000,000. How much was this a pound?

150. In 1872 the United States exported 16,363,975 gal. of crude oil; in a recent year it exported 134,892,120 gal. What was the percentage of increase?

151. The world's production of petroleum in one year was recently 5,000,000,000 gallons, of which the United States produced $2\frac{1}{2}$ billions and Russia $2\frac{1}{4}$ billions. How many gallons were produced by all other countries?

152. In a certain leap year there was built on an average 1 oil tank a day; the tanks averaged in capacity 30,000 bbl. What was the total tankage added in this year?

153. The average cost of the tanks mentioned in Exercise 152 was $8,500 each. What did these tanks cost?

154. What is the value of 1,500 tons of steel rails at $28 a ton?

155. The following list shows the annual capacities of the leading steel manufactories. Find the total capacity:

Carnegie	2,000,000 tons	National..........	2,580,000 tons
Federal Steel ...	2,310,000 “	Am. Steel and Wire	935,000 “

156. In 1890 the value of the steel exports of this country was $27,000,000; their value in 1900 had increased 480%. What was this value? In 1897 the value was $62,737,000. What was the percentage of increase from 1897 to 1900?

157. The area of the United States is 3,029,600 sq. mi. The coal fields underlie $\frac{1}{16}$ of the country. How many square miles of coal fields are there?

153. In a recent year, 258,372 T. of bituminous, or soft, coal were mined in the United States, taken from the following regions:

Appalachian field	65%	Western field	12%
Central field	15%	Other fields	8%

How many tons were produced by each?

159. When goods listed at $45 are purchased at a discount of $\frac{1}{3}$, and 20%, what is their cost?

What is the interest on:

160. $500 at $3\frac{1}{2}$% for 1 yr. 8 mo. 6 da.?

161. $625 at 4% for 2 yr. 10 mo. 15 da.?

162. $1200 at $2\frac{1}{2}$% for 5 yr. 6 mo. 28 da.?

163. $985 at 3% for 4 yr. 0 mo. 17 da.?

164. $725 at $4\frac{1}{2}$% for 5 yr. 3 mo. 0 da.?

Fill out the blanks for cylinders:

	165.	**166.**	**167.**	**168.**	**169.**	**170.**
Volume:	( )	( )	( )	200	640	800
Altitude:	12	20	30	20	16	( )
Area of base:	16	25	14	( )	( )	25

Determine the numbers to fill the blanks:

VOLUME OF PYRAMID.	AREA OF BASE.	ALTITUDE.
171. 432 cu. ft.	( )	12 ft.
172. 96 cu. in.	36 sq. in.	( )
173. ( )	225 sq. ft.	15 ft.

174. What is the volume of a cone whose base contains 18 sq. ft. and whose altitude is 27 ft.?

175. How is the lateral area of a cone found? What is the lateral area of a cone of radius 6 in. and slant height 20 in.?

176. Find the area of the surface of a sphere of radius 3 in.

SUPPLEMENT

THE METRIC SYSTEM

1. The **metric system** of measures is called international because it has been adopted by the following 43 governments:*

- Argentina
- Austria-Hungary and territories
- Belgium
- Bolivia
- Brazil
- Bulgaria
- Central America
- China (28 ports)
- Chili
- Colombia
- Costa Rica
- Cuba
- Ecuador
- Egypt
- Finland
- France
- French Colonies (including Madagascar)
- Germany
- Greece
- Guatemala
- Haiti
- Holland and dependencies
- Honduras
- Italy and dependencies
- Japan
- Java
- Mauritius and dependencies
- Mexico
- Nicaragua
- Norway and Sweden
- Ottoman Empire
- Peru
- Philippines
- Porto Rico
- Portugal, Azores, and Madeira
- Roumania
- Russia
- Salvador
- San Domingo
- Servia
- Spain and colonies
- Switzerland
- Uruguay

In the United States and Great Britain the use of the system is permitted by law, but not made compulsory.

It has not been recognized by law in Morocco, Denmark, Persia, Siam, the interior of China, and among the African races.

Although the international system of measures has not yet been adopted by the United States, our large and growing international commerce requires acquaintance with it. The system is already used to a large extent in the United States by manufacturers for export and by scientists.

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* The statements of this page and a few of the exercises following are taken from the report of the Committee on Coinage, Weights and Measures of the 57th Congress.

2. A unit of each denomination is always 10 times as large as the unit of the next lower denomination. This makes the computations conform to the scale of 10, and hence the system is often called the *decimal* system.

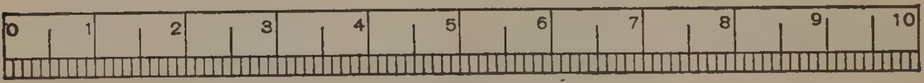
Linear Measure

3. The common units are the **meter**, the **centimeter**, the **millimeter**, and the **kilometer**.

The meter is 39.37 in. long and is used in quite the same way as the yard is used in this country.

LINEAR MEASURE

10 millimeters = 1 centimeter
10 centimeters = 1 decimeter
10 decimeters = 1 meter
10 meters = 1 dekameter
10 dekameters = 1 hektometer
10 hektometers = 1 kilometer



1 DECIMETER.

The following are *standard* abbreviations:

mm. for millimeter
cm. " centimeter
m. for meter
km. " kilometer
dm. for decimeter

TABLE OF EQUIVALENTS

1 yard = .91 meters
1 mile = 1.61 kilometers
1 meter = 39.37 inches
1 kilometer = .62 miles

Only one unit is commonly used to express a magnitude. For small lengths the millimeter is used. A board would be described as 23 millimeters thick, not 2 centimeters 3 millimeters, just as we say a 10-qt. bucket, not a 2-gal. 2-qt. bucket. For greater lengths, centimeters, meters, and kilometers are the usual units. Thus:

36 cm. is read 36 centimeters,
7.32 m. " " 7 and 32 hundredths meters,
56.47 km. " " 56 and 47 hundredths kilometers.

ORAL EXERCISES

Read the following:

1. 3.5 m.; 84 mm.; 27 cm.

2. A board is 18 mm. thick.

3. A picture is 27 cm. by 32 cm.

4. A remnant of silk contains 3.27 m.

5. From Paris to Calais is 295.32 km.

6. A foot is 30.5 cm. long.

WRITTEN EXERCISES

1. What is the cost of 7 m. of cloth at $1.80 a meter?

2. What is the cost of 9.2 m. of ribbon at $.35 a meter?

3. What is the cost of 17.46 m. of lace at $.55 a meter?

4. How many square meters of carpet will it take to cover a floor 6 m. long and 5 m. wide? One 7.2 m. long and 3.45 m. wide?

5. A rope is 5.36 m. long. How many cm. long is it?

6. How many cm. are there in 16 m.? In 60 mm.?

7. Which divisions of the scale on the opposite page are centimeters? Which are millimeters?

8. How many rounds are there in a ladder 12 m. long when the rounds are 40 cm. apart?

9. How many meters of ribbon would be required to make 35 badges 20 cm. in length?

10. If the current of a river flows 2,160 m. an hour, how many meters does it flow in a minute?

11. 1 meter = ( ) decimeters? 1 m. = ( ) cm.?

12. 1 cm. = ( ) mm.? 1 dm. = ( ) mm.? 1 mm. = .001 of a ( )? 1 cm. = .01 of a ( )? 1 dm. = .1 of a ( )?

13. What are the common international units of length?

14. Express a decimeter in inches. A millimeter.

15. The United States postal service accepts packages not over 30 cm. in length, 20 cm. in width, and 10 cm. in thickness. Express the dimensions in inches.

16. Express the following barometric readings in centimeters: 29.8 in., 30.1 in., 28.6 in.

17. Taking 1 m. = $3\frac{1}{4}$ ft., how many meters in a mile (to nearest meter)?

18. The distance from Paris to Rouen is 136 km. Express this distance in miles (to nearest mile).

Square Measure

4. The common units are the **square meter** (sq. m.), **square centimeter** (sq. cm.), the **square millimeter** (sq. mm.), and the **square dekameter,** the **square hektometer,** and the **square kilometer** (sq. km.).

The square dekameter and the square hektometer when used to measure land are called, respectively, the **are** and the **hektare** (ha.).

SQUARE MEASURE

100 square millimeters = 1 square centimeter
100 square centimeters = 1 square decimeter
100 square decimeters = 1 square meter
100 square meters = 1 square dekameter
100 square dekameters = 1 square hektometer
100 square hektometers = 1 square kilometer

TABLE OF EQUIVALENTS

1 square yard = .84 square meters
1 acre = .405 hektares

1 square meter = 1.2 square yards
1 hektare = 2.47 acres

WRITTEN EXERCISES

1. How many square millimeters are there in 2 sq. cm.? How many square centimeters are there in 4 sq. m.? How many square millimeters in 1 sq. m.?

2. How many square meters are there in 1 are? How many ares are there in 2 ha.?

3. How many hektares are there in a square kilometer?

4. If a square centimeter of gold leaf is worth 3 cts., what is the cost of 10 sq. m.?

5. 30 ha. are about how many acres? 50 hektares are about how many acres?

6. How many square meters are there in the surface of a street 20 m. wide and 5 km. long?

7. A floor is 5.2 m. by 8.4 m. How many tiles 7 cm. by 13 cm. would be required to pave it?

8. A lot contains 2.7 ares; the owner refused an offer of $1,800 an are for it; afterward he sold it for $20 a square meter. Did he gain or lose, and how much?

9. The distance from Paris to Orleans is 121 km.; the railway is a double track, and the rails are 4 m. long. How many rails are used?

10. The roadbed of the railway just mentioned is 15 m wide. How many hektares does it contain?

Cubic Measure

5. The cubes of the various linear units are used as measures of volume. The most important are the **cubic meter** (cu. m.), the **cubic decimeter** (cu. dm.), the **cubic centimeter** (cu. cm.), and the **cubic millimeter** (cu. mm.).

CUBIC MEASURE

1000 cubic millimeters = 1 cubic centimeter
1000 cubic centimeters = 1 cubic decimeter
1000 cubic decimeters = 1 cubic meter
1000 cubic meters = 1 cubic dekameter
1000 cubic dekameters = 1 cubic hektometer
1000 cubic hektometers = 1 cubic kilometer

The cubic meter is called a **stere** when used in measuring wood.

TABLE OF EQUIVALENTS

1 cubic in. = 16.4 cubic centimeters
1 cubic ft. = 28.3 cubic decimeters
1 cubic yd. = .76 cubic meters

1 cubic centimeter = .061 cubic in.
1 cubic decimeter = .035 cubic ft.
1 cubic meter = 1.31 cubic yd.

WRITTEN EXERCISES

1. If a cubic meter of granite weighs 7 metric tons, what is the weight of a block 5.5 m. long, 3.75 m. wide, and 2.4 m. thick?

2. How many cubic meters of earth are there in an excavation 4 m. long, 6.5 m. wide, and 2.2 m. deep?

3. How many wagon loads of earth of 1 cu. m. each are removed in excavating a cellar 5 m. long, 4.5 m. wide, and 2.8 m. deep?

4. How many cubic centimeters are there in a package 30 cm. by 20 cm. by 10 cm.?

5. How many steres of wood are there in a pile 10.5 m. long, 1 m. wide, and 2.5 m. high? How many cords?

6. At $7.50 a cubic meter what is the cost of a bridge pier 10.5 m. wide, 3 m. thick, and 13 m. high?

7. How many cubic meters are there in the capacity of a car 10 m. long, 2.5 m. high, and 2 m. wide? How many cubic feet?

8. On ocean steamers $\frac{1}{2}$ cubic meter is sometimes allowed for a passenger's baggage. A traveler has a box 42 cm. by 48 cm. by 76 cm., a trunk 32 cm. by 80 cm. by 50 cm., a chest 45 cm. by 52 cm. by 62 cm., and a box 30 cm. by 40 cm. by 105 cm. Has he excess baggage? If so, how much? What is the charge for the excess at $10 a cubic meter?

9. A block of granite was 10.4 ft. long, 3.5 ft. wide, and 2.7 ft. thick. How many meters long, wide, and thick was it? How many cu. m. in its volume? (Use 1 ft. = .3033 m.)

10. How many steres of wood are there in a cord of 4-ft. wood?

Measures of Capacity

6. The **liter** (l.), **hektoliter** (hl.), and **kiloliter** (kl.) are the common units for liquid and dry measures. The capacity of the liter is 1 cubic decimeter.

In countries where the metric system is in use, the liter corresponds to the English quart measure. That is, just as milk is sold in New York City by the quart, it is sold in Paris by the liter.

MEASURES OF CAPACITY

10 centiliters = 1 deciliter
10 deciliters = 1 liter
10 liters = 1 dekaliter
10 dekaliters = 1 hektoliter
10 hektoliters = 1 kiloliter

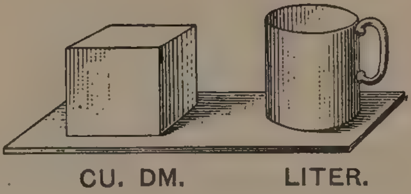


TABLE OF EQUIVALENTS

1 dry quart = 1.1 liters
1 liquid quart = .95 liter
1 bushel = .35 hektoliter

1 liter = .91 dry quart
1 liter = 1.06 liquid quart
1 hektoliter = 2.84 bushels

WRITTEN EXERCISES

1. How many liters of water are there in a tank 3 dm. by 18 dm. by 7.5 dm.?

2. A water tank has a capacity of 1 cu. m. How many liters does it hold?

3. How many hektoliters of grain are there in a bin 4 m. by 5 m. by 2 m.?

4. A grocer bought beans at $2 a hektoliter and sold them at 3 cts. a liter. What per cent did he gain?

5. A milkman sold 5 kl. of milk in a month at 5 cts. a liter. To how much did the sales amount?

6. A tank 1 m. wide, 5.5 m. long, and 20 cm. deep is full of water. How many liters does it hold? How many quarts?

7. If the rainfall is 11 cm. for a certain month, how many cubic centimeters of water fall on an are of land? How many kiloliters? How many gallons?

8. 2.5 kl. of olive oil was put up in bottles holding .5 l. each. How many bottles were used?

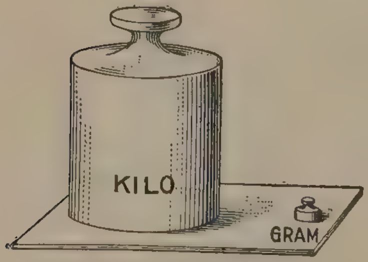
9. A cylindrical tank of radius .52 m. is filled with water to a depth of 2.58 m. How many liters of water are there in the tank? How many quarts in the tank?

10. 2 deciliters are what part of a liter? Of 7 dekaliters? Of 5 hektoliters?

Measures of Weight

7. The weight of a liter of water at a fixed temperature is taken as a unit of weight and is called a **kilogram** or simply a **kilo.** The common units of weight are the **gram** (g.), the **centigram** (cg.), and the **kilogram** (kg.)

The kilo is about 2.2 lb. and is used in the same manner that the pound is used in the United States.



MEASURES OF WEIGHT

10 centigrams = 1 decigram
10 decigrams = 1 gram
10 grams = 1 dekagram
10 dekagrams = 1 hektogram
10 hektograms = 1 kilogram
1000 kilograms = a metric ton = the weight of 1 cu. m. of water.

The weights most commonly used are the kilo and the gram.

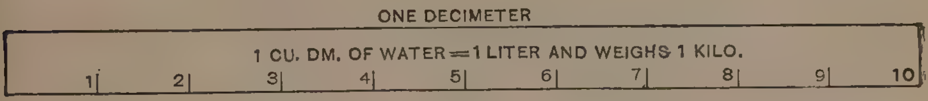
TABLE OF EQUIVALENTS

1 pound avoirdupois = .45 kilogram
1 kilogram = 2.2 pounds avoirdupois
1 ton = .91 metric ton
1 metric ton = 1.1 ton

WRITTEN EXERCISES

1. .5 of a kilogram is how many grams?

2. A cubic decimeter of water weighs how many grams? How many kilograms?



3. Alcohol is .83 as heavy as an equal volume of water. A liter of alcohol weighs how many grams? What part of a kilogram? How many pounds?

4. A tank holding 5 cu. m. of water weighs how many metric tons? How many English tons?

5. A tank whose volume is 1 cu. m. contains how many liters? How many kilos does the water weigh? How many pounds?

6. If the weight of ice is .92 of that of water per volume, what is the weight in kilos of a block of ice .2 m. by .2 m. by 5 m.? What is the weight in pounds?

7. Petroleum is .7 as heavy as water per volume. What is the weight in pounds of 1 cu. m. of petroleum?

8. Express 5 centigrams as grams, 3 decigrams as grams, and add the results.

9. Express as kilos, 6 hektograms + 4 dekagrams + 3 grams.

10. Express as hektograms, 9 dekagrams + 5 grams + 8 centigrams.

11. A cubic foot of water weighs 1000 oz. How many grams is this? How many kilograms?

12. A tank 40 ft. long, 10 ft. wide, and 5 ft. deep holds how many cubic feet of water? How many pounds would the water weigh? How many kilograms?

REVIEW

1. If the metric system were used altogether in this country, what points of superiority would it have over the Anglo-American system?

2. A meter being about $1\frac{1}{12}$ yd., how many square yards are there in a square meter?

3. What is the cost of making a cement sidewalk .875 km. long, 3 m. wide at 65 cts. a square meter?

4. 1 cu. dm. of water weighs about 2.2 lb. What is the weight of the water in a tank containing 5 cu. m.?

5. 4 steres make a wagon load of wood. How many loads of wood are there in a pile $10\frac{1}{2}$ m. long, 1 m. wide, and $2\frac{3}{4}$ m. high?

6. The Eiffel tower is 300 m. high. About how many feet high is it?

7. Since a cu. dm. of water weighs 2.2 lb. and granite is 2.7 times as heavy as water, what is the weight of a granite block 2 m. by 3 m. by $5\frac{1}{2}$ m.?

8. Copper is 8.8 times as heavy as water per volume. A cube of copper of edge 10 cm. weighs how many kilograms?

9. How many square meters are there in the floor of a room 6 m. long and 5 m. wide? How many tiles 1 dm. square will it take to cover it?

10. The weight of a liter of hydrogen is .07 of the weight of a liter of air, and 1 liter of air weighs 1.293 g. What is the weight of 1 liter of hydrogen?

11. A cylindrical granite shaft is 2 m. in diameter and 10 m. high. If the stone is 2.7 times as heavy as water per volume, what is the weight of the shaft?

12. The Singer building in New York City is 612 ft. high. How many meters high is it?

THE INTERNATIONAL SYSTEM AND THE ANGLO-AMERICAN SYSTEM CONTRASTED

1. *Add:* ANGLO-AMERICAN.

YD.	FT.	IN.
12	2	7
9	1	8
14	0	11
6	2	5
7	1	10
51	0	5

Add: INTERNATIONAL.

METERS.
12.27
9.18
14.11
6.25
7.20
49.01

2. *Reduce to inches:*	*Reduce to millimeters:*
3 mi.	5 kilometers
167 rods	6 hektometers
3 yd.	7 dekameters
2 ft.	1 meter
9 in.	5 decimeters
	1 centimeter
	2 millimeters
Ans., 223,287 in.	Ans., 5,671,512 mm. (by inspection).

3. Find the contents of a tank:

8 ft. 4 in. long	2.5 m. long
5 ft. wide	1.52 m. wide
2 ft. 5 in. deep	80 cm. deep
Ans., 100 cu. ft. 1,200 cu. in.	Ans., 3.04 cu. m.

4. What is the pressure on the bottom when the tank is filled with water?

Ans., 1.049 lb. per sq. in.	Ans., 80 g. per sq. cm. (No computation needed, since 1 cu. cm. of water weighs 1 g., and the tank is 80 cm. deep.)

5. Find how much the tank holds:

In gallons: (1 gal. = 231 cu. in.).	In liters:
Ans., $753\frac{57}{231}$ gal.	Ans., 3,040 l.

6. Find the weight of water filling the above tank.

Ans., 3 tons 293 lb.	Ans., 3.04 metric tons (by inspection).

FOREIGN MONEYS

8. The prices at which foreign money is actually bought and sold will vary slightly from the following official values, fixed by the United States Treasury. The market price is called the **rate of exchange.**

TABLE OF VALUES OF FOREIGN MONEYS

COUNTRY.	MONETARY UNIT.	VALUE IN UNITED STATES GOLD.
Canada	Dollar	$1.00
Great Britain	Pound sterling	4.8665
France	Franc	.193
Belgium	Franc	.193
Switzerland	Franc	.193
Italy	Lira	.193
Spain	Peseta	.193
Greece	Drachma	.193
Germany	Mark	.238
Austria-Hungary	Crown	.203
Cuba	Peso	.926
Mexico	Dollar (silver)	.443

Currency of Great Britain

1 pound sterling	= 20 shillings	£1	= 20*s.*
1 shilling	= 12 pence	1*s.*	= 12*d.*
1 penny	= 4 farthings	1*d.*	= 4 far.

9. The rate of exchange for sterling is usually expressed by stating in United States money the amount for which £1 sterling can be bought or sold.

Thus, "sterling exchange 4.88" means that $4.88 is paid for £1.

Fractions of the smallest current coin, as the cent of United States money, are to be rejected if less than ½, otherwise to be counted as a whole unit.

WRITTEN EXERCISES

1. When sterling exchange is 4.88, what will a draft on London for £200 cost?

2. If sterling exchange is quoted at $4.87\frac{1}{2}$, how much English money can be procured for $1000?

3. A merchant owes a London firm £1,421 14*s.* 6*d.* (usually written, £1,421–14–6). What must he pay for a draft for this amount at $4.87\frac{1}{2}$?

4. The English Parliament has voted the King an annual income of £470,000. Make an approximate estimate of the equivalent in United States money, taking £1 = $5.

5. The income of the government of the United Kingdom for a recent year is given in the table. Fill the blank:

Taxes	£98,496,000
Post-offices	13,192,000
Telegraphs	3,390,000
Remainder	( )
Total	£128,633,000

6. Make an approximate estimate of the equivalent of each item of the above table in United States money.

7. Determine the equivalent of each item to the nearest cent in United States money, using £1 = $4.8665.

8. The savings banks deposits average as follows *per capita* of population in the countries named:

	£	*s.*	*d.*		£	*s.*	*d.*
Denmark	15	11	$6\frac{1}{4}$	United States	6	4	$10\frac{1}{2}$
Switzerland	13	0	$2\frac{3}{4}$	Austria-Hungary	5	8	$3\frac{3}{4}$
Germany	7	10	$6\frac{3}{4}$	France	4	8	$8\frac{1}{2}$
Norway	7	8	$7\frac{1}{2}$	Great Britain	4	2	$5\frac{3}{4}$
Australia	7	6	$4\frac{3}{4}$	Canada	2	5	$2\frac{3}{4}$

Express each amount in United States money.

9. Express in United States money the items of the following table, and fill the blanks:

GREAT BRITAIN.	IMPORTS FROM.	EXPORTS TO.
British Possessions..........	£99,433,955	£83,426,761
United States..............	126,062,155	14,716,489
France....................	51,396,793	13,706,246
Germany..................	28,534,159	22,525,987
Others....................	( )	( )
Total.................	£470,378,583	£233,359,987

Currency of France

10. The unit is the **franc** (fr.); it is divided into 100 centimes (pronounced *sonteem*). 100 centimes equal 1 franc.

Hence, hundredths of a franc are read centimes.

Thus, 14.25 fr. = 14 francs, 25 centimes; 0.60 fr. = 60 centimes.

It is customary in France and Germany to write a zero before the decimal point when the number is a pure decimal.

11. The price of French money is usually quoted by stating how many francs are worth one dollar.

Thus, if francs are quoted at $5.15\frac{1}{2}$, 5 francs and $15\frac{1}{2}$ centimes are equivalent to $1. For approximate estimates, 1 franc = 20 cts., 5 francs = $1.

WRITTEN EXERCISES

Find in francs the total cost of the following purchases:

1. 23.64 meters of silk at 7.25 francs per meter.

2. 8.5 meters of velvet at 9.95 francs per meter.

3. 127.85 meters of ribbon at 0.60 franc per meter.

Solve:

4. At 5.15 how many francs can be procured for $500?

5. How much will a draft for 2,000 francs cost at $5.15\frac{3}{8}$?

6. A traveler exchanged $15.80 for francs at 20 cts. per franc. How many francs did he get?

7. A teacher sent to Gauthier, a publisher in Paris, for some books. The bill was 47.45 francs. How many dollars did the teacher remit, exchange being 5.15½?

8. When a franc is worth 19.3 cts., how many dollars will a draft for 455.5 francs cost?

9. How many francs will $9.55 buy when exchange is quoted at 5.15⅜?

10. Imports and exports of France in a recent year were:

COUNTRY.	IMPORTS. *Million francs.*	EXPORTS. *Million francs.*
British Isles	669	1,239
United States	460	254
Germany	412	460
Others	( )	( )
Total	4,409	4,079

In each case what per cent of the total was the item for the United States? Supply the missing numbers.

11. Express in dollars each number in Exercise 10.

Currency of Germany

12. The unit is the Reichsmark or **Mark** (M.). It is divided into 100 Pfennigs (pfgs.). 1 mark = 100 pfgs.

Hence, hundredths of a mark are read pfennigs.

Thus, 4.01 M. = 4 marks and 1 pfennig. 0.70 M. = 70 pfennigs.

13. The rate of exchange for German money is usually stated by naming in cents the equivalent of 4 marks.

Thus, when marks are quoted at 95⅞, 4 marks are sold for 95⅞ cts.

For approximate estimates, take one mark as 25 cts.

WRITTEN EXERCISES

Find the total cost in marks of the following purchases:

1. 8.6 meters of calico at 0.45 M. per meter.

2. 14.5 meters of muslin at 0.65 M. per meter.

3. 12 meters of cloth at 5.28 M. per meter.

4. 0.6 kilos of butter at 3.30 M. per kilo.

5. Make approximate estimates of the equivalents in United States money of the prices and of the results of Exercises 1 to 4.

Solve:

6. When marks are quoted at $95\frac{1}{2}$, what will be the cost of a draft on Berlin for 20,000 M.?

7. If marks cost $95\frac{3}{4}$, how many marks can be procured for \$1,200?

8. The following are the expenditures of the German Empire in a recent year:

	MARKS
Army	665,850,576
Navy	172,207,975
Postal and telegraph	355,910,050
Railways	79,925,610
Administration	586,358,408

Estimate the equivalent amounts of United States money for each item.

9. Compute the amount in dollars equivalent to each number in Exercise 8 when exchange is $95\frac{7}{8}$.

Comparison of United States and Foreign Moneys

14. To find the approximate equivalents of amounts of foreign money in United States money, use the following:

One pound = \$5.00.
One shilling = \$ .25.
One mark (M.) = \$.25.
One franc (fr.) = \$.20.

WRITTEN EXERCISES

1. An American in London sees the following prices in the shop windows: hats, 5/9 (5*s.* 9*d.*); gloves, 2/11½; suit, 65/0; overcoat, 2–10–6 (£2 10*s.* 6*d.*); necktie, 1/5½. What would be the approximate equivalents in United States money?

2. The following purchases were made in London:

2½ yd. lace at 18/6 a yard.
7⅜ yd. velvet at 13/9 a yard.
16 yd. silk at 10/3 a yard.
50 yd. ribbon at 1/3 a yard.

Find the total cost in English money and find the approximate cost of each item in United States money.

3. Find the exact cost of the whole purchase mentioned in Exercise 2, exchange being quoted at $4.88⅝.

4. A man offered a sovereign (1 pound) in payment of a book and received 12*s.* 6*d.* in change. What did the book cost?

5. A person imported some books from Berlin; the bill was 200 M. About how many dollars was this?

6. A traveler paid 15.50 fr. for a railway ticket. About how many dollars and cents did the ticket cost?

7. A man's hotel bill in Paris was 67 fr. About how many dollars was this?

8. A man spent 1,300 M. while sightseeing in Germany. About how many dollars was this?

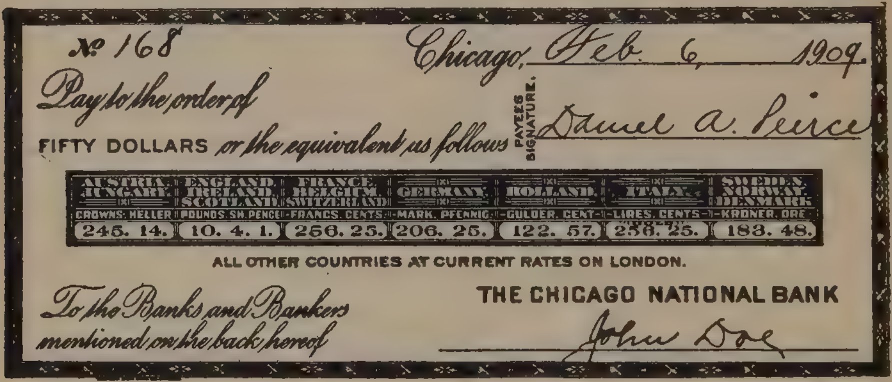
9. A traveler bought an Oriental rug for 240 M. Find the approximate equivalent of the price in United States money. Find the exact equivalent, using 23.8 cts. per mark. Add 40% for duty and find the total cost of the rug.

10. It cost 100,000 fr. to lay out one of the national parks of France. How many dollars was this?

11. A traveler had 215 francs and 90 centimes when he went from Paris to Berlin; he exchanged his money for marks and pfennigs. How many did he receive?

Money Orders

15. Travelers in foreign countries often carry **bank** or **express money orders** instead of cash. The following is a bank money order:



№ 168 Chicago, Feb. 6, 1909.

Pay to the order of

FIFTY DOLLARS or the equivalent as follows

PAYEES SIGNATURE. Daniel A. Peirce

AUSTRIA HUNGARY	ENGLAND. IRELAND. SCOTLAND.	FRANCE. BELGIUM. SWITZERLAND	GERMANY.	HOLLAND.	ITALY.	SWEDEN. NORWAY. DENMARK
CROWNS. HELLER	POUNDS. SH. PENCE	FRANCS. CENTS.	MARK. PFENNIG.	GULDER. CENT.	LIRES. CENTS.	KRONER. ORE.
245. 14.	10. 4. 1.	256. 25.	206. 25.	122. 57.	256. 25.	183. 48.

ALL OTHER COUNTRIES AT CURRENT RATES ON LONDON.

To the Banks and Bankers
mentioned on the back hereof

THE CHICAGO NATIONAL BANK
John Doe

This order cost the purchaser, Mr. Peirce, \$50. The amount of money that he will receive if he presents it at any of the banks in England named on the back of the order is shown on its face to be £10 4*s*. 1*d*. Thus, because of the rate of exchange, the traveler does not always receive the equivalent of \$50 when he cashes the order.

WRITTEN EXERCISES

1. When £1 of English money is worth \$4.866, what are £10 4*s*. 1*d*. worth?

2. The cash value of the above order in Germany is 206.25 marks. At 23.8 cts. a mark, find the equivalent of this amount in United States money.

3. The cash value of the same order in France is 256.25 francs. At 19.3 cts. a franc, express this amount in dollars.

4. Find in English money the cash value of 10 fifty-dollar express money orders at the rate given in the above order. Find similarly their cash value in Paris. In Berlin.

5. Find in United States money the equivalent of the order shown above, if cashed in Austria, the crown being 20.3 cts.

ALGEBRA

Definitions

16. An expression that does not contain within itself the sign of addition or of subtraction is called a **monomial.**

Thus, 7, $\frac{26}{5}$, $\sqrt{18}$, a, a^2c, $\frac{ab}{4^2}$, $xy \div a$ are monomials.

17. Two monomials connected by the sign + or the sign − are called a **binomial.**

Thus, $7-4$, $8+6$, $a+b$, and $c-5d^2$ are binomials.

18. Three monomials connected in any way by addition or subtraction signs are called a **trinomial.**

Thus, $axy - 2bc + 4mn$ is a trinomial.

19. The numbers separated by the sign + or − are called **terms** of the expression.

Thus, $6+8+4$, $6+8-4$, $12-7-3$, $a+5b-c$ are trinomials. 6, 8, and 4 are the terms of $6+8+4$; and a, $5b$, c are the terms of $a+5b-c$.

20. The **parenthesis,** ( ), may be used to group several terms into one expression. The absence of a sign between a number and a parenthesis indicates multiplication.

The following examples illustrate the method of reading expressions containing parentheses:

$4(8-3)$	is read "4 times the binomial, $8-3$."
$a(b-c+d)$	" " "a times the trinomial, $b-c+d$."
$(a+b)(4-3)$	" " "The binomial, $a+b$, times the binomial, $4-3$."
$(8+6-2) \div 4$	" " "The trinomial, $8+6-2$, divided by 4."
$(b+6)^2$	" " "The square of the binomial, $b+6$."
$(a-b+c)^2$	" " "The cube of the binomial, $a-b+c$."

ORAL EXERCISES

Read the following:

1. $(17 + 15) \div (9 - 3 + 10)$.

2. $(a + b) - (c - d)$.

3. $(a + b - c) + (b - d)$.

4. $(a + b)\ (b - c)^2$.

5. $(12 - 7)^2$.

6. $(x + y) - c$.

7. $(x + 1)^2$.

8. $(a^2 + b^2)^2$.

Select the monomials; the binomials; the trinomials:

9. $a + b - c$.

10. $x - y + z + w$.

11. $2g - 5 + x^2$.

12. $4x^2 + 7$.

13. m.

14. $gt^2 + a$.

15. $a + b + c + d$.

16. $a + 5b - c$.

17. $5x^2 + ay^3$.

Addition and Subtraction

ORAL EXERCISES

1. $3a + 4a = ?$ If a is 1, what is the value of $3a$? Of $4a$? Of $3a + 4a$?

2. $6b + 5b = ?$ If b is 1, what is the value of $6b$? Of $5b$? Of $6b + 5b$?

3. If $a = 1$ and $b = 1$, what is the value of $3a + 6b$? What is the value of $4a + 7b$?

Find the value of each of these expressions, when each letter = 1:

4. $a + 2b$.

5. $a + b + c$.

6. $b + c + 2a$.

7. $2a + 2b$.

8. $a + 2b + c$.

9. $b + d + 3c$.

10. $3a + b$.

11. $2a + 3b + 3c$.

12. $c + 4d + 2a$.

13. Find the numbers to fill the blanks: What is the sum of $3a + 4b$ and $2a + 6b$?

$$\begin{array}{rr} 3a + 4b & 7 \\ 2a + 6b & 8 \\ \hline (\ )a + (\ )b & 15 \end{array}$$

14. When $a = 1$ and $b = 1$, what is the value of each addend in Exercise 13? What is the sum of these values? What is the value of the sum? Explain how to test the work of adding.

Add and test by letting $a=b=c=1$:

15. $$\begin{array}{r} a + b + c \\ 2a + b + 3c \\ \hline \end{array}$$

16. $$\begin{array}{r} a + 2b + c \\ a \qquad + 6c \\ \hline \end{array}$$

17. $$\begin{array}{r} 6a + b + c \\ 2b + c \\ \hline \end{array}$$

18. Find the numbers to fill the blanks in this subtraction.

Let $a = 1$, $b = 1$; how is the work tested?

$$\begin{array}{rr} 4a + 7b & 11 \\ 2a + 5b & 7 \\ \hline (\ )a + (\ )b & 4 \end{array}$$

WRITTEN EXERCISES

Add and test:

1. $$\begin{array}{r} a + 3b \\ 10a + 11b \\ \hline \end{array}$$

2. $$\begin{array}{r} a + b + c \\ m \qquad + 6c \\ \hline \end{array}$$

3. $$\begin{array}{r} 16a + 4b \\ 4b + 16a \\ \hline \end{array}$$

4. $$\begin{array}{r} 5a + 9b \\ 5a + 3b \\ \hline \end{array}$$

5. $$\begin{array}{r} 4a \qquad + c \\ 2a + b + c \\ \hline \end{array}$$

6. $$\begin{array}{r} 6a + b \\ 3a + 2b + c \\ \hline \end{array}$$

7. $$\begin{array}{r} m + n + p \\ 2m + 5n + 9p \\ \hline \end{array}$$

8. $$\begin{array}{r} 45m + 3n + p \\ 5m + 2n + p \\ \hline \end{array}$$

9. $$\begin{array}{r} 6m + 4n + p \\ 4m + 6n + 9p \\ \hline \end{array}$$

Subtract and test:

10. $$\begin{array}{r} 6a + 10b \\ 2a + 5b \\ \hline \end{array}$$

11. $$\begin{array}{r} 5a + 6b \\ a + 3b \\ \hline \end{array}$$

12. $$\begin{array}{r} 10a + 5b \\ 9a + 5b \\ \hline \end{array}$$

13. $$\begin{array}{r} 17a + 9b \\ 17a + b \\ \hline \end{array}$$

14. $$\begin{array}{r} 2a + 5b + 4c \\ a + b + c \\ \hline \end{array}$$

15. $$\begin{array}{r} 2a + b + 7c \\ a + b + 5c \\ \hline \end{array}$$

16. $$\begin{array}{r} 10a + 6b + 5c \\ 5a + b + 5c \\ \hline \end{array}$$

17. $$\begin{array}{r} 25 + n + 7p \\ 5 + n + p \\ \hline \end{array}$$

18. $$\begin{array}{r} 3x + 7y + 11z \\ 2x + y + 4z \\ \hline \end{array}$$

19. $$\begin{array}{r} 45m + 3n + 12p \\ 5m + n + 2p \\ \hline \end{array}$$

20. $$\begin{array}{r} 16a + 2b + 2c \\ 12a + b \qquad \\ \hline \end{array}$$

21. $$\begin{array}{r} 17m + 6n + 2p \\ 9m + n + 2p \\ \hline \end{array}$$

Solve:

22. A fruit grower picked $2a$ bu. of apples, $3b$ bu. of peaches, and $4c$ bu. of plums in one season. Indicate the sum of these numbers to show how many bushels of fruit he picked.

23. The same man picked $4a$ bu. of apples, b bu. of peaches, and $3c$ bu. of plums the next season. How many bushels of fruit did he pick? How many bushels did he pick in the two seasons?

24. For a house a builder used of three kinds of lumber $50a$ ft., $10b$ ft., $12c$ ft. respectively; for another house he used of the same kinds $75a$ ft., $15b$ ft., and $10c$ ft. How many feet did he use in all?

25. A grain dealer had $8m$ bu. of oats, $10n$ bu. of wheat, and $7p$ bu. of rye, and sold $6m$ bu. of oats, $5n$ bu. of wheat, and $7p$ bu. of rye. How many bushels of grain had he left?

Multiplication

21. The product of two or more letters is indicated by placing them next to each other without a sign of equation.

Thus, the product of a, b, and c, or $a \times b \times c$, is written abc.

22. The number written above and at the right of a number to show how many times it is taken as a factor is called an **exponent.**

Thus, in c^2, the ^{2} shows that c^2 means $c \times c$ or cc; also in 3^2y^3 the ^{2} and ^{3} show that $3^2y^3 = 3 \times 3 \times y \times y \times y$.

WRITTEN EXERCISES

Write the equivalent forms by using exponents:

1. 9 (or 3×3).
2. 8 (or $2 \times 2 \times 2$).
3. 81.
4. $8aa$.
5. 25.
6. 16.
7. bb.
8. $16bbb$.
9. 49.
10. $sssss$.
11. ccc.
12. 27 (or $3 \times 3 \times 3$).
13. 100.
14. $aayyy$.
15. $100aabb$.

Indicate by use of exponents the number of:

16. Days in 7 weeks.
17. Things in a gross.
18. Months in 12 years.
19. Seconds in an hour.
20. Cubic inches in a cubic foot.
21. Years in 100 centuries.

Multiply:

22. $\begin{array}{r} 4a \\ \underline{3a} \end{array}$

23. $\begin{array}{r} 5m \\ \underline{8m} \end{array}$

24. $\begin{array}{r} 5b \\ \underline{2b} \end{array}$

25. $\begin{array}{r} 6b^2 \\ \underline{3b^2} \end{array}$

26. $\begin{array}{r} 8c^2 \\ \underline{3c} \end{array}$

27. $\begin{array}{r} 4x \\ \underline{2x^2} \end{array}$

28. $\begin{array}{r} 4x^3 \\ \underline{2x^2} \end{array}$

29. $\begin{array}{r} 40s \\ \underline{3s^2} \end{array}$

23. The following examples illustrate the process of multiplying literal expressions of more than one term.

$$\begin{array}{lr} a + 7b & 8 \\ \underline{a + b} & \underline{2} \\ a^2 + 7ab & \\ \underline{\phantom{a^2 + {}} ab + 7b^2} & \\ a^2 + 8ab + 7b^2 & 16 \end{array} \qquad\qquad \begin{array}{lr} a + 2 & 3 \\ \underline{a + b} & \underline{2} \\ a^2 + 2a & \\ \underline{ + ab + 2b} & \\ a^2 + 2a + ab + 2b & 6 \end{array}$$

We multiply the multiplicand by each term of the multiplier, place like terms in column, and add.

The numbers at the right are the values of the corresponding expressions when $a = 1$, $b = 1$. These serve to check the work. Thus, in the first example, $2 \times 8 = 16$; and in the second, $2 \times 3 = 6$.

WRITTEN EXERCISES

Multiply and test the work by giving values to the letters:

1. $\begin{array}{r} 3a + x \\ \underline{a + b} \end{array}$

2. $\begin{array}{r} x + 1 \\ \underline{x + 1} \end{array}$

3. $\begin{array}{r} s + 6 \\ \underline{s + 3} \end{array}$

4. $\begin{array}{r} 4x + 5 \\ \underline{a + x} \end{array}$

5. $\begin{array}{r} n + 2 \\ \underline{n + 2} \end{array}$

6. $\begin{array}{r} 1 + m \\ \underline{2 + m^2} \end{array}$

7. $\begin{array}{r} 2 + a \\ \underline{b + x} \end{array}$

8. $\begin{array}{r} z + 5 \\ \underline{z + 5} \end{array}$

9. $\begin{array}{r} a + x \\ \underline{b + y} \end{array}$

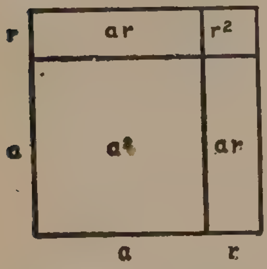
10. $\begin{array}{r} y + 3 \\ \underline{3y + 2} \end{array}$

11. $\begin{array}{r} x + y \\ \underline{x + y} \end{array}$

12. $\begin{array}{r} x^2 + 2 \\ \underline{x + 1} \end{array}$

Solve:

13. Point out in the figure that the square on $a + r$ is the square on a plus the square on r plus twice the rectangle of a and r.



14. Multiply $a + r$ by $a + r$.

15. Replace a by 30, and r by 2, throughout the work of Exercise 14. What number have you squared?

16. What is the result if $a = 40$, $r = 3$? What is the square of 43?

24. The result of Exercise 14 says:

The square of the sum of any two numbers is the square of the first plus twice the product of the first and the second, plus the square of the second.

1. Square 17 by writing it $10 + 7$.

2. Find similarly: 24^2; 13^2; 41^2; 25^2; 42^2; 33^2.

Multiply:

3. $x + 10$ by $x + 20$.

4. $2a + 1$ by $2b + 3$.

5. $a + 1$ by $b + 1$.

6. $3x + y$ by $3y + x$.

Division

25. In division we have the product and one factor given to find the other factor.

Thus, in $40ax \div 20a$, $40ax$ is the product of $20a$ and another factor, which is evidently $2x$.

ORAL EXERCISES

State the quotients in the following cases of division:

	1.	2.	3.	4.	5.
Dividend:	$36x$	15	$15a$	$14b$	xy
Divisor:	9	5	5	7	y

Find the numbers to fill the blanks:

	6.	**7.**	**8.**	**9.**	**10.**
Dividend:	( )	( )	$21ay$	$42t$	$18ab$
Divisor:	$3a$	$9y$	( )	$7t$	$9a$
Quotient:	$4b$	$2x$	$7a$	( )	( )

26. The following example illustrates the process of dividing literal expressions of more than one term.

Example: Divide $a^2 + 3ab + 2b^2$ by $a + b$.

$$\begin{array}{r|l} & a \quad + 2b \\ \hline a + b & a^2 + 3ab + 2b^2 \\ & \underline{a^2 + \ \ ab} \\ & \qquad 2ab + 2b^2 \\ & \qquad \underline{2ab + 2b^2} \end{array}$$

Divide the first term of the dividend by the first term of the divisor; the quotient is a. Multiply the entire divisor by a and subtract, as in the case of numbers. Divide the first term of the remainder by the first term of the divisor again; the quotient is $2b$. Multiply and subtract, as in the case of numbers.

Test: Compare the product of the divisor and quotient with the dividend.

WRITTEN EXERCISES

Divide and test:

1. $c^2 + 2cd + d^2$ by $c + d$.
2. $mn + np + nq$ by n.
3. $mp + p^2 + pq$ by p.
4. $4a^2 + 4ab + b^2$ by $2a + b$.
5. $a^2 + 4ab + 4b^2$ by $a + 2b$.
6. $3a^2 + 7ab + 2b^2$ by $3a + b$.
7. $x^2 + 5xy + 6y^2$ by $x + 2y$.
8. $m^2 + 3mn + 2n^2$ by $m + n$.

Factors and Roots

27. A product may be given, and it may be required to find the factors; that is, a set of numbers whose product is the given number. This is called **factoring** the given expression. It may usually be done in various ways.

Thus, $28 = 4 \times 7$, or 2×14, or $2 \times 2 \times 7$, or 28×1.

Also, $4ax = 4 \times a \times x$, or $4a \times x$, or $a \times 4x$, or $2 \times 2a \times x$, or $4 \times ax$.

28. An **integral factor** is one that is not in the form of a fraction. By factor is meant integral factor, unless otherwise stated.

ORAL EXERCISES

Find two different sets of factors for each of the following:

1. $6a$.
2. $6abc$.
3. $14a^2$.
4. $15axy$.
5. $8a^2b$.
6. $48x$.
7. $7ay$.
8. $14br^2$.
9. $21a^3x^2$.

Regarding $3a$ *as one of two factors, state the other factor of each of the following expressions:*

10. $6a$.
11. $3ac$.
12. $15a^3$.
13. $9ab$.
14. $12az$.
15. $24ay^2$.
16. $30axy$.
17. $21a^2$.
18. $15ax^4$.

Regarding x *as one of two factors, state the other factor of each of the following expressions:*

19. $4x$.
20. $10ax$.
21. $2bx$.
22. mx.
23. x^2.
24. $10axy$.
25. $5x^2z$.
26. x^3.
27. $1x$.
28. $6a^2x$.
29. $7ax^2$.
30. $2rx^4$.

29. The symbol $\sqrt{}$ means "the square root of."

Thus, $\sqrt{196} = 14$.

Similarly, $400 = 2^4 \times 5^2$; hence $\sqrt{400}$ is $2^2 \times 5$, or 20.

Also, $169a^2b^4 = 13^2a^2b^4$; hence $\sqrt{169a^2b^4}$ is $13ab^2$.

ORAL EXERCISES

State the square root of:

1. 25.
2. 144.
3. $16a^4$.
4. $121m^4$.
5. a^2.
6. 81.
7. $49x^2$.
8. $225c^4$.
9. b^6.
10. a^2b^2.
11. $64b^2$.
12. $16a^4c^6$.

30. When there are so many factors in the numerical part that it is not easy to separate them into two equal groups by inspection the factors may first be written out in full, and then grouped.

Thus, $7056 = 4 \times 4 \times 3 \times 3 \times 7 \times 7$
$= (4 \times 3 \times 7)(4 \times 3 \times 7) = 84^2$.

Therefore, $\sqrt{7056} = 84$.

The work of finding the factors may be arranged thus:

$$\begin{array}{r} 4\underline{)7056} \\ 4\underline{)1764} \\ 3\underline{)441} \\ 3\underline{)147} \\ 7\underline{)49} \\ 7 \end{array}$$

WRITTEN EXERCISES

1. Find: $\sqrt{625}$; $\sqrt{11{,}025}$; $\sqrt{1024b^8}$.

2. Find: $\sqrt{256a^4b^4}$; $\sqrt{3025a^2}$; $\sqrt{169a^4b^6}$.

Find the square root of:

3. $625m^2p^4$.
4. $1{,}225a^6b^6$.
5. $256x^2y^2$.
6. $361p^8q^8$.
7. $144s^8t^6$.
8. $196m^6n^4$.
9. $4{,}225m^2n^{10}$.
10. $441x^2y^{12}$.

Order of Operations

31. When not otherwise indicated, multiplications and divisions are performed before additions and subtractions.

Thus, $3 + 7 \times 2$ is taken to mean $3 + 14$, or 17.

Similarly, $4 + 12 \div 2$ means $4 + 6$, or 10.

Also, $5 + 14 \div 7 - 3 = 5 + 2 - 3 = 7 - 3 = 4$.

ORAL EXERCISES

Perform the operations indicated:

1. $12 - 6 \div 3$.
2. $6 \times 4 - 3$.
3. $6 \times 5 - 12 + 7$.
4. $8 + 2 \times 7$.
5. $40r - 36r \div 2r$.
6. $14x \times 3x - 2x \times 10x$.
7. $30y + 2y \times y \div 3y$.
8. $30 + 5 \times 8$.
9. $2 + 8 \times 4 - 3$.
10. $4 \times 9 - 10 \div 5$.
11. $12p - 16p \div 4p$.
12. $52p \div 26p + 5p$.
13. $34y + 6y \div 2y - 17y$.
14. $40s + 2s \div 7s + 4s$.

32. To indicate that additions and subtractions are to be performed first, the parenthesis is used.

Thus, in $(3 + 7) \times 2$, the parenthesis shows that 3 and 7 are first to be added, then the result multiplied by 2. It indicates that what is within the parenthesis is to be treated as a single expression.

Similarly, $(4 + 12) \div 2$ means $16 \div 2 = 8$.

Perform the operations indicated:

1. $(12a - 6a) \div 3a$.
2. $(2c + 8c) \times (4c - 3c)$.
3. $(16 - 12) \div 4$.
4. $(40c - 36c) \div 2c$.
5. $8b \times (30b + 5b)$.
6. $6h \times (5h + 7h - 10h)$.
7. $(84x + 6x) \div (22x - 17x)$.
8. $(8m + 12m) \div (7m - 2m)$.

Equations

33. Two equal expressions connected by the sign of equality are called an **equation.**

Thus, $3x + 10 = 40$. $ax + b = c$ are equations.

The expressions connected by the equality sign are called **the members** of the equation.

34. If the same number is added to each member of an equation or subtracted from each member, the result is an equation equal to the first.

Similarly, if each member of an equation is multiplied or divided by the same number, the result is an equation.

Thus, $3x + 10 = 40$ becomes $3x = 30$ by subtracting 10 from each member.

Also, $3x = 30$ becomes $x = 10$ by dividing each member by 3.

ORAL EXERCISES

1. In $2x + 5 = 25$, subtract 5 from both members. What is the result?

2. In $2x = 30$, divide both members by 2. What is the value of x?

3. Find x in $2x + 5 = 25$. Also in $3x + 10 = 40$.

Find the value of the letter in each equation.

4. $5x + 5 = 25$.

5. $8x + 1 = 17$.

6. $2y + 12 = 24$.

7. $6 + x = 15$.

8. $20 = 4 + x$.

9. $3z + 2 = 14$.

35. The equation is used in solving problems.

EXAMPLE: Three times a number and 2 more is 14. Find the number.

Solution: Let x = the number.

Then, $3x + 2 = 14$. $x = \frac{14 - 2}{3} = 4$.

WRITTEN EXERCISES

1. Five times a number less 6 is 29. What is the number?

2. A house and lot, sold at a gain of 10%, brought $2,200. What was its former cost?

Plan: (1) Let x be the cost of the property.
(2) Then 110% of x, or $1.10x$, is the selling price.
(3) Therefore, $1.10x = \$2,200$. (4) Therefore, $x = \$(\quad)$.

3. A hardware dealer sold a furnace for $180 at a gain of 5%. What did the furnace cost him?

4. A merchant sold a damaged carpet for $42.50 at a loss of 15%. What did the carpet cost him?

5. A collector remitted $475 after deducting a fee of 5%. How many dollars did he collect?

6. The amount of a certain principal at 4% simple interest for 1 yr. was $416. What was the principal?

7. A sum of money less 20% is $960. What is the sum of money?

8. $\frac{2}{3}$ of the height of a staff plus 5 ft. is 25 ft. What is the height of the staff?

9. A commission merchant remitted $1,900 after deducting a commission of 5%. On how much did he reckon his commission?

10. A man's salary was increased $\frac{1}{5}$ of itself; he then received $1,800. What was his salary before the increase?

11. George thought of a number, doubled it, added 15, divided by 5 and obtained 5. What was the number?

12. A capital is invested thus: $\frac{1}{2}$ at $5\frac{1}{2}\%$, $\frac{1}{3}$ at 6%, and the rest at $4\frac{1}{2}\%$; the annual income is $605. What is the capital?

Plan: (1) Let x be the capital in dollars.
(2) Then, the parts invested are $\frac{1}{2}x$, $\frac{1}{3}x$, $\frac{1}{6}x$.
(3) $.055 \times \frac{1}{2}x + .06 \times \frac{1}{3}x + .045 \times \frac{1}{6}x = \$(\quad)$, the income.
(4) Therefore, $.0275x + .02x + .0075x = (\quad)\,x = \$605$.
(5) Therefore, $x = \$(\quad)$.

13. By investing $\frac{1}{4}$ of his capital at 3% and the rest at 6%, Mr. White received $10 a year more than by investing the whole at 5%. What was his capital?

14. A man sold some of his land at $400 an acre. He invested the proceeds at 5% and found his daily (365 da. = 1 yr.) income from the investment to be $1.50. How many acres did he sell?

15. The price of wheat was $78\frac{1}{2}$ cts. on Wednesday. It rose a certain amount on Thursday and half as much on Friday, when the price was 80 cts. What was the price on Thursday?

16. A merchant bought 2 canisters of coffee, 50 lb. each, for $23.25. One kind cost $4\frac{1}{2}$ cts. a pound more than the other. Find the price per pound of each.

17. Indicate by use of parentheses, that if twice the cost (c) of a lot be diminished by $100 and the remainder multiplied by 3, the result will be the cost (h) of the house.

18. Goods are sold for $1,620 at a loss of 10%. What did they cost?

REVIEW

Regarding 2y *as one of two factors, state the other factor in each of the following expressions:*

1. $4ay$. **2.** $20y$. **3.** $2y^2$. **4.** $30xy$. **5.** $6ay^3$.

Perform the operations indicated:

6. $85a + 15a \div 5a$.

7. $6z \times (8z + 3z)$.

8. $(85x + 15x) \div 5x$.

9. $8a + 12b \div 4$.

10. $6y \times 8y + 3y$.

11. $(8a + 12b) \div 4$.

Add:

12. $\begin{array}{r} 4a + 2 \\ \underline{7a + 9} \end{array}$ **13.** $\begin{array}{r} 2x + 6y \\ \underline{3x + 12y} \end{array}$ **14.** $\begin{array}{r} a + 5b \\ \underline{a + 12b} \end{array}$ **15.** $\begin{array}{r} 1 + a \\ \underline{2 + 5a} \end{array}$

Subtract:

16. $\begin{array}{r} 12m + 16 \\ \underline{3m + 1} \end{array}$ **17.** $\begin{array}{r} 1 + 12q \\ \underline{1 + 4q} \end{array}$ **18.** $\begin{array}{r} 6x + 27q \\ \underline{5x + 18q} \end{array}$ **19.** $\begin{array}{r} a + 9b + 5c \\ \underline{a + 2b } \end{array}$

Indicate by use of exponents:

20. The area of a square $5a$ ft. on a side.

21. The number of cubic feet in a cube $3x$ ft. on an edge.

22. The price of 16 lb. of fine wire at $16a$ cts. an ounce.

23. The number of inches in $36x$ yd.

Multiply:

24. $\begin{array}{r} 2x + 1 \\ \underline{3x + 4} \end{array}$ **25.** $\begin{array}{r} 5a + b \\ \underline{6a + 3b} \end{array}$ **26.** $\begin{array}{r} a + m \\ \underline{a + m} \end{array}$ **27.** $\begin{array}{r} 2a - c \\ \underline{a - c} \end{array}$

Find the square root by factoring:

28. $3136x^2$. **29.** $28224m^2$. **30.** $9216a^2$. **31.** a^4x^8.

Solve:

32. $2x + 8 = 20$.

33. $3x + 3 = 18$.

34. $x + 17 = 19$.

35. $4x + 1 = 17$.

36. $x + 9 = 29$.

37. $5x + 6 = 31$.

CUBE ROOT

36. The **cube root** of a given number is a number whose cube is the given number. If the number can be factored into 3 sets of equal factors, the cube root can be found by factoring.

EXAMPLE: Since

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3.$$
$$= (2 \times 3) \times (2 \times 3) \times (2 \times 3).$$
$$\sqrt[3]{216} = 2 \times 3 = 6.$$

WRITTEN EXERCISES

Find by factoring the cube roots of:

1. 512.
2. 1,728.
3. 3,375.
4. 13,824.
5. 157,464.
6. 262,144.
7. 592,704.
8. 1,157,625.

37. The following reasoning shows how to find the number of places in the integral part of the cube root of any number:

Since $10^3 = 1000$, if a number is smaller than 1000 its cube root is smaller than 10, and if the number is greater than 1000 its cube root is greater than 10. Likewise, since $100^3 = 1{,}000{,}000$, if a number is smaller than 1,000,000 its cube root is smaller than 100, and so on.

Hence if a number is pointed off from the decimal point into periods of 3 digits each (possibly fewer in the left period) then each period to the left of the decimal point corresponds to a digit of the integral part of the root.

By trial, the largest integer whose cube is less than the left period can be found. This will be the first figure of the root.

EXAMPLE: Find the cube root of 283′ 627.

The left period is 283. By trial we find $6^3 = 216$, and $7^3 = 343$. Hence the first figure of the result is 6. Since there are two periods, this is the number of tens in the root.

WRITTEN EXERCISES

Find the first figure of the cube root of:

1. 28,431. **2.** 2,841,869. **3.** 97,821.4863. **4.** 489,763.834.

38. The calculation by which the cube root of any number may be computed is shown by the following examples:

Extract the cube root of 79,507.

(A)

	Root	4 3
	Number	79'507
		64
		15 507
Trial divisor:		
$300 \times 4^2 =$	4800	
Corrections:		
$30 \times 4 \times 3 =$	360	
$3^2 =$	9	
Complete divisor:	5169	15 507
		0

1. *Find by inspection the largest integer whose cube does not exceed the left period.*

(In Example A this is 4.)

2. *Subtract the cube of this number from the left period, and to the remainder annex the next period.*

3. *Use 300 times the square of the part of the root already found as a trial divisor.*

(In Example A the trial divisor is 4800.)

4. *Use the first figure of the quotient* (3 in Example A) *as the next digit of the root. Call this the quotient.*

5. *Correct the trial divisor by adding to it:* (1) *30 times the product of the part of the root already found and the quotient;* (2) *the square of the quotient. The sum is the complete divisor.*

(In Example A, the corrections are 360 and 9, and the complete divisor is 5169.)

6. *Multiply the complete divisor by the quotient, and subtract the product from the result of step 2. If the remainder is not zero, annex the next period to it, and repeat steps 3 to 5.*

The process is continued until the remainder zero, or until the desired degree of accuracy, is reached.

If the product of the complete divisor and the quotient is larger than the number from which it is to be subtracted, try the next smaller integer as quotient.

(B)

Root 1 2 . 7 +

Number 2′070′.000

Trial divisor, $300 \times 1^2 = 300$

Corrections, $30 \times 1 \times 2 = 60$ — 1

$2^2 = \underline{4}$ — 1 070

Complete divisor, 364 $2 \times 364 =$ 728

342 000

Trial divisor, $300 \times \overline{12}^2 = 43200$

Corrections, $30 \times 12 \times 7 = 2520$

$7^2 = \underline{49}$

Complete divisor, 45769 $7 \times 45769 =$ 320 383

21 617

In Problem B, the first trial divisor, 300, gives the quotient 3; the corrections are 90 and 9, or 99; the complete divisor is 399; and the product of 399 and 3 is 1,197, which is larger than 1,070. Hence 2 is taken as the quotient.

WRITTEN EXERCISES

Find at least three figures of the cube roots of the following, unless the root comes out exact with fewer figures:

1. 4,096.

2. 12,167.

3. 132,651.

4. 13,106.

5. 185,193.

6. 279,609.

7. 2,924.207.

8. 13,343.2831.

9. 6,206,504.77.

10. A cubical reservoir is to be built to store 10,000,000 cu. ft. of water. Find the length of an edge of the reservoir, to the nearest foot.

11. A certain cubical block of stone weighs 8,562 lb. If one cu. in. of the stone weighs 6 oz., what is the length of an edge of the block to the nearest .1 inch?

12. The cubical base for a monument is built of brick 2 × 4 × 8 in.; 592,704 bricks are used. Find an edge of the base in feet.

13. At $4.05 per cu. yd. it cost $7,597.95 to prepare a cubical excavation for the foundation of an office building. Find the dimension of the excavation in feet.

TABLES OF DENOMINATE NUMBERS

Linear Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
16.5 feet or 5.5 yards = 1 rod (rd.)
320 rods = 1 mile (mi.)
1,760 yards = 1 mile
5,280 feet = 1 mile
6 feet = 1 fathom

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
231 cu. in. = 1 gal.
31½ gal. = 1 bbl.

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)
32 quarts = 1 bu.
2,150.4 cu. in. = 1 bu.

United States Money

10 cents (¢) = 1 dime
10 dimes = 1 dollar
100 cents = 1 dollar ($)
10 dollars = 1 eagle
20 dollars = 1 double eagle

Measure of Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days or 12 months (mo.) = 1 year (yr.)
10 years = 1 decade
10 decades = 1 century

Table of Counting

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross (gt. gro.)
24 sheets of paper = 1 quire
20 quires, or 480 sheets, = 1 ream

Avoirdupois Weight

7,000 grains (gr.) = 1 pound (lb.)
16 ounces (oz.) = 1 pound
100 pounds = 1 hundredweight
2,000 pounds = 1 ton (T.)
2,240 pounds = 1 gross ton

Angular Measure

60 seconds (″) = 1 minute (′)
60′ = 1 degree (°)
90° = 1 right angle (rt. ∠)
180° = 1 straight angle (st. ∠)
360° = 4 right angles

Surveyors' Measures of Length

7.92 inches = 1 link (li.)
100 links = 1 chain (ch.)
80 chains = 1 mile

The public lands of the United States are surveyed by the chain. Surveyors and engineers now generally use a steel tape graduated in feet and inches or in feet and tenths of a foot.

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet (sq. ft.) = 1 square yard (sq. yd.)
30¼ sq. yd. = 1 square rod (sq. rd.)
160 sq. rd. = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
1 sq. mi. = a section
36 sq. mi. = a township
100 sq. ft. = a square (of roof, etc.)

Cubic Measure

1,728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet (cu. ft.) = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord
1 cubic yard = 1 load (of earth, etc.)
24¾ cubic feet = 1 perch (stone)

Apothecaries' Weight

20 grains (gr.) = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb.)
5,760 grains = 1 pound

The table of apothecaries' weight is used in selling drugs at retail; but the metric system is taking its place with pharmacists.

Troy Weight

24 grains (gr.) = 1 pennyweight (pwt. or dwt.)
20 pennyweights = 1 troy ounce
12 troy ounces = 1 troy pound
480 grains = 1 troy ounce
5,760 grains = 1 troy pound

Troy weight is used in weighing precious metals.

For the metric system and moneys of foreign countries see the Supplement, pp. 417–434.

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  Measure of, 122.
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(12)

ANSWERS TO
THE APPLETON ARITHMETICS
GRAMMAR-SCHOOL BOOK

Page 2

1. 265,125.
2. 520,075.
3. 801,502.
4. 3,500,009.
5. 628.35.
6. 2,400.24.
7. LXXVI.
8. M.
9. MCMVIII.
10. XLIII.
11. D.
12. MDCCLXXVI.
13. XVII.
14. CXXV.
15. MCMX.
16. XXV.
17. C.
18. MDCXX.
19. MDCCCLIV.
20. MCCLXV.
21. MCDXCII.

Page 4

1. 3.
3. 225.
4. 168 hr.
5. 364 da.
6. 1,088.
7. 23,577.
8. 38,400.
9. 1,845.
10. 27,735.
11. 21,900.
12. 4,970.
13. 17,700.
14. 72,611.
15. 40,710.
16. 174,794.
17. 161,893.
18. $322.
19. $506.97.
20. $22,112.24.

Page 6

1. 81.
2. 110.
3. $6.90.
4. $20\frac{10}{13}$.
5. $35\frac{4}{11}$.
6. 91.
7. 11.
8. $9.14.
9. $21\frac{2}{17}$.
10. $135\frac{11}{12}$.
11. 69.
12. 401.
13. $6.01.
14. $22\frac{1}{21}$.
15. $70\frac{8}{25}$.
16. 56.
17. 150.
18. $1.50.
19. $201\frac{6}{7}$.
20. $200\frac{1}{2}$.
21. 71.
22. 2,457.
23. 144.
24. 812.
25. 151.
26. 200.
27. 1,331.
28. 2,814.

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Page 7

1. $1\frac{1}{12}$.
2. $1\frac{1}{2}$.
3. $1\frac{7}{15}$.
4. $1\frac{5}{12}$.
5. $\frac{9}{10}$.
6. $1\frac{1}{8}$.
7. $1\frac{7}{12}$.
8. $\frac{14}{15}$.
9. $\frac{1}{8}$.
10. $\frac{7}{30}$.
11. $\frac{11}{24}$.
12. $\frac{1}{12}$.
13. $\frac{3}{10}$.
14. $\frac{1}{8}$.
15. $\frac{1}{15}$.
16. $\frac{1}{30}$.
17. $\frac{5}{6}$ of a day.
18. $1\frac{7}{24}$ hr.
19. $\frac{11}{30}$.
20. $\frac{1}{6}$.
21. $\frac{2}{9}$.
22. 2.
23. $6\frac{3}{4}$.
24. $1\frac{1}{2}$.
25. $5\frac{2}{5}$.
26. $1\frac{2}{3}$.
27. $7\frac{1}{9}$.
28. $2\frac{5}{8}$.
29. $1\frac{5}{7}$.
30. $6\frac{2}{5}$.
31. 4.
32. $2\frac{4}{7}$.
33. $4\frac{4}{9}$.
34. 8 hr.
35. 20.
36. $\frac{1}{5}$.
37. $\frac{3}{7}$
38. $\frac{3}{10}$.
39. $\frac{5}{24}$.
40. $\frac{2}{9}$.
41. $\frac{1}{8}$.
42. $\frac{2}{15}$.
43. $\frac{2}{11}$.

Page 8

44. 15.97.
45. 588.5
46. 160.19.
47. 131.60.
48. 34.1.
49. 30.85.
50. 127.4.
51. 1.25.
52. 162.
53. 42.84.
54. 439.68.
55. 625.67.
56. .15.
57. .20.
58. 4.1.
59. 9.51.
60. .5; .25; .5; .75; .2; .6; .8; .4.
61. 215.4 mi.
62. 10.5 A.
63. 57.2 yd.

Page 9

9. 19 ft. 8 in.
10. 13 rd. 8.5 ft.
11. 12 gal. 1 qt.
12. 38 min. 8 sec.
13. 16 bu. 2 pk.
14. 50 wk. 4 da.
15. 23 lb. 2 oz.
16. 2 gal. 3 qt.
17. 24 wk. 4 da.

Page 10

2. \$2.25; \$3.75; \$1.28; \$10.80; total, \$18.08.
3. 4 yd.
4. $\frac{1}{4}$; $\frac{1}{2}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{4}$.
5. \$12.75.

1. 10 bales; 100 bales.
2. 50,000; \$50.
3. \$1,875.

Page 11

4. \$70.
5. \$100.
6. \$150.
8. 140 yd.
9. 10 ft.; 20 ft.; 120 ft.; 140 ft.
10. $\frac{1}{2}$ in.; $\frac{1}{4}$ in.; $\frac{3}{4}$ in.; $1\frac{1}{2}$ in.; $1\frac{7}{8}$ in.
11. 168 hr.
12. 480 min.
13. 240 sec.; $\frac{1}{15}$ hr.
14. $\frac{7}{24}$ of a year.
15. \$2.40.
16. \$11.25.
17. 48.
18. 140.
19. \$38,400.

Page 12

20. 40 ft.; 75 ft.; 230 ft.
21. 30 ft.; 65 ft.; 5 ft.
22. 3,000 sq. ft.
27. \$2.34.
28. \$1.52.

Page 13

29. 25 lb.
30. 200; 800.
31. 1,600.
32. \$64.
33. \$19.65.
34. \$495.
35. \$288.
38. \$694.92.

Page 14

39. 300 lb.
40. $2\frac{1}{4}$ T.
41. 100 lb. nitrate of soda ; 200 lb. phosphate ; 200 lb. ground bone ; 200 lb. potash.
42. 700 lb.
43. 18 lb. ; $\frac{1}{25}$.
44. 9,000 lb.; 1,800 lb.
45. 30 T.
46. $666\frac{2}{3}$.
47. $62\frac{1}{2}$ lb.
48. 3,750 lb.
49. 125,000 lb.

Page 16

1. 8,000,000.
2. 85,000,000.
3. 4,275,000.
4. 62,308,279.
5. 5,698,010.
6. 2,031,107.
7. 20,400,002.

Page 19

1. 97,669,189.
2. 371,570 sq. mi. ; 27,130,814.
3. 472,240 sq. mi. ; 9,950,000.
4. 426,765 sq. mi. ; 35,332,382.
5. 323,570 sq. mi. ; 3,450,000.
6. 553,325 sq. mi. ; 26,022,057.

Page 20

1. $505.08.
2. $1,692.60.
3. $1,720.55.
4. $1,695.24

Page 21

1. $274.91.
2. $259.44.
3. $372.15.
4. $672.78.
5. 61,056.
6. 23,648.
7. 21,258.
8. 38,781.

Page 22

9. $585.55.
10. $155.75.
11. $303.81.
12. $301.91.
13. 31,172.
14. 538,455.
15. 76,051.
16. 37,586.
17. $324,500.
18. $114,000.
19. 25,439.
20. 216,610 sq. mi.
21. 325,104 sq. mi.

Page 24

1. $1.35 ; $2.55 ; $3.16 ; $1.29 ; $2.05 ; $3.20.

	Dollars	Cents	Cents		Dollars	Cents	Cents
2.	2	30	5	**8.**	8	50	3
3.	3	20	3	**9.**		10	7
4.	5	70	5			70	5
5.	1	10	8		1		5
6.	3	20	9		2	20	
7.	9	90	9				8

10. $4.50.
11. $3.25.
12. $43.60.
13. $.45.
14. $49.95.

Page 26

1. 11,610.
2. 22,127.
3. 43,152.
4. 4,350.
5. 47,775.

Page 27

6. 213,161.
7. 1,080,864.
8. 3,005,415.
9. 4,732,910.
10. 963,248.
11. 432,208.
12. 447,692.
13. 3,589,821.
14. 5,535 T.
15. 26,875 yd.
16. 4,290 mi.
17. 661,920 lb.

Page 28

1. 340 ; 300 ; 180 ; 240 ; 960.
2. 600 ; 680 ; 800 ; 3,440 ; 2,160.
3. 900 ; 1,500 ; 1,700 ; 4,800 ; 12,500.
4. 7,500 ; 4,500 ; 2,700 ; 6,000 ; 12,000.
5. 340 T. ; 500 T. ; 260 T. ; 560 T.
6. 760 bu.; 960 bu.; 1,320 bu.; 2,600 bu.
7. 2,700 ft. ; 4,500 ft. ; 8,700 ft.

Page 29

1. $70,665.
2. 36,846 gal.
3. 311,304 qt.
4. 24,000 min.
5. $175,200.
6. 17,280 in.
7. 378,840 lb.
8. 626,200 lb.
9. 187,200 ft.
10. 61,908 yd.
11. 777,777 hr.
12. 264,600 T.

1. $74.40.
2. $450.
3. $611.80.
4. $366.25.
5. $755.37.
6. $671.40.
7. $4,136.48.
8. $271.13.
9. $6,990.50.
10. $16.66.
11. $9.60.
12. $172.22.
13. $631.80.
14. $940.80.
15. $4,410.

Page 31

1. 7^2.
2. 12^2.
3. 3^2.
4. 12^2.
5. 10^2.

1. 10,350 sq. ft.
2. 113,400 cu. ft.
3. $1,750.
4. $840.
5. $350.
6. $1,837.50.
7. 4,200.
8. $4,200.

Page 32

4. 23.
5. 43.
6. 42.
7 41.
8. 81.
9. 61.
10. 71.
11. 91.
12. 81.
13. 202.
14. 10.
15. 81.
16. 101.
17. 61.
18. 50.
19. 20.

Page 33

1. 39.
2. 72.
3. $41\frac{1}{5}$.
4. 16.
5. 89.
6. 64.
7. 92.
8. $136\frac{2}{3}$.
9. 322.
10. 508.
11. $18 ; $72.
12. 86 ft. ; 60 mi.
13. 9 sq. in. ; 144 cu. in.
14. 12 wk.
15. 150.
16. 72 rows, 2 plants.

Page 34

2. $12\frac{2}{3}$ yd.
3. $10\frac{1}{2}$ bu.
4. $31\frac{1}{4}$ gal.
5. $52\frac{1}{7}$ wk.
6. $21\frac{1}{3}$ ft.
7. 20 lb.

Page 37

1. $4\frac{3}{5}$; $12\frac{9}{10}$; $38\frac{1}{2}$; $62\frac{1}{10}$; $128\frac{3}{5}$; $195\frac{1}{2}$.
2. $29\frac{9}{25}$; $9\frac{9}{20}$; $1\frac{43}{50}$; $6\frac{1}{4}$; $10\frac{3}{50}$; $46\frac{9}{100}$.
3. \$ $65\frac{4}{5}$.

Page 38

1. 38.
2. 65.
3. $68\frac{16}{19}$.
4. $626\frac{5}{46}$.
5. 377.
6. $40\frac{16}{33}$.
7. 300.
8. $173\frac{4}{7}$.
9. $299\frac{211}{223}$.
10. 729.
11. $2240\frac{74}{137}$.
12. $2235\frac{349}{365}$.
13. $981\frac{30}{101}$.
14. $2209\frac{103}{104}$.
15. $1013\frac{27}{133}$.
16. 128.
17. 192 A.
18. $2,405.
19. $\frac{43}{80}$.
20. 3,982.
21. 189,900 mi.
22. $1,041\frac{2}{3}$ mi.; $17\frac{13}{36}$ mi.
23. 139 gro. T.
24. 110 mi.

Page 39

1. $15.04.
2. $6.25.
3. \$ $3.50\frac{1}{6}$.
4. \$ $7.18\frac{3}{23}$.
5. $3.77.
6. $13.52.
7. $13.05.
8. $.38.
9. $8.14.
10. $30.11.
11. $27.01.
12. $3.10.
13. $55.75.
14. $126.50.
15. $43.27.
16. $1.40.

Page 41

1. $31.
2. $108.
3. $342.78.
4. $37.50.
5. $345.70.
6. $91.34.
7. $548.15.

Page 42

1. 18¢; $43.93.

Page 43

2. 3¢; 3¢; 5¢.
   5¢; 8¢; 10¢.
3. $60.15; $60.40.
4. $83.55.
5. 10¢.
6. 10¢.
7. $53.35; $53.55.
8. $1.50.
9. 12¢.
10. 12¢.
11. 10¢.
12. 8¢.

Page 45

1. $6.15.
2. $3.95.
3. $11.75.
4. $3.10.
5. $12.95.
6. $20.35.
7. $4.80.
8. $11.95.
9. $3.30.
10. $6.90.
11. $4.15.
12. $17.
13. $1.90.
14. $12.10.
15. $7.45.
16. $10.55.

Page 46

1. 2¢; 6¢; 10¢; 4¢; 4¢; 6¢; 14¢.
2. $42.19.
3. 2¢.
4. $1,035.16.
5. 12 oz.
6. 3 oz., or less than 3 oz., and over 2 oz.
7. 36¢.
8. $2.88; $10.
9. $25.

Page 48

1. 2, 3, 5, 7.
2. 2, 2, 2, 2, 2, 2, 5, 7.
3. 3,809.
4. 2, 5, 7, 7, 11.
5. 2, 2, 2, 2, 2, 3, 3.
6. 5, 5, 5, 13.
7. 3, 3, 3, 5, 11.
8. 3, 3, 3, 3, 3, 3, 5.
9. 5, 5, 5, 5.
10. 2, 3, 3, 11, 11.
11. 2, 2, 2, 5, 5, 5.
12. 3, 7, 101.
13. 2, 3, 197.
14. 2, 2, 2, 2, 5, 7, 11.
15. 2, 5, 5, 5, 5, 5.

Page 50

7. 3, or 9.
8. 3, 5, or 15.
9. 2, 3, 4, 6, or 12.
10. 11.
11. 2, 4, 8.
12. 7.
13. 5.
14. 13.
15. $\frac{2}{4}$; $\frac{4}{6}$; $\frac{6}{8}$; $\frac{5}{10}$; $\frac{4}{100}$; $\frac{25}{100}$.

1. 9.
2. 11.
3. 6.
4. 64.
5. 5.
6. 12.
7. 25.
8. 7.
9. 144.

Page 51

10. 6.
11. 28.
12. 11.
13. 1,024.
14. 21.
15. 6.
16. 250.
17. 8.

Page 52

1. 252.
2. 216.
3. 154.
4. 252.
5. 28.
6. 352.
7. 108.
8. 300.
9. 288.
10. 180.
11. $1.50.

Page 53

1. 240.
2. 150.
3. 594.
4. 192.
5. 180.
6. 441.
7. 150.
8. 847.

1. $\frac{7}{8}$.
2. 30.
3. 21.
4. 13,500.

Page 55

1. 3,270,065.

2. 25,090,120.

3. 87,087,049.

4. 20,040,140.

5. XIX; XXVI; XXXV; XXXIX; XLIII; LVI; XLIX; C; XCVII; LXIII; XLV; XC; LXXV.

6. 0, 1, 4, 9, 16, 25, 36, 49, 64, 81.

7. 12; 16.

8. 1,694; 175,812.

9. $\frac{77}{3240}$.

10. $2\frac{2}{3}$.

11. $2,531.40.

12. $322.

13. $27,673.

14. 42.

Page 56

15. $20.

16. $125.

17. 708 lb.

18. Neck, $0.72; chuck, $7.80; prime of rib, $9.52; porterhouse, $18.40; sirloin, $5.44; rump, $1.96; round, $14.88; plate, $4.48; flank, $.88; shin, $2.00; shank, $.60; total, $66.68.

19. Chuck; flank; porterhouse; porterhouse.

20. 74.3 lb.; $5.48.

21. Leg; rib; leg; leg.

Page 57

22. Rose, 20,196.40 lb.; Nora, 19,598.90 lb.

23. 3,569.60 lb.; 266.18 lb.

24. Rose, 4,925.65+ qt.; Nora, 3,373.65+ qt.

25. Rose, $246.28; Nora, $168.68. Rose, $90.37; Nora, $47.78.

26. 6 lb.; $1.20.

27. 75 lb.

28. $37\frac{1}{2}$ lb.; $12.

Page 58

29. $36,964,663.50.

30. $300,000.

31. 50,000.

32. $62.50.

33. $1,636.

34. $15,000,000.

35. 385.

36. 45,600.

37. 148,750.

38. 97,000.

39. 7,720.

Page 60

1. $\frac{25}{36}$. **2.** $\frac{17}{85}$. **3.** $\frac{30}{101}$. **4.** $\frac{300}{1000}$. **5.** $\frac{5}{500}$. **6.** $\frac{201}{301}$. **7.** $\frac{205}{3510}$. **8.** $59\frac{19}{91}$.

Page 62

1. $\frac{13}{5}$.

2. $\frac{29}{7}$.

3. $\frac{83}{8}$.

4. $\frac{13}{12}$.

5. $\frac{23}{3}$.

6. $\frac{84}{5}$.

7. $\frac{5}{4}$.

8. $\frac{29}{3}$.

9. $\frac{202}{5}$.

10. $\frac{27}{20}$.

11. $\frac{21}{16}$.

12. $\frac{19}{13}$.

13. $\frac{10}{3}$.

14. $\frac{58}{9}$.

15. $\frac{25}{2}$.

16. $\frac{23}{8}$.

17. $\frac{53}{9}$.

18. $\frac{73}{10}$.

19. $\frac{9}{5}$.

20. $\frac{8}{3}$.

21. $\frac{17}{2}$.

22. $\frac{51}{4}$.

23. $\frac{82}{5}$.

24. $\frac{56}{3}$.

25. $\frac{43}{8}$.

26. $\frac{49}{5}$.

27. $\frac{59}{3}$.

28. $\frac{25}{8}$.

29. $\frac{35}{17}$.

30. $\frac{81}{2}$.

Page 63

1. $\frac{8}{12}$, $\frac{8}{9}$, $\frac{8}{8}$, $\frac{8}{7}$, $\frac{8}{3}$.
2. $\frac{10}{13}$, $\frac{10}{11}$, $\frac{10}{10}$, $\frac{10}{4}$, $\frac{10}{2}$.
3. .7, $\frac{7}{9}$, $\frac{7}{5}$, $\frac{7}{3}$, $\frac{7}{2}$.
4. $\frac{9}{20}$, $\frac{9}{16}$, .9, $\frac{9}{5}$, $\frac{9}{4}$.
5. $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{8}{8}$, $\frac{11}{8}$.

6. England, Denmark, Germany, Venezuela.

7. Connecticut, Georgia, New Hampshire, Pennsylvania, Rhode Island, North Carolina, South Carolina, Massachusetts.

Page 64

8. New Hampshire, New York, Massachusetts.

Page 66

1. $\frac{2}{5}$.
2. $\frac{8}{27}$.
3. $\frac{1}{16}$.
4. $\frac{1}{13}$.
5. $\frac{1}{12}$.
6. $\frac{2}{13}$.
7. $\frac{1}{3}$.
8. $\frac{1}{3}$.
9. $\frac{4}{5}$.
10. $\frac{3}{43}$.
11. $\frac{8}{21}$.
12. $\frac{2}{9}$.
13. $\frac{1}{14}$.
14. $\frac{1}{8}$.
15. $\frac{9}{25}$.
16. $\frac{3}{8}$.
17. $\frac{7}{180}$.
18. $\frac{4}{13}$.
19. $\frac{1}{3}$.
20. $\frac{35}{41}$.
21. $\frac{1}{6}$.
22. $\frac{6}{23}$.
23. $\frac{1}{25}$.
24. $\frac{11}{13}$.
25. $\frac{3}{25}$.
26. $\frac{1}{5}$.
27. $\frac{5}{8}$.
28. $\frac{2}{3}$.
29. $\frac{7}{138}$.
30. $\frac{3}{20}$.
31. $\frac{1}{16}$.
32. $\frac{2}{7}$.
33. $\frac{4}{7}$.
34. $\frac{1}{11}$.
35. $\frac{1}{9}$.

Page 68

1. $\frac{20}{64}$, $\frac{4}{64}$.
2. $\frac{14}{42}$, $\frac{24}{42}$, $\frac{105}{42}$.
3. $\frac{48}{28}$, $\frac{28}{28}$, $\frac{7}{28}$.
4. $\frac{4}{16}$, $\frac{6}{16}$, $\frac{56}{16}$.
5. $\frac{10}{18}$, $\frac{36}{18}$, $\frac{33}{18}$.
6. $\frac{24}{40}$, $\frac{36}{40}$, $\frac{120}{40}$.
7. 2, 3, 5.
8. 32, 42, 28, 8, 18, 20.

Page 70

1. $1\frac{17}{24}$. 2. $\frac{7}{9}$. 3. $\frac{15}{16}$. 4. $1\frac{5}{18}$. 5. $\frac{29}{35}$. 6. $1\frac{23}{36}$. 7. $1\frac{1}{4}$. 8. $1\frac{5}{6}$.
9. $1\frac{17}{45}$. 10. $\frac{25}{36}$. 11. $1\frac{1}{60}$. 12. $1\frac{7}{16}$. 13. $1\frac{5}{6}$. 14. $1\frac{7}{10}$. 15. $1\frac{11}{48}$.
16. $1\frac{13}{24}$. 17. $1\frac{7}{20}$. 18. $1\frac{89}{90}$. 19. $2\frac{1}{2}$. 20. $2\frac{23}{40}$. 21. $1\frac{17}{36}$. 22. $2\frac{5}{8}$.
23. $2\frac{17}{20}$. 24. $1\frac{143}{168}$. 25. $2\frac{1}{8}$. 26. $1\frac{49}{90}$. 27. $2\frac{5}{36}$. 28. $1\frac{11}{12}$. 29. $\frac{3}{8}$.

Page 71

30. $1\frac{1}{2}$ mi. 31. $1\frac{8}{15}$ ft. 32. $\frac{41}{48}$ yd. 33. $\frac{11}{12}$. 34. $\frac{33}{40}$. 35. $\frac{21}{64}$. 36. $\frac{193}{216}$.

Page 72

1. $4\frac{3}{4}$.
2. $1\frac{11}{21}$.
3. $\frac{1}{25}$.
4. $\frac{1}{8}$.
5. $\frac{9}{16}$.
6. $\frac{5}{24}$.
7. $\frac{1}{10}$.
8. $\frac{5}{16}$.
9. $\frac{17}{25}$.
10. $\frac{31}{198}$.
11. $\frac{13}{32}$.
12. $\frac{4}{9}$.
13. $\frac{1}{30}$.
14. $\frac{5}{36}$.
15. $\frac{11}{20}$.
16. $1\frac{5}{8}$.
17. $\frac{5}{16}$.
18. $\frac{1}{36}$.
19. $\frac{5}{48}$.
20. $\frac{3}{10}$.
21. $\frac{19}{24}$.
22. $\frac{1}{18}$.
23. $\frac{7}{60}$.
24. $\frac{6}{25}$.
25. $\frac{3}{5}$.
26. $\frac{1}{8}$.
27. $\frac{101}{168}$.
28. $\frac{5}{8}$ yd.
29. $\frac{5}{16}$ gal.
30. $\frac{17}{60}$.
31. $\frac{4}{5}$.

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32. $\frac{23}{32}$ yd. 33. $\frac{10}{27}$. 34. $\frac{3}{8}$; $\frac{19}{24}$. 35. Frank, $\frac{1}{50}$ more. 36. Roy, $\frac{1}{32}$ bu. more.

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1. $4\frac{3}{5}$.
2. $2\frac{1}{4}$.
3. $1\frac{3}{8}$.
4. 1.
5. 2.
6. $2\frac{6}{11}$.
7. $1\frac{1}{13}$.
8. $2\frac{11}{15}$.
9. $9\frac{5}{12}$.
10. $6\frac{3}{10}$.
11. $68\frac{1}{6}$.
12. $13\frac{13}{15}$.
13. $2\frac{59}{60}$.
14. $84\frac{53}{60}$.
15. $6\frac{11}{12}$.
16. $11\frac{11}{24}$.
17. $134\frac{33}{80}$.
18. $7\frac{19}{24}$.
19. $9\frac{3}{16}$.
20. $88\frac{49}{90}$.
21. $7\frac{7}{12}$.
22. $8\frac{35}{36}$.
23. $10\frac{3}{8}$.
24. $10\frac{1}{4}$.
25. $28\frac{1}{12}$.
26. $113\frac{31}{48}$.
27. $34\frac{1}{4}$.
28. $8\frac{3}{16}$.
29. $42\frac{11}{40}$.
30. $8\frac{1}{4}$.
31. $15\frac{1}{5}$.
32. $20\frac{11}{16}$.
33. $44\frac{23}{48}$.
34. $18\frac{2}{9}$.
35. $16\frac{37}{60}$.
36. $21\frac{1}{12}$.
37. $17\frac{3}{4}$ mi.; $7\frac{3}{4}$ mi.
38. $2\frac{1}{3}$ yr.; $20\frac{7}{12}$ yr.; $32\frac{1}{3}$ yr.

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39. $25\frac{4}{5}$ gr.
40. $116\frac{1}{10}$ gr.

1. $8\frac{1}{2}$.
2. $9\frac{5}{6}$.
3. $3\frac{4}{15}$.
4. $3\frac{1}{12}$.
5. $3\frac{11}{28}$.
6. $12\frac{14}{15}$.
7. $\frac{11}{12}$.
8. $2\frac{5}{18}$.
9. $3\frac{1}{10}$.
10. $1\frac{7}{12}$.
11. 13.
12. $10\frac{12}{85}$.
13. $1\frac{13}{14}$.
14. $\frac{19}{24}$.
15. $5\frac{1}{10}$.
16. $15\frac{33}{40}$.
17. $23\frac{3}{7}$.
18. $26\frac{25}{33}$.
19. $7\frac{1}{4}$.
20. $17\frac{28}{45}$.
21. $25\frac{18}{25}$.
22. $31\frac{4}{15}$.
23. $6\frac{4}{11}$.
24. $34\frac{9}{14}$.
25. $44\frac{7}{18}$.
26. $62\frac{55}{64}$.
27. $73\frac{14}{27}$.

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28. $4\frac{7}{8}$ yd.
29. Charles, $\frac{3}{4}$ mi. farther.
30. $9\frac{5}{8}$ lb.
31. $1\frac{3}{8}$ yd.
32. $25\frac{17}{40}$ lb.
33. $2\frac{2}{3}$ gal.
34. $89\frac{1}{24}$ A.
35. $2\frac{3}{4}$ mi.
36. $1\frac{5}{6}$ hr.
37. $112\frac{7}{8}$ A., $139\frac{3}{16}$ A., $354\frac{11}{16}$ A.
38. $92\frac{5}{6}$ yd.
39. $2\frac{59}{80}$ mi.

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1. 45.
2. $2\frac{14}{19}$.
3. $14\frac{7}{12}$.
4. 252.
5. $2\frac{4}{5}$.
6. $11\frac{1}{2}$.
7. $9\frac{1}{2}$.
8. 50.
9. $11\frac{1}{3}$.
10. $5\frac{25}{72}$.
11. $5\frac{5}{8}$.
12. $9\frac{2}{3}$.
13. 91.
14. 48.
15. $\frac{35}{72}$.
16. 102.
17. $24\frac{1}{2}$.
18. 110.
19. $22\frac{4}{5}$.
20. 51.
21. 6 yd.
22. $53\frac{2}{3}$ ft.
23. 5 hr.
24. \$15.
25. \$15.53.
26. \$121.88.
27. \$2.50.
28. \$1.10.
29. \$15.12.
30. \$3.
31. \$70.83.
32. \$37.33.
33. \$1.54.
34. \$50.70.

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1. 350.
2. $24\frac{1}{2}$.
3. $156\frac{3}{14}$.
4. $116\frac{2}{3}$.
5. 220.
6. 8.
7. $268\frac{4}{5}$.
8. $106\frac{2}{3}$.
9. 180.
10. $122\frac{1}{2}$.
11. $14\frac{2}{3}$.
12. 33.
13. $588\frac{4}{5}$.
14. $\frac{1}{2}$.
15. 153.
16. 675.
17. 720.
18. 168.
19. 288.
20. $128\frac{4}{7}$.
21. $266\frac{2}{3}$.
22. 720.
23. $100\frac{5}{21}$.
24. 80.

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25. 100 mi.; 50 mi.; 25 mi.; $12\frac{1}{2}$ mi.; 10 mi.
26. 260 mi.
27. 120 mi.
28. 75 mi.
29. 60 mi.
30. 140 mi.
31. 140 mi.
32. 140 mi.

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7. $\frac{15}{32}$. **8.** $\frac{8}{25}$. **9.** $\frac{7}{16}$. **10.** $\frac{4}{9}$. **11.** $\frac{16}{25}$. **12.** $\frac{1}{4}$. **13.** $\frac{5}{8}$. **14.** $\frac{25}{36}$. **15.** $\frac{4}{21}$.

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1. $\frac{15}{56}$.
2. $\frac{50}{63}$.
3. 6.
4. $\frac{6}{11}$.
5. $23\frac{4}{5}$.
6. $\frac{3}{10}$.
7. $\frac{8}{45}$.
8. $1\frac{13}{15}$.
9. $141\frac{3}{4}$.
10. $\frac{2}{3}$ qt.
11. $\frac{1}{25}$ sq. in.
12. $\frac{8}{25}$ sq. in.
16. $\frac{3}{5}$ mi.

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17. $\frac{1}{2}$. **18.** $1\frac{4}{23}$. **19.** $8\frac{1}{10}$ sq. in. **20.** $15\frac{3}{25}$ sq. ft. **21.** $1\frac{40}{81}$ sq. in. **22.** $55\frac{5}{49}$ sq. in.

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1. 2,303.
2. 1,030.
3. 1,624.
4. 140.
5. 1,469.
6. 616.
7. 2,700.
8. $892\frac{1}{2}$.
9. 1,300.
10. $392\frac{1}{2}$.
11. 1,214.
12. 1,076.
13. 5,640.
14. 600.
15. 1,368.
16. 1,179.
17. \$3.40.
18. (1) \$17.85; (5) \$21;
(2) \$15.75; (6) \$17.85;
(3) \$22.58; (7) \$15.23;
(4) \$20.48; (8) \$15.23.
19. \$3.78.
20. $67\frac{1}{2}$ yd.
21. \$21.87$\frac{1}{2}$.
22. $668\frac{1}{2}$ mi.
23. $115\frac{5}{8}$ rd.
24. $55\frac{1}{3}$ mi.
25. \$33.33;
\$5.63;
total cost,
\$38.96.

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1. $21\frac{17}{20}$.
2. $33\frac{1}{3}$.
3. $304\frac{19}{22}$.
4. $116\frac{23}{32}$.
5. $308\frac{1}{5}$.
6. $349\frac{3}{8}$.
7. $257\frac{44}{45}$.
8. $16\frac{19}{24}$.
9. $316\frac{17}{25}$.
10. $2,206\frac{28}{75}$.
11. $30\frac{11}{96}$.
12. $129\frac{14}{25}$.
13. $481\frac{13}{80}$.
14. $376\frac{27}{91}$.
15. $338\frac{19}{24}$.
16. $225\frac{75}{98}$.
17. $1,926\frac{21}{320}$.
18. $1,282\frac{127}{300}$.
19. $86\frac{17}{56}$.
20. $1,229\frac{3}{20}$.
21. $15\frac{91}{108}$.
22. $1,759\frac{25}{98}$.
23. $573\frac{23}{49}$.
24. $1,292\frac{43}{56}$.
25. $828\frac{11}{20}$.
26. $206\frac{99}{128}$.
27. $380\frac{115}{256}$.
28. $111\frac{8}{9}$ sq. yd.
29. $202\frac{25}{32}$ sq. ft.
30. $17\frac{5}{12}$ mi.
31. $64\frac{7}{72}$ A.

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5. $1\frac{1}{2}$; $4\frac{1}{2}$; 3.
6. $1\frac{1}{4}$; $7\frac{1}{2}$; $3\frac{3}{4}$.
7. $2\frac{1}{2}$; 10; 5.
8. $1\frac{2}{3}$; $13\frac{1}{3}$; $6\frac{2}{3}$.

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1. $6\frac{2}{5}$.
2. $19\frac{4}{5}$.
3. $3\frac{3}{4}$.
4. $28\frac{4}{5}$.
5. $15\frac{3}{7}$.
6. $5\frac{5}{6}$.
7. 21.
8. 200.
9. 21.
10. $12\frac{4}{9}$.
11. $8\frac{3}{4}$.
12. 147.
13. $16\frac{1}{2}$.
14. $13\frac{1}{3}$.
15. $7\frac{7}{8}$.
16. 75.
17. 16.
18. 16.
19. 75.
20. 15.
21. $326\frac{2}{3}$.

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1. $\frac{8}{9}$.
2. $\frac{15}{22}$.
3. $\frac{4}{7}$.
4. $\frac{36}{65}$.
5. $\frac{20}{21}$.
6. $1\frac{1}{35}$.
7. $1\frac{7}{9}$.
8. $\frac{45}{128}$.
9. $\frac{15}{16}$.
10. $\frac{9}{22}$.
11. $1\frac{1}{10}$.
12. $\frac{2}{3}$.
13. $1\frac{1}{9}$.
14. $\frac{5}{12}$.
15. $\frac{25}{64}$.
16. $1\frac{71}{105}$.

1. $\frac{4}{81}$.
2. $\frac{1}{160}$.
3. $\frac{1}{132}$.
4. $\frac{1}{48}$.
5. $\frac{1}{40}$.
6. $\frac{1}{560}$.
7. $\frac{1}{75}$.
8. $\frac{1}{50}$.
9. $\frac{1}{120}$.
10. $\frac{1}{96}$.
11. $\frac{2}{25}$.
12. $\frac{1}{288}$.
13. $\frac{11}{140}$ min.
14. $\frac{3}{20}$ min.
15. $\frac{1}{160}$ bbl.
16. $\frac{3}{112}$ mi.

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1. $\frac{13}{14}$.
2. $6\frac{6}{7}$.
3. $2\frac{4}{9}$.
4. $\frac{261}{536}$.
5. $3\frac{1}{2}$.
6. $\frac{7}{50}$.
7. $3\frac{13}{36}$.
8. $\frac{225}{286}$.
9. $1\frac{4}{5}$.
10. $5\frac{6}{19}$.
11. $3\frac{4}{33}$.
12. $\frac{11}{39}$.
13. 6.
14. 2.
15. 12; $\frac{1}{12}$ in.; 6; 18; 3; 15.
16. 3.
17. 8.

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1. $\frac{1}{2}$.
2. $\frac{3}{8}$; $\frac{3}{25}$; $\frac{1}{18}$.
3. $\frac{1}{9}$; $\frac{3}{100}$; $\frac{1}{64}$.
4. $\frac{1}{15}$; $\frac{3}{80}$; $\frac{1}{61}$; $\frac{1}{7}$.
5. $\frac{9}{160}$.
6. $\frac{9}{40}$.
7. $\frac{8}{64}$.
8. $\frac{7}{13}$.
9. $\frac{5}{11}$.
10. $\frac{43}{63}$.

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1. $562\frac{1}{2}$ A.
2. 1,760 yd.
3. 142 mi.
4. 540 bu.
5. $15,216.
6. $4,125.

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1. $\frac{9}{70}$.
2. 4.
3. $1\frac{19}{77}$.
4. $\frac{115}{309}$.
5. $2\frac{11}{103}$.
6. $\frac{1744}{22275}$.
7. $678\frac{1}{3}$.
8. $4\frac{163}{1161}$.

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1. $\frac{2}{3}$.
2. $\frac{1}{5}$.
3. $\frac{1}{3}$.
4. $\frac{2}{3}$.
5. $\frac{1}{5}$.
6. $\frac{5}{19}$.
7. $\frac{8}{121}$.
8. $\frac{3}{10}$.
9. $\frac{3}{5}$.
10. $\frac{4}{21}$.
11. $\frac{23}{42}$.
12. $\frac{19}{42}$.
13. $\frac{1}{2}$.
14. $\frac{3}{8}$.
15. $\frac{3}{5}$.
16. $\frac{2}{3}$.
17. $\frac{11}{20}$.
18. $\frac{2}{11}$.
19. $\frac{2}{15}$.
20. $\frac{1}{9}$.
21. $\frac{3}{4}$.
22. $\frac{11}{32}$.
23. $2\frac{7}{45}$.
24. $1\frac{35}{48}$.
25. $\frac{37}{51}$.
26. 161.
27. $1\frac{1}{2}$.
28. $1\frac{1}{6}$.
29. $1\frac{47}{102}$.
30. $1\frac{11}{12}$.
31. $1\frac{2}{5}$.
32. $100\frac{13}{16}$.
33. $\frac{95}{144}$.
34. $\frac{59}{60}$.
35. $1\frac{103}{400}$.
36. $\frac{147}{150}$.
37. $1\frac{121}{300}$.
38. $176\frac{1}{10}$.
39. $\frac{23}{30}$.
40. $1\frac{5}{21}$.
41. $\frac{1}{72}$.
42. 0.
43. $53\frac{3}{20}$.
44. $10\frac{181}{332}$.

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45. $\frac{37}{56}$.
46. $\frac{19}{36}$.
47. $43\frac{11}{12}$.
48. $1\frac{1}{6}$.
49. $\frac{41}{81}$.
50. $\frac{26}{99}$.
51. $3\frac{1}{2}$.
52. 95.
53. $23\frac{1}{3}$.
54. $27\frac{12}{19}$.
55. 6.
56. 24.
57. $\frac{15}{32}$; $\frac{3}{8}$; $\frac{5}{12}$; $\frac{1}{2}$; $\frac{25}{48}$; $\frac{35}{64}$; $\frac{15}{64}$.
58. $\frac{9}{35}$; $\frac{9}{28}$; $\frac{2}{7}$; $\frac{9}{56}$; $\frac{3}{8}$; $\frac{12}{35}$; $\frac{3}{8}$.
59. $\frac{1}{100}$.
60. $\frac{1}{50}$.
61. $133\frac{1}{3}$.
62. $112\frac{1}{2}$.
63. $1\frac{5}{9}$.
64. $\frac{8}{343}$.
65. $\frac{4}{1445}$.
66. $1,012\frac{1}{2}$.
67. $41\frac{2}{3}$.
68. $3\frac{1}{3}$.
69. $\frac{1}{144}$.
70. $\frac{1}{144}$.
71. 256.
72. $581\frac{9}{11}$.
73. $\frac{5}{8}$.
74. $\frac{1}{6}$; $\frac{1}{9}$; $\frac{1}{16}$.
75. $\frac{12}{49}$; $\frac{1}{3}$; $\frac{3}{20}$.
76. $36\frac{4}{7}$.
77. $113\frac{2}{5}$.
78. $\frac{7}{4050}$.
79. $\frac{10400}{30247}$.
80. 24 in.
81. $6\frac{1}{2}$ in.

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82. 17 in.
83. $8\frac{5}{8}$ in.
84. $\frac{1}{2}$ in.
85. $4\frac{3}{8}$ in.
86. $9\frac{25}{48}$ yd.
87. $431\frac{23}{45}$ yd.
88. $32\frac{583}{696}$ ft.
89. $709\frac{17}{18}$ in.

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90. $144\frac{1}{6}$ ft.
91. $1\frac{23}{40}$ mi.
92. $9\frac{27}{40}$ T.
93. $143\frac{5}{32}$ A.
94. $5\frac{1}{4}$ ft.
95. $99\frac{11}{12}$ lb.
96. 310 bu.
97. $\frac{74}{531}$.
98. $193\frac{3}{80}$ mi.
99. $2,891.
100. 38 yd.
101. $123.75.

Page 100

1. .7.
2. 19.9.
3. .11.
4. .35.
5. 12.39.
6. 1.01.
7. 20.2.
8. 22.2.
9. .05.
10. .99.
11. .40.
12. 5.05.
13. .3.
14. .5.
15. 1.4.
16. 3.9.
17. 1.1.
18. 4.5.
19. .1.
20. .6.
21. .9.
22. .8.
23. .7.
24. .4.
25. .03.
26. .25.
27. .55.
28. .10.
29. .09.
30. .05.
31. .15.
32. 1.75.
49. $\frac{3}{10}$.
50. $\frac{15}{10}$
51. $\frac{15}{100}$.
52. $\frac{9}{100}$.
53. $\frac{125}{100}$.
54. $\frac{86}{100}$.
55. $\frac{71}{10}$.
56. $\frac{1}{10}$.
57. $\frac{109}{100}$.
58. $\frac{99}{100}$.
59. $\frac{101}{100}$.
60. $\frac{45}{100}$.
61. $\frac{234}{10}$.
62. $\frac{111}{10}$.
63. $\frac{1008}{100}$.
64. $\frac{175}{100}$.
65. $\frac{29}{100}$.
66. $\frac{117}{100}$.
67. $\frac{36}{10}$.
68. $\frac{203}{10}$.
69. $\frac{5}{100}$.
70. $\frac{5625}{100}$.
71. $\frac{575}{100}$.
72. $\frac{290}{100}$.

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1. .017.
2. .843.
3. .125.
4. .009.
5. .085.
6. .066.
7. .995.
8. 100.005.
9. 200.022.
10. 1.3 ; 4 ; .3.
11. 10.5 ; .40.
12. .30 ; .50 ; .70 ; .60 ; 8.00 ; 15.00.
13. .050 ; .010 ; .300 ; .080 ; 10.000 ; 25.000.
14. .045.
15. .125.
16. .222.
17. .15.
18. 5.6.
19. .009.
20. .809.
21. .999.
22. 2.1.
23. 10.6.
24. .001.
25. .205.
26. .400.
27. .11.
28. 2.09.
41. $10\frac{9}{10}$.
42. $\frac{110}{1000}$.
43. $\frac{15}{1000}$.
44. $\frac{225}{1000}$.
45. $21\frac{1}{1000}$.
46. $121\frac{121}{1000}$.
47. $1\frac{6}{1000}$.
48. $46\frac{46}{1000}$.
49. $\frac{5}{1000}$.
50. $1\frac{9}{100}$.
51. $\frac{1}{1000}$.
52. $\frac{365}{1000}$.
53. $50\frac{7}{100}$.
54. $\frac{44}{1000}$.
55. $\frac{76}{1000}$.
56. $\frac{625}{1000}$.

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1. $5.30.
2. $.44.
3. 14.23 mi.
4. 4.03 gal.
5. 89.30 yd.
6. 830.63 mi.
7. $.023.
8. 24.87 ft.
9. 125.70 gal.
10. 99.78 ft.
11. 1.760 ft.
12. 6.278 mi.

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1. 4.5.
2. 3.6.
3. 80.8.
4. 3.24.
5. 56.25.
6. .064.
7. 3.650.

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8. 6.3.
9. 43.5.
10. 1.33.
11. 6.3.
12. 4,292.84.
13. 35.1.
14. .54.
15. 4.8.
16. 1,028.2.
17. 402.27.
18. 3.
19. 10.
20. 4,825.4.
21. .036.
22. 674.865.
23. 230.4.
24. 15,607.8.
25. 28.50.
26. 11.025.
27. $392.288.
28. $8.80.
29. 175.92 ft.
30. 551 ft.
31. 51 in.
32. 202.09 mi.
33. $53.30.
34. 971.472 mi.
35. Yes.
36. 281.875 in.

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37. 4,419.15 ft.

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1. 3.
2. 201.2.
3. 7.
4. 298.
5. 9.
6. 1,967.5.
7. 11.
8. 4,003.
9. 119.3.
10. 2,871.
11. 25.
12. 1,365.
13. 50.
14. 1.
15. 125.
16. 222.5.
17. 113.
18. 67.5.
19. 405.
20. 312.5.

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1. Total, $17.55.
2. $2.46.

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3. $8.02.
4. $11.34.
5. $3.30.
6. $95.75.
7. $25.47.
8. $193.36.
9. $1,046.35.

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1. .6.
2. 4.1.
3. .21.
4. .013.
5. .35$\frac{5}{9}$.
6. .7.
7. .03$\frac{4}{7}$.
8. .5.
9. .8.
10. .07.
11. .50$\frac{5}{6}$.
12. .31.
13. .42$\frac{6}{7}$.
14. 1.05$\frac{1}{3}$.
15. .52.
16. 4.13$\frac{1}{3}$.
17. .06$\frac{1}{2}$.
18. .43$\frac{6}{7}$.
19. 2.76$\frac{74}{81}$.
20. 2.93$\frac{6}{17}$.
21. 2.12$\frac{7}{19}$.
22. 2.01$\frac{1}{7}$.
23. 1.31$\frac{6}{13}$.
24. .04$\frac{5}{9}$.
25. .22$\frac{1}{11}$.
26. 1.19$\frac{8}{15}$.
27. .85$\frac{1}{12}$.
28. .133$\frac{2}{3}$.
29. .091$\frac{7}{11}$.

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30. $2.15.
31. $48.50.
32. $.50$\frac{5}{13}$.
33. 3.6 yd.
34. .9 cu. yd.
35. 30.8 yd.
36. $16.35.
37. $9.50.

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1. $\frac{3}{10}$.
2. $\frac{2}{5}$.
3. $\frac{4}{5}$.
4. $\frac{5}{8}$.
5. 9$\frac{3}{4}$.
6. $\frac{9}{20}$.
7. $\frac{3}{25}$.
8. $\frac{1}{100}$.
9. $\frac{2}{5}$.
10. 17$\frac{1}{2}$.
11. $\frac{1}{8}$.
12. 4$\frac{1}{4}$.
13. $\frac{3}{40}$.
14. 4$\frac{1}{20}$.
15. $\frac{3}{8}$.
16. $\frac{4}{25}$.
17. $\frac{1}{4}$.
18. $\frac{1}{125}$.

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1. .4.
2. .75.
3. .24.
4. 1.5.
5. .625.
6. .18.
7. .8.
8. .75.
9. 1.125.
10. 3.25.
11. .25.
12. .9.
13. .32.
14. 2.56.
15. .255.
16. .475.
17. .44.
18. .15.

1. .33$\frac{1}{3}$.
2. .62$\frac{1}{2}$.
3. .09$\frac{1}{11}$.
4. 1.66$\frac{2}{3}$.
5. .12$\frac{1}{2}$.
6. .85$\frac{5}{7}$.
7. .63$\frac{7}{11}$.
8. 5.83$\frac{1}{3}$.
9. .16$\frac{2}{3}$.
10. .37$\frac{1}{2}$.
11. .08$\frac{1}{3}$.
12. 2.58$\frac{1}{3}$.
13. .83$\frac{1}{3}$.
14. .87$\frac{1}{2}$.
15. .41$\frac{2}{3}$.
16. 9.83$\frac{1}{3}$.
17. 12.87$\frac{1}{2}$.
18. 7.75.
19. 8.62$\frac{1}{2}$.
20. 16.66$\frac{2}{3}$.

Page 116

1. 70.
2. $186.
3. $15.
4. 80.5 lb.
5. 102.
6. 12.12 ft.
7. 17 mi.
8. 203.1.
9. $14.
10. 1; 3.
11. $5.10.
12. $20.
13. $35.
14. $8.
15. $13.
16. $6.
17. $9.

Page 118

1. $\frac{3}{5}$; 4$\frac{9}{25}$; $\frac{3}{100}$; $\frac{141}{200}$; $\frac{3}{40}$; $\frac{1}{2}$; $\frac{1}{150}$.
2. .6; .5; .875; .28; 5.
3. .9; .06; .8; .008$\frac{7}{17}$; 1.14$\frac{2}{7}$; .182$\frac{2}{11}$.
4. .66$\frac{2}{3}$; .75; .87$\frac{1}{2}$; .08$\frac{1}{3}$; 2.41$\frac{2}{3}$; .36$\frac{4}{11}$.
5. 85.41 ft.
6. 11.6 bu.
7. 19.01 gal.; 190.1 gal.
8. 70.13 ft.
9. 227.52 mi.
10. $46.90.
11. 749.07 in.
12. 9.6 ft.
13. 41.1 in.

Page 119

14. $.50.
15. $.50.
16. $1; $1; $.65.
18. $1.60.
19. $38.22.

		BREADSTUFFS	MEATS	CLOTHING
20.	2d year	$ .083 increase	$ .471 increase	$ .206 increase
	3d year	$2.196 increase	$ .165 increase	$ .855 increase
	4th year	$ .70 increase	$ .294 increase	$ .358 increase
	5th year	$1.415 increase	$ .918 increase	$1.303 increase
	6th year	$ .006 increase	$ .524 increase	$1.226 decrease
	7th year	$5.63 increase	$2.198 increase	$ .435 increase
	8th year	$3.061 decrease	$2.359 decrease	$1.603 increase

21. $.57.
22. $9.32.
23. $4.87.
24. $1.86.
25. $3,027.50.
26. $299.

Page 122

1. 20 bbl. **2.** 25 bu. **3.** 180. **4.** 126. **5.** 63. **6.** $23\frac{1}{2}$. **7.** 75.

Page 124

1. 6 lb.
2. $1\frac{1}{4}$ T.
3. 152 oz.
4. 392 oz.
5. $7\frac{1}{2}$ lb.
6. $2\frac{1}{2}$ long T.
7. 16 lb.
8. 117 lb.
9. 56 T.
10. 109,200 lb.

1. 240 pwt.
2. $\frac{3}{4}$ pure gold.
3. $9\frac{3}{5}$ gr.
4. 24 sc.
5. 96 dr.
6. 60 gr.
7. 12.
8. 120 sc. 40 dr.
9. 14,400 gr. troy; 14,400 gr. apoth.
10. 1,240 gr.
11. 7 pwt.
12. 480 gr.
13. 40.

Page 127

1. 3 cords.
2. 23,040 A.
3. 9,504 cu. in.
4. 4.58 A.
5. $12.96.
6. 7,920 ft.
7. 27,648 cu. in.
8. $272\frac{1}{4}$ sq. yd.
9. 3,240 cu. yd.

Page 129

1. 90°; 60°; 45°.
2. 45°.
3. 45°.
4. 360°; 180°; 90; 60°; 30°.
5. 60°; 150°.
6. 330″; 230″; 3,600″.
7. 110′; 340′.
8. 90°.
9. 90°; 360°.
10. 120°.
11. 180°.
12. 5,400′.
13. 7,200′.

Page 130

1. 258 in.
2. 66 oz.
3. 94.5 ft.
4. 3,017 lb.
5. 1,824 units.
6. 740 sq. rd.
7. 28 pt.
8. 52 qt.
9. 94 pt.
10. 106″.
11. 38 ft.
12. 90 cu. ft.

Page 131

1. 23 lb.
2. 50 gal.
3. 85 bu.
4. 1 hr.
5. 364 da.
6. $6\frac{1}{4}$ gross.
7. 2 cu. ft.
8. $233\frac{1}{3}$ gal.
9. $36\frac{7}{12}$ da.
10. $15\frac{5}{9}$ yd.
11. $\frac{7}{30}$ hr.
12. $6\frac{17}{18}$ yd.
13. $2\frac{27}{88}$ mi.
14. 1 cu. ft.
15. $2\frac{7}{9}$ sq. ft.
16. 592,000 oz.
17. 26,520 hr.
18. 2 sq. rd.

Page 132

1. 256 lb. 6 oz.
2. 38 gal. 1 qt.
3. 30 yd. 1 ft. 3 in.
4. 86 mi. 80 rd.
5. 63 bu. 1 pk.
6. 15 wk.
7. 69 rd. 1 ft.
8. 120 gal. 3 qt.
9. 22 wk. 4 da. 1 hr. 45 min.
10. 126 bu. 2 pk. 5 qt.
11. 46 yd. 2 ft. 10 in.
12. 10 bu. 2 pk. 6 qt.
13. 12 gal. 3 qt.
14. 19 mi. 120 rd.
15. 2 wk. 6 da.

Page 133

16. 31 lb. $14\frac{3}{4}$ oz.
17. 45 bu. 1 pk.
18. 8 rd. $14\frac{1}{4}$ ft.
19. 40 gal. 2 qt. 1 pt.
20. 58 bu. 4 qt.
21. 4 gal. 2 qt.
22. 20 yd. 11 in.

1. 298 lb.
2. 1,012 ft. 6 in.
3. 548 gal.
4. 1,012 da. 20 hr. 20 min.
5. 125 wk. 6 da.
6. 93 mi. 2,910 ft.
7. 7,149 yd. 1 ft.
8. 2,700 gro. 6 doz.
9. 3,979 ft. 3 in.
10. 4,010 T.
11. 35 gal. 1 qt.
12. 366 rd. 13 ft. 6 in.

Page 134

1. 10 ft. $\frac{2}{3}$ in.
2. 55 lb. $9\frac{5}{9}$ oz.
3. 13 da. 9 hr. 8 min.
4. 138 rd. 8 ft. 3 in.
5. 2 sq. ft. $15\frac{1}{3}$ sq. in.
6. 43 yd. 2 ft. 4 in.
7. 11 rd. $4\frac{11}{16}$ ft.
8. 41 min. $31\frac{3}{7}$ sec.
9. 146 sq. rd.
10. $82\frac{14}{27}$ cu. in.
11. 20 rd. 4 ft. $1\frac{1}{2}$ in.
12. 7 gal. 2 qt.
13. 5 rd. 1 yd. $6\frac{2}{3}$ in.

Page 135

1. $10\frac{23}{34}$.
2. $9\frac{2}{9}$.
3. $12\frac{12}{17}$.
4. $7\frac{31}{35}$.
5. $74\frac{8}{11}$.
6. $2\frac{97}{144}$.
7. $5\frac{3}{10}$.
8. $32\frac{1}{3}$.
9. 21.
10. $2\frac{27}{80}$.
11. $5\frac{62}{65}$.
12. $4,889\frac{1}{15}$.
13. $9\frac{11}{17}$.
14. $28\frac{1}{22}$.
15. 152.
16. 59.
17. $36\frac{2}{3}$ ft.
18. $2\frac{19}{40}$ T.
19. 11.
20. 145.

Page 137

1. 12.
2. 78.75 sq. yd.
3. 41.25 sq. ft.
4. 529 sq. in.
5. 236.25 sq. ft.
6. 6,000 sq. ft. ; $666\frac{2}{3}$ sq. yd.
7. 77,469 sq. yd.
8. 2 in.
9. 66 sq. ft.
10. $195\frac{5}{16}$ A.
11. $74.75.
12. 5,280 ft.
13. $416.67.
14. 10 rd.
15. $1,111\frac{1}{9}$ sq. yd.
16. 3,528 sq. ft.

Page 138

1. 6 sq. in.
2. $13\frac{1}{2}$ sq. ft.
3. $1\frac{1}{4}$ sq. yd.
4. $10\frac{1}{2}$ sq. ft.
5. $23\frac{5}{8}$ sq. rd. ; 39 sq. yd. ; $1\frac{7}{8}$ sq. in. ; $14\frac{11}{28}$ sq. ft.
6. 1 A.

Page 139

1. $\frac{s \times a}{2}$.
2. 32 sq. in.
3. 6,768 sq. in.
4. 221 sq. ft.
5. $30.
6. $104.76.

Page 140

7. 240 sq. ft.
8. 1000 sq. in.
9. 300 sq. yd.
10. 360 sq. rd.
11. 28 sq. in.

Page 142

1. 4 ; 20 yd.
3. $31\frac{1}{3}$ yd.

Page 143

4. 14 yd. **6.** Bedroom, $ .80; dining-room, $ 1.60; sitting-room, $ 1.20; total, $ 3.60. **8.** 12,096.
5. $ 4.33. **9.** $ 35.88.
7. $ 9 for side walls; $ 12.60, cost including ceiling paper.

Page 144

1. $164\frac{1}{16}$. **3.** $419\frac{1}{21}$ loads. **5.** $1{,}012\frac{1}{2}$ cu. ft. **7.** 450,000 cu. ft.
2. $2,126.67. **4.** 750 cu. ft. **6.** 5,600 cu. yd.

Page 145

1. 57.75 cu. in. **2.** $67.20\frac{1}{16}$ cu. in.
3. The dry quart is $9.45\frac{1}{16}$ cu. in. larger than the liquid quart.
4. 1.244+ cu. ft. **7.** 6 times. **9.** $1{,}558\frac{2}{7}$ cu. in.; $6\frac{402}{539}$ gal.
5. 154.34+ bu. **8.** $66\frac{121}{144}$ cu. ft. **10.** $351\frac{214}{311}$ bu.
6. 585 times.

Page 146

1. 3 cords. **2.** 25 cords; 32 cords; 23 cords; 81 cords. **3.** $3.50.

Page 147

1. 40 bd. ft. **6.** $18.69. **10.** $55.
2. 336 bd. ft. **7.** $15.25. **11.** $39.20.
3. 2,800 bd. ft. **8.** $30.80. **12.** $102\frac{1}{4}$ ft.
4. $533\frac{1}{3}$ bd. ft. **9.** $48.84. **13.** 4,480 bd. ft.
5. $ 8.62.

Page 148

1. $81.29. **2.** 83 cu. yd. **3.** $264\frac{4}{9}$ cu. ft. **4.** $264\frac{16}{81}$ cu. yd.

Page 149

5. $66\frac{26}{27}$ cu. yd. **6.** $354\frac{22}{27}$ cu. ft. **7.** 228 cu. ft. **8.** 1,512 cu. in.

Page 150

1. 17 yr. 1 mo. 4 da. **3.** 67 yr. 9 mo. 22 da. **5.** 1,759.
2. 51 yr. 8 mo. 20 da. **4.** 69 yr. 6 mo. 17 da. **6.** 56 yr. 2 mo. 2 da.

Page 151

1. 127 gal. 2 qt.
2. 28 bu. 2 pk. 2 qt.
3. 7 mi. 59 rd. 15 ft. 9 in.
4. 43 T. 501 lb. 3 oz.
5. 103 sq. yd. 1 sq. ft. 116 sq. in.
6. 483 cu. yd. 21 cu. ft. 204 cu. in.
7. 9 mi. 1 rd. 15 ft. 7 in.
8. 619 T. 908 lb. 10 oz.
9. 8 bu. 3 pk. 3 qt.
10. 22 gal. 2 qt. 1 pt.
11. 114 sq. yd. 7 sq. ft. 124 sq. in.
12. 269 cu. yd. 1 cu. ft. 808 cu. in.
13. 1 sq. yd. 6 sq. ft. $83\frac{7}{15}$ sq. in.
14. 255 rd. 8 ft. $11\frac{5}{11}$ in.

Page 152

15. 400 sq. ft.
16. 890 sq. ft.; 600 sq. ft.
23. 7,875 cu. ft.; $2,479.17.
24. 567 cu. ft. oxygen; 2,133 cu. ft. nitrogen.
17. $17\frac{7}{9}$ yd., or 3 strips each 20 ft. long.
18. 21.
19. 720 A.
20. 96 ¢.
25. 72 times; $3\frac{3}{5}$ min.
21. 480 bd. ft. $2.88.
22. 640 cu. in.

Page 153

26. 18 hr.
27. $3,141\frac{9}{11}$ gal.
28. $311,114\frac{1}{216}$ cu. ft.
29. 1,542.8 + bu.
30. 128 cords.
31. $42\frac{7}{9}$ yd.
32. 86,400 sec.
33. 2,100 min.
34. 750 cu. ft.
35. $91\frac{1}{3}$ cu. in.
36. 11 cu. ft.
37. 680 cu. in.
38. (1) 1,456 cu. ft. (2) 4,122 cu. ft. (3) 3,210 cu. ft. (4) 1,937 cu. ft. (5) 2,312 cu. ft.

Page 155

9. 285 mi.
10. $1.60.

Page 156

11. $.80.
12. 78 millions.
13. $40; $21.33.
14. $8.40; $6.30.
15. 28.
16. 1,700,000.
17. 320,000; 200,000.
18. 270,000 T.
19. 990.
20. 444 ft.

Page 157

1. $39.10. **2.** $72. **3.** 134,000 T. **4.** $142,105\frac{5}{19}$ T. **5.** 4 rolls; $2. **6.** $1.60.

Page 158

7. $4,050.
8. $56,000.
9. $3,150.

Page 159

1. 176 cu. ft.
2. 47.125 sq. in.
3. $19\frac{1}{4}$ sq. yd.
4. 11 ¢.
5. 4 ¢.
6. 432.
7. $2.56.
8. $12\frac{4}{7}$ hr.

Page 160

1. $.017.
2. Maintenance, $2,870,000; equipment, $1,629,000; transportation, $5,548,000; general expenses, $454,000.

Page 162

2. Mary, $4\frac{3}{7}$ da.; Helen, $4\frac{3}{7}$ da.; Leslie, $4\frac{3}{7}$ da.; Charles, 5 da.; James, $4\frac{3}{7}$ da.
3. 1st wk., $4\frac{3}{5}$; 2d wk., $4\frac{3}{5}$; 3d wk., $4\frac{3}{5}$; 4th wk., 4; 5th wk., 5; 6th wk., $4\frac{2}{5}$; 7th wk., $4\frac{3}{5}$. **4.** $25\frac{41}{46}$; $21\frac{108}{109}$. **5.** $23\frac{28}{155}$.

Page 163

6. $717\frac{3}{5}$. **7.** 900,000.

Page 164

1. $\frac{1}{2}$ mi. long; $\frac{1}{4}$ mi. wide. **2.** $96,000.

Page 165

3. 50. **4.** $7,000. **5.** 4 ft.

Page 166

3. 4.3 in.
4. .25; .4; .2; 1.5; 1; .75; .2; .6. The diagram represents the table graphically. .6.

Page 167

6. .8. **7.** 40 ft. **8.** 30 ft.; 30 ft.; 20 ft.
9. A, 20 ft., 15 ft.; B, 20 ft., 20 ft.; C, 20 ft., 10 ft.; D, 30 ft., 10 ft.; E, 20 ft., 20 ft.

Page 168

5. 2,805,156. **6.** 46,306.56+ sq. mi. **7.** 37.99+. **8.** 490.275 mi.

Page 169

9. $6.75; $$.18\frac{3}{4}$; $.56. **10.** $7.97.
11. A, $5\frac{23}{32}$ da.; B, $5\frac{27}{32}$ da.; C, $3\frac{9}{16}$ da.; D, $5\frac{15}{32}$ da.
12. A, $7\frac{5}{8}$ hr.; B, $7\frac{19}{24}$ hr.; C, $4\frac{3}{4}$ hr.; D, $7\frac{7}{24}$.
13. A, $7.14; B, $8.77; C, $7.13; D, $9.57.
14. A, $1.19; B, $1.46; C, $1.19; D, $1.60.
15. $32.61. **16.** $19\frac{1}{2}$ hr. **17.** $\frac{13}{14}$; $\frac{13}{14}$.
18. 48,000 lb. copper; 2,250 lb. silver; $2\frac{1}{12}$ lb. gold.

Page 171

1. 30.005. **3.** 100.0263. **5.** 96.000497. **7.** .34346.
2. 87.025. **4.** 408.1879. **6.** .00475. **8.** .924563.

1. $\frac{5}{8}$. **5.** $\frac{301}{4000}$. **9.** $\frac{913}{20000}$. **13.** $\frac{19}{4000}$.
2. $\frac{5}{16}$. **6.** $\frac{51}{400}$. **10.** $\frac{3}{800}$. **14.** $\frac{503}{4000}$.
3. $\frac{113}{2000}$. **7.** $\frac{1}{16}$. **11.** $\frac{7}{8}$. **15.** $\frac{3203}{8000}$.
4. $\frac{3}{400}$. **8.** $\frac{5}{32}$. **12.** $\frac{1}{400}$. **16.** $\frac{51}{4000}$.

Page 172

1. .1875.
2. .46666+.
3. .7.
4. .24242+.
5. .88888+.
6. .91666+.
7. .36363+.
8. .53333+.
9. .66666+.
10. .41666+.
11. .69230+.
12. .77777+.
13. .85714+.
14. .45.
15. .20833+.
16. .9375.

1. 99.342.
2. 76.2911.
3. 45.7722.
4. 51.4084.

Page 173

5. 201.0842.
6. 1.88588.
7. 2.362093.
8. 6.80330.
9. 2.8174.
10. .30720.
11. 274.28.
12. .9651.
13. .07605.
14. 491.634.
15. 6.5667.
16. .124289.
17. 8.5439.
18. .35.
19. 6.25 ft.
20. 3.5 ft.
21. $.0275.
22. $.0095.

Page 175

1. 12.4.
2. 70.07.
3. 25.025.
4. 507.755.
5. 94.824.
6. 58.4375.
7. 39.0625.
8. 3,699.63.
9. 55.65.
10. 9,039.
11. 72.09.
12. 7.75.
13. 20.736.
14. 166,777.5.
15. 10,110.10.
16. 745.008.
17. 769.2.
18. 28.098.
19. 485.74.
20. 36.531.
21. 769.23.
22. 2,656.2.
23. 374.
24. 35.7035.
25. 250.25 sq. ft.
26. 10.5625 sq. yd.
27. $2.25.
28. $3.02.
29. $444.60.
30. 185.25 mi.
31. $.83.
32. $6.69.
33. 142 ft.
34. 735.18$\frac{1}{8}$ mi.

Page 177

1. .75.
2. 2.5.
3. 200$\frac{10}{71}$.
4. 2,180.
5. 52.9.
6. .51.
7. 6.42.
8. 58.
9. 798$\frac{2}{111}$.
10. 49$\frac{3}{41}$.
11. 51.
12. 3.02.
13. 8.4.
14. 207$\frac{23}{235}$.
15. 3,201.
16. 9.166+.
17. 6.868+.
18. 7.003+.
19. .695+.
20. 273.602+.

21. 5.738+.
22. 1.756+.
23. 900 mi.
24. 40.156+ hr.
25. 320 rd.; 154 rd.
26. $\frac{1}{2}$ sec.
27. 200 in.
28. 7.5 sq. rd.
29. 55.4 mi.

Page 181

1. 40.
2. 125.
3. 920.
4. 2,410.
5. 79.98.
6. 100.
7. 1000.
8. 787.5.
9. 450.
10. 76.8.
11. 1,200.
12. 720.
13. 211.64.
14. 224.
15. 86.67.
16. 384,440 bu.

Page 182

17. \$44; \$220; \$22; \$11.
18. 75 lb.; 3,360 lb.
19. 648.
20. 40; 40%.
21. \$8,250.
22. $18\frac{2}{3}$ A.
23. 1368.75 T.

Page 183

1. 50%; $33\frac{1}{3}$%; 24%; $9\frac{3}{5}$%; 10%; $4\frac{4}{5}$%.
2. $33\frac{1}{3}$%; 20%; $16\frac{2}{3}$%; 14%; 5.6%; 4%; 2%.
3. 50%; $33\frac{1}{3}$%; $16\frac{2}{3}$%; 5%; $4\frac{6}{11}$%; 14%; $3\frac{1}{2}$%.
4. 100%; 50%; $12\frac{1}{2}$%; 25%; $19\frac{3}{13}$%; $13\frac{8}{9}$; 10%.
5. $33\frac{1}{3}$%; 80%; 100%; 200%; 400%; 800%; 1000%; 1,600%.
6. 25%.

Page 184

7. 15%.
8. 20%.
9. $23\frac{1}{13}$%.
10. 20%.
11. 70%.

Page 185

1. 300.
2. 500.
3. 150.
4. 28.
5. 60.
6. 300.
7. 140 lb.
8. 60 lb.
9. \$7.33.
10. \$1.20.
11. $4{,}215\frac{5}{93}$ bu.
12. $8\frac{2}{11}$ oz.

Page 186

1. 1.7078.
2. 202.363.
3. 21.4117.
4. .55946.

Page 187

5. 27.046.
6. 13.0099.
7. .836457.
8. 366.606.
9. $\frac{7}{8}$.
10. $\frac{251}{400}$.
11. $\frac{3}{400}$.
12. $\frac{1}{8000}$.
13. $\frac{1999}{2000}$.
14. $\frac{1267}{2000}$.
15. $\frac{1111}{1250}$.
16. $\frac{17}{20000}$.
17. .625.
18. .4666+.
19. .8333+.
20. .55.
21. .5625.
22. .7272+.
23. .4687+.
24. .1406+.
25. 250.974.
26. 674.928.
27. 838,761.75.
28. 213.5567.
29. .087783.
30. .004455.
31. 1.8551+.
32. 11.0687+.
33. 1.3317+.
34. 28,944.
35. .9009+.
36. 70.1666+.
37. \$4,815.34.
38. 263.1 rd.
39. 2.159 oz.
40. \$11.25.
41. \$6.075.
42. \$26.4375.
43. 8.

Page 188

44. $33\frac{1}{3}$%; 2,500,000; 500,000; 800,000; 3,000,000.
45. Nitrogen, \$.40; phosphoric acid, \$.45; potash, \$.075.
46. 8 lb.; $8\frac{1}{2}$ lb.
47. Land surface, 39,603.96+ sq. mi.; water surface, 396.03+ sq. mi.
48. 9,300 sq. mi.
49. 8 lb. 4 oz.; $82\frac{1}{2}$%.
50. 46.8 lb.
51. 24.5 lb.

Page 189

52. $\frac{1}{4}$; 25%; 18 lb.

53.

	1st	2d	3d	4th	5th	6th
Gravel,	25%	$27\frac{7}{9}$%	$37\frac{1}{2}$%	$23\frac{17}{21}$%	$16\frac{2}{3}$%	30%
Sand,	$41\frac{2}{3}$%	$44\frac{4}{9}$%	$44\frac{4}{9}$%	$42\frac{6}{7}$%	$27\frac{7}{9}$%	$44\frac{1}{6}$%
Gravel and sand,	$66\frac{2}{3}$%	$72\frac{2}{9}$%	$81\frac{17}{18}$%	$66\frac{2}{3}$%	$44\frac{4}{9}$%	$74\frac{1}{6}$%

54. $16\frac{7}{8}$%. **56.** $1.26\frac{9}{16}$ T. gravel; $2\frac{13}{16}$ T. cement; $4.35\frac{15}{16}$ T. sand.

55. 49%. **57.** 46.5 oz. **58.** 6.2 oz.

Page 190

59. 26.2 oz. **60.** 910; 4,550. **61.** .526 lb.; 18.936 lb.

62. Mutton, 3.45 lb.; veal, 3.85 lb.; rice, 2 lb.; eggs, 3.275 lb.; flour, 2.85 lb.

63. Pork, 8.288 lb.; fish, .064 lb.; eggs, 2.976 lb.; rice, .096 lb.; beans, .576 lb.; flour, .32 lb.

64. Beef, 1.69 lb.; mutton, 1.03 lb.; eggs, 1.21 lb.; flour, 7.56 lb.; rice, 7.94 lb.; corn, 2.04 lb.

Page 191

65. 10 ft. **66.** 6 ft.; 8 ft. **67.** $\frac{1}{2}$ ft. **68.** 5 ft. in 100 ft. **69.** 5%.

70. Rise of 12 ft. in 100 ft.; rise of 5 ft. in 100 ft.; rise of $\frac{1}{2}$ ft. in 100 ft.; rise of $\frac{2}{3}$ ft. in 100 ft. **71.** 52.8 ft.; 528 ft.; 633.6 ft.

72. 1 ft.; 2.3 ft.; $3\frac{1}{3}$ ft.; 2 ft.; 2.5 ft.; 4 ft. **73.** 2.3%.

Page 193

1. $6\frac{1}{4}$%. **4.** 36%. **7.** 20%. **10.** $1.60. **13.** $1,875.
2. $4,000. **5.** 25%. **8.** $33\frac{1}{3}$%. **11.** $7.50. **14.** 10%.
3. $50. **6.** 40%. **9.** 10%. **12.** $100. **15.** 25%.
16. 120 million. **17.** $7\frac{1}{2}$ million bales, or 3,750 million lb.

Page 194

18. 50%; 25%. **19.** Gain, $250; loss, $50. **20.** $250.

Page 195

1. $375. **4.** $570. **7.** $25. **10.** $79.20; 26.4%.
2. $686. **5.** $3,120. **8.** 4%. **11.** $168.50.
3. 8%. **6.** $150; 5%. **9.** $289.50.

Page 196

1. $13.50; $35. **2.** $3.76. **3.** $1.40. **4.** 5%. **5.** 10%. **6.** $9.

Page 197

7. $20.
8. $7.70.
9. $4.20.
10. $816.53.
11. $10.50.
12. $2\frac{18}{41}\%$.
13. $100.
14. $53.39.
15. $20.80.
16. $1.70.
17. $7.50.
18. $6.
19. $5.
20. $1.50.
21. 25%.
22. $33\frac{1}{3}\%$.

Page 198

1. $8.
2. $25.
3. $28.98.
4. $225.
5. The interest on $100 for 4 yr. at 5% is greater by $2.00.
6. $1000.
7. $100.

Page 199

8. $43.
9. $6; $12; $18; $24; $30; $60; $93.
10. $4\frac{1}{2}$ yr.
11. 2 yr.; 5 yr.; $3\frac{1}{2}$ yr.; $1\frac{1}{2}$ yr.
12. $35.
13. $550; $27.50.

1. $1,236.25; $1,505; $1,881.25.
2. $658.225.
3. $379.
4. $3,375; $5,000.

Page 201

2. $692.50.
3. $440.625.
4. $1,722.

1. $12; $4; $5.
2. $7; $11; $31; $47.
3. $24.
4. $10.
5. $18.
6. $4.
7. $12.
8. $20.
9. $6.
10. $14.
11. $22.
12. $8.
13. $16.
14. $24.

Page 202

15. $16.
16. $33.75.
17. $168.
18. $96.80.
19. $24.75.
20. $56.25.
21. $15.75.
22. $95.29.
23. $195.
24. $31.69.
25. $6.
26. $15.26.
27. $86.63.
28. $117.

Page 203

1. $490.30; $455.30.
2. $132.87.

Page 204

3. 222.
4. $195.14.

1. $.90; $90.90.
2. $2.05.
3. $1.30; $131.30.
4. $504.58.

Page 205

5. $.79; $2.14; $182.93.

1. $10.
2. $7,500.
3. $18.75; $\frac{1}{4}\%$.
4. $.012.
5. $120.

Page 206

6. $15,000.
7. $1.20.
8. $5.

1. $510.
2. $5,000.
3. $45.
4. $19.95.
5. $100; $190.

Page 207

7. $271,867,260.
8. From liquors; from miscellanies; $141,189,488; $179,684,121.
9. 4.9+%; 19.+%.

1. $29.70.
2. $29.25.
3. $21.52½.
4. $41.80.
5. $29.70.
6. $66.
7. $120.

Page 208

8. $245.
9. $55.50.

Page 209

1. $300.
2. $49.50.

Page 210

1. $31.
2. $20.83; $145.83; $395.83.
3. 10¢; $11.70; $7.80.
4. $13.83.
5. $20,400.

Page 213

1. $83.33.
2. $98.
3. $30.28.
4. $30.34.
5. $46.40.
6. $142.19.
7. $20.83.
8. $165.21.
9. $150.
10. $158.57.
11. $630.63.
12. $1,001.
13. 16⅔% gain.
14. 33⅓% gain.
15. 24.9+% loss.
16. 17⅓% gain.
17. $33.33.
18. 5.5%.
19. $241.92.

Page 214

20. $440.
21. $700.
22. $650.
23. $24.75.
24. $56.25.
25. $15.75.
26. $195.89.
27. $30.
28. $29.75.
29. $83.33.
30. $548.25.
31. $999.96, or $1,000.
32. $192.50.
33. $88.38.
34. $237.86.
35. $57.75.

Page 215

6. 32.028.
7. .019.
8. 5.139.
9. 1.001.
10. .408.
11. 400.008.
12. 9.352.
13. 82.321.
14. .01; .005; .200; .025; .015; .986; .100.
15. 447,194.
16. 347,317.
17. 4,041.56.
18. 5,174.87.

Page 216

19. 76,896.
20. 179,892.
21. 979,267.2.
22. .003818.
23. 199.12.
24. 17.641.
25. 165.906.
26. .009858.
27. 799.998.
28. 15.486.
29. 166.9128.
30. .00000864.
31. $62,758.0317.
32. $.374656.
33. $50,165.65.
34. $9,082.25.
35. $223,429.626.
36. 63,226.641.
37. 59,822.362.
38. 79,689.50.
39. 3.93.
40. .01.
41. .01.
42. 535.27.
43. 02.
44. 4.10.
45. 18.
46. 1.96.
47. .50.
48. 13.98.
49. 7,846.222.
50. 1.989.
51. 693,425.
52. 13,676.778.
53. .376
54. $2\frac{41}{120}$.
55. $2\frac{1}{12}$.
56. $\frac{65}{72}$.
57. $1\frac{49}{60}$.
58. $1\frac{15}{28}$.
59. $1\frac{49}{90}$.
60. $\frac{269}{360}$.
61. $\frac{341}{360}$.
62. $1\frac{7}{20}$.
63. $1\frac{17}{180}$.
64. $1\frac{31}{45}$.
65. $1\frac{71}{126}$.
66. $\frac{7}{24}$.
67. $\frac{7}{36}$.
68. $\frac{5}{14}$.
69. $\frac{1}{2}$.
70. $\frac{19}{72}$.
71. $\frac{22}{63}$.
72. $\frac{2}{3}$.
73. $\frac{36}{119}$.
74. $\frac{22}{75}$.
75. $\frac{15}{76}$.
76. $\frac{1}{2}$.
77. $\frac{8}{65}$.
78. $\frac{137}{187}$.
79. $\frac{137}{228}$.
80. $\frac{33}{56}$.
81. $\frac{82}{385}$.

Page 217

82. $6\frac{25}{36}$.
83. $3\frac{1}{2}$.
84. $2\frac{1}{30}$.
85. $11\frac{1}{36}$.
86. $29\frac{17}{72}$.
87. $\frac{4}{7}$.
88. $\frac{5}{42}$.
89. $\frac{5}{9}$.
90. $\frac{8}{45}$.
91. $\frac{35}{48}$.
92. $\frac{3}{56}$.
93. $\frac{8}{63}$.
94. $\frac{8}{33}$.
95. $\frac{27}{50}$.
96. $\frac{20}{91}$.
97. $\frac{9}{176}$.
98. $\frac{12}{119}$.
99. $\frac{55}{108}$.
100. $\frac{39}{136}$.
101. $5\frac{1}{3}$.
102. $2,516\frac{1}{2}$.
103. $6,350\frac{7}{8}$.
104. 6,970.
105. $7,488\frac{4}{9}$.
106. $9,865\frac{1}{3}$.
107. $26,875\frac{4}{5}$.
108. $883\frac{19}{32}$.
109. $5,883\frac{1}{3}$.
110. $\frac{2}{9}$.
111. $\frac{1}{24}$.
112. $\frac{2}{21}$.
113. $\frac{3}{20}$.
114. $\frac{2}{35}$.
115. $\frac{5}{36}$.
116. $1\frac{7}{17}$.
117. $3\frac{3}{4}$.
118. $1\frac{2}{7}$.
119. $1\frac{1}{44}$.
120. $1\frac{12}{65}$.
121. $2\frac{1}{22}$.
122. $\frac{27}{52}$.
123. $3\frac{9}{25}$.
124. $\frac{117}{136}$.
125. $1\frac{25}{52}$.
126. $1\frac{29}{102}$.
127. $1\frac{145}{442}$.
128. $3\frac{3}{8}$.
129. $1\frac{6}{7}$.
130. $13\frac{1}{3}$.
131. $8\frac{1}{8}$.
132. $9\frac{16}{21}$.
133. $9\frac{2}{11}$.
134. $9\frac{15}{19}$.
135. $31\frac{1}{3}$.
136. $47\frac{17}{18}$.
137. $7\frac{1}{4}$.
138. $25\frac{3}{5}$.
139. $27\frac{13}{14}$.
140. $14\frac{1}{6}$.
141. $4\frac{3}{8}$.
142. $4\frac{1}{6}$.
143. $8\frac{1}{2}$.
144. $\frac{337}{3}$.
145. $\frac{851}{4}$.
146. $\frac{409}{10}$.
147. $\frac{1279}{12}$.
148. $\frac{791}{27}$.
149. $\frac{1803}{32}$.
150. $\frac{2417}{12}$.
151. $\frac{5769}{32}$.
152. $\frac{6417}{32}$.
153. $\frac{3024}{5}$.
154. $\frac{1505}{3}$.
155. $\frac{401}{3}$.
156. 15 rd. 4 ft. 1 in.
157. 27 mi. 1 rd. 4 ft. 6 in.
158. 37 gal. 2 qt.
159. 282 lb. 11 oz.

Page 218

160. 44 sq. yd. 1 sq. ft. 134 sq. in.
161. 72 cu. yd. 25 cu. ft. 92 cu. in.
162. 163 gal. 1 pt.
163. 66 bu. 3 pk. 6 qt.
164. 373 hr. 31 min. 40 sec.
165. 26,093 lb. 12 oz.
166. 312 rd. 4 yd. 2 ft. 6 in.
167. 1,292 mi. 38 rd. 13 ft.
168. 1 rd. 6 ft. $4\frac{2}{3}$ in.

169. $5\frac{3}{7}$.
170. 3 lb. 6 oz.
171. $20\frac{2}{3}$.
172. $3\frac{205}{1116}$.
173. 4.
174. 36.
175. 8.
176. One.
177. 720.
178. 864.
179. $\frac{13}{2000}$.
180. $100\frac{1}{8}$.
181. $\frac{51}{400}$.
182. $\frac{321}{400}$.
183. $\frac{3}{40000}$.
184. $\frac{20011}{100000}$.
185. $6\frac{1}{4}$.
186. $\frac{9009}{10000}$.
187. .66666.
188. .625.
189. .88888.
190. .4375.
191. .64.
192. .22727.
193. .46875.
194. .27083.
195. .81818.
196. .07812.
197. .86821.
198. .00099.

Page 219

1. 50.7 bu.
2. 131.25 lb.
3. 1020 lb.
4. 1,200.
5. $29\frac{1}{2}$ yd.; $28.91.
6. 60,169 lb.
7. $100,000,000.
8. $.67$\frac{3}{8}$.
9. 5.17 hr.
10. .73 mi.
11. $\frac{1}{108}$; $\frac{1}{9}$; $\frac{13}{108}$; $\frac{2}{9}$.

Page 220

12. $6\frac{2}{3}$.
13. 60; 100; 340.
14. 240; 2,400; 3,840.
15. 35; 5; 45.
16. 100; 900; 70; 3.
17. 150; 300; 600.
18. 200; 400; 160; 10.
19. $.40; 20%.
20. 5,600.
21. $2; $2.04.
22. $11.76.
23. $3.81.
24. $2.33.
25. $277.75.
26. $6.89.
27. $3.16.
28. $8.69.
29. 441 sq. in.
30. 558 mi.

Page 221

31. 231 mi.
32. 2,860 lb.
33. 153 sq. rd.
34. 78 lb.; 390 lb.; 195 lb.
35. 2 lb.; $1\frac{1}{2}$ lb.; 5 lb.; $6\frac{1}{4}$ lb.
36. 185 lb.
37. 33 in.; 10 in.
38. 29.885+ in.
39. $76\frac{1}{3}$°.
40. 2 : 3; 1st, $2,400; 2d, $3,600.
41. 2 ¢.
42. 480 bd. ft.; $2.88.

Page 222

43. 12°.
44. $3\frac{3}{4}$%.
45. 36 yr. 4 mo. 23 da.
46. $173\frac{1}{3}$ lb.
47. 720 A.
48. $100.
49. $105.
50. $49.40.
51. $105.83.
52. $26\frac{1}{2}$ yd.
53. $4.67.
54. $1,476.

Page 223

56. $51\frac{8}{17}$%; $70\frac{5}{11}$%; $44\frac{44}{49}$%.
57. 435,000,000 bu.; $22,500,000.
58. $$1.37\frac{1}{2}$; $$.14\frac{18}{59}$.
59. $6,537.50.
60. $2.25.
61. 346.2912 cu. in. of oxygen; 1209.9456 cu. in. of nitrogen; 9.94% of carbonic acid gas.
62. 21.12 sec.
63. 1.32 sec.; 14.89+ sec.

Page 224

64. 2 min.
65. 1,920 bricks; $12.96.
66. 5,280 ft.; 63,360 in.; .03 in.
67. 49 : 46, or $1\frac{3}{46}$.
69. 12,000,000 lb.
70. 3 min. 25 sec.
71. $107.136.
72. $5,672,059.

Page 225

73. $111, including ceilings.
74. San Diego, 1.56; San Francisco, $3.31\frac{5}{6}$; Sitka, $8.59\frac{1}{2}$; Salt Lake City, $1.75\frac{5}{6}$; New York, $3.58\frac{5}{6}$.
75. 40,455 bu.
76. $\frac{2}{9}$.
77. Two brothers, $6,000 each; 3d brother, $2,000; cousin, $4,000.
78. $\frac{1}{2}$ sq. ft.
79. $\frac{1}{38}$; $\frac{3}{76}$.
80. $21\frac{7}{9}$ lb.

Page 226

81. $5.52.
82. 12.
83. 847 lb.
84. $435.
85. 17 ft. 4 in.
86. 25; 25; 75; 75.
87. 16 ft.
88. $9\frac{7}{8}$ mi.
89. $6.25.
90. 9,600 A.
91. $1.74.
92. $615,485.

Page 227

93. $12.10.
94. 12,000 lb.; 6 T.
95. 913 + mi.
96. 640 rd.
97. 21,120 ft.
98. 10,560.
99. $124.74.
101. Altitude, 4 in.; area, 12 sq. in.
102. North; west; south; south; west.
103. 120°; 150°.
104. $52.08.

Page 228

105. $15.19.
107. $3.38; $11.67; $47.20; $84.39; $93.56.
108. 1,540 da.
109. 144 cu. yd.
110. 7.
111. $1.03.
112. $2.03; $6.97; $10.17.
113. $21.04.

Page 229

114. Cheaper by express.
115. Express; freight; freight; freight.
116. 52 lb.
117. Cheaper by express; $.12.
118. $4.13.
119. 18¢.

Page 230

121. $.001.
122. 14,400 ft.; $14.40.
123. Deduct 35¢.
124. $18.08.
125. 110 ft.
126. $.003; $.018; $.54.

Page 234

1. 2,083,017.
2. 5,106,200.
3. 17,300,625.
4. 375,406.125.
5. 6,805,456,279.
16. 200,000,000; 400,000,000; 790,000,000.
17. 1,000,000,000.

Page 235

1. 1,987.705; 55.3034.
2. 346,242.
3. $735,573.07.
4. $226,537.54.

Page 236

1. 185,404.
2. 19,000.
3. 256,982.
4. 7,203.
5. 80,266.
6. 11,864.
7. 757,689.
8. 5,146.

Page 237

9. 512,849.
10. 158.4.
11. 479.653.
12. 114.74.
13. 2,227,259.
14. 179.53.
15. 1,749.126.
16. 15.120.
17. $8,684,219.40.

Page 238

1. 5,424.
2. 60.34.
3. 6,099.
4. 7,854.37.
5. 79.905.
6. 666.22.
7. 4,328.74.
8. 400.92.
9. 117.38.
10. 999.01.
11. 1,990.13.
12. 1,011.118.
13. $19,754,824; $4,372,629.
14. Liberty National, $1,818,600.

Page 239

15. N. Y. Produce Exchange, $1,283,100; New Amsterdam National, $498,200; State Bank, $889,000; Fourteenth Street Bank, $316,400.
16. $10,393,900; $4,354,500; $3,769,200; $12,067,000; $6,049,000.
17. 22,439; 40,608.

Page 240

1. $4,815.
2. $11,495.
3. $2,892.
4. $2,932.

Page 241

5. $75; $87.50; $219; $861.30; $4,410; $1,095; $328.90; $956.25; $1,097.60; $1,584.74; $11,475; $40,650.
6. 541.80.
7. 21,760.
8. 437.03.
9. 500.04.
10. 19,680.3
11. 62,264.
12. 11.2518.
13. 5,742.87.
14. 36,169.6.
15. 75.6851.
16. 1,688.4.
17. 25.375.
18. 2,280.
19. 11.286.
20. 2,496.9.

Page 243

1. 207; 486; 684; 873.
2. $4.23.
3. $34.29.
4. 1,386; 2,673; 5,841; 8,316.
5. $51.48.
6. $15.84.
7. 3,996; 50,949; 83,916; 25,974; 717,282.
8. 686; 1,470; 3,528; 4,802; 7,056.
9. $1,176.
10. 597; 1,791; 10,348; 13,731; 16,119; 9,154.
11. $209.58.
12. 1,188; 2,376; 18,414; 9,306.
13. $13.86.
14. 894; 4,768; 11,622; 16,986.
15. $47.68.
16. $33.83.
17. 10,374; 20,748; 1,197; 6,783.
18. $167.52.

Page 245

1. $108.
2. $49.
3. $48.
4. $8.80.
5. $6.75.
6. $37.33.
7. $1.87.
8. $56.
9. $21.88.

Page 247

1. 72 ; 320 ; 9,000.
2. 5^2 ; 2×5^2 ; 3×5^2 ; 2×3^2 ; 3^3 ; $3^2 \times 7$; $2^3 \times 3^2$; $2 \times 3^2 \times 5$.
3. 6×10^2.
4. 3×10^2; 9×10^2 ; 10^3 ; 5×10^3.
5. 3×10^5; 9×10^5; 95×10^4 ; 895×10^3 ; 10^6 ; 9×10^6 ; 129×10^6 ; 1236×10^6.
7. 1492×10^6 ; 1206×10^6 ; 79×10^7 ; 515×10^6; 107×10^6.

Page 248

1. 45.
2. .55.
3. $118\frac{7}{12}$.
4. 41.
5. $201.05\frac{1}{4}$.
6. 2,001.
7. 230.
8. 77.
9. 3,002.
10. $204\frac{10}{31}$.
11. $264\frac{27}{29}$.
12. 5.
13. .83.
14. 208.
15. 1,331.
16. $977\frac{13}{29}$.
17. 7.47.
18. $20.05\frac{2}{7}$.

Page 249

1. $2.97.
2. $1.25.
3. $.98.
4. $.36.
5. $1.79.
6. $8.48.
7. $.08.
8. $5.82.
9. $.08.
10. $1.11.
11. $3.36.
12. $9.75.
13. $1,111,111.11.
14. $625,000.
15. $2,112,676.06.
16. $655,737.70.
17. $146,341.46.

Page 250

18. 1870 value = $866.66 ; 1880 value = $736.84 ; 1890 value = $840 ; 1900 value = $1,058.82.
19. 1840 population = 17,084,766 ; 1850 population = 23,181,895 ; 1860 population = 31,445,999 ; 1870 population = 38,558,713 ; 1880 population = 50,139,029 ; 1890 population = 62,647,695 ; 1900 population = 76,319,437.

Page 252

1. 80 yd.
2. 120.
3. 9.
4. 28.
5. 45.
6. 57.
7. 18.
8. 18.
9. 9.
10. 9.
11. 60.
12. 150 A.
13. 72.
14. 24 cords.

Page 254

1. 2 × 23.
2. 2 × 2 × 3 × 3 × 3.
3. 5 × 5 × 5.
4. 2 × 19.
5. 2 × 2 × 2 × 2 × 2 × 2 × 7 × 5.
6. 2 × 5 × 13 × 31.
7. 1 × 211.
8. 3 × 3 × 3 × 313.
9. 3 × 37.
10. 2 × 2 × 2 × 2 × 3 × 3.

11. 5 × 5 × 5 × 5.
12. 3 × 17.
13. 2 × 2 × 2 × 3 × 79.
14. 7 × 3 × 37.
15. 2 × 2 × 2 × 2 × 7 × 3 × 3.
16. 2 × 2 × 479.
17. 37 × 1.
18. 2 × 2 × 3 × 23.
19. 5 × 11 × 11.
20. 2 × 2 × 2 × 5 × 5 × 5.
21. 2 × 2 × 5 × 5 × 19.
22. 3 × 271.
23. 2 × 3 × 5 × 233.
24. 3 × 811.
25. 3 × 3 × 17 × 61.
26. 2 × 2 × 491.
27. 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3.
28. 2 × 2 × 2 × 2 × 2 × 2 × 5 × 5.
29. 2 × 2 × 3 × 3 × 53.
30. 2 × 2 × 3 × 11 × 11.
31. 13 × 367.
32. 7 × 7 × 13 × 11.

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1. 26.
2. 18.
3. 14.
4. 19.
5. 12.
6. 5.
7. 12.
8. 15.
9. 25.
10. 21.
11. 7.
12. 5.

Page 256

1. 168.
2. 2,016.
3. 1,694.
4. 2,484.
5. 270.
6. 1,815.
7. 43,095.
8. 1,728.
9. 564,480.
10. 625.
11. 360.
12. 1,728.

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2. 3,225,000 bales; 967,500 bales; 1,612,500 bales; 645,000 bales; 50%.
4. 40; 18; 12. Iron, 40,000,000 T.; petroleum, 18,000,000 T.; salt 12,000,000 T.

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5. Corn, 25.2 + bu.; wheat, 522,799,200 bu.; oats, 26,976,000 A.
6. $289\frac{1}{3}$ ft.; $147\frac{7}{12}$ ft.; $93\frac{2}{3}$ ft.
7. 2.8+ T.; 3.5+ T.; 1.7+ T.; 1.5+ T.; 2.6+ T.; 2.0+ T.
8. $12\frac{5}{8}$ in.; 36 in.; $10\frac{1}{2}$ in.; $67\frac{3}{4}$ in.; $7\frac{1}{4}$ in.
9. 73 in.; 70 in.; 50 in.; 34 in.; 25 in.

Page 261

1. $3^2 \times 2$; $3^2 \times 2^2$; 3^3.
2. 2×5^2; $2^3 \times 5^2$; $2^3 \times 5^3$; $2^3 \times 5^4$; $2^6 \times 5^3$.
3 2,210 T.
4. $5,495,000.
5. Passenger coach, $5,300; $5.290.20; sleeping car. $14,300; $14,263.20; dining car, $10,900; $10,932.51; freight car, $500; $533.16
6. $1{,}008{,}533\frac{1}{3}$ sq. mi.
7. $645{,}461{,}333\frac{1}{3}$.
8. 40,945,965 A.; 2,537,516 A.; 2,607,089 A.

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9. \$18,514.04. **12.** 36,239,000,000 ft.
10. \$104,806,200. **13.** 325 million.
11. \$453,466,387.20.
14. Returns per acre, \$30.50; expenses per acre, \$19.95; net profit, \$10.55; net profit from 25 A., \$263.75.
15. 305($304\frac{12}{17}$). **16.** \$9.33. **17.** 2,262 thousand ft.; $12\frac{3}{7}$.

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18. \$48,562.50. **22.** $17\frac{11}{32}$ da. **26.** 800. **29.** \$243,960.
19. \$$53,333\frac{1}{3}$. **23.** 78($77\frac{1}{12}$). **27.** \$35,700. **30.** 625.
20. 69,375 bu. **24.** $15,416\frac{2}{3}$ bbl. **28.** 321. **31.** \$200,000.
21. \$41,625. **25.** 3.

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32. \$196.90. **35.** $70\frac{27}{73}$ T.; **36.** \$229.41. **39.** \$630; \$420.
33. \$33.30. \$78.02; \$14.19. **38.** $836\frac{428}{777}$ yr. **40.** 16,000 A.
34. $12\frac{58}{73}$ T.

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41. 696,960,000. **43.** $177\frac{7}{9}$ da. **45.** \$3,580,000. **47.** 81 million T.
42. 2,400,000 T. **44.** \$1,860,000. **46** 300,000 T. **48.** 22,800,000,000 lb.

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1. 70. **7.** $\frac{693}{756}$, $\frac{63}{756}$, $\frac{35}{756}$. **13.** $\frac{225}{1000}$, $\frac{18}{1000}$, $\frac{9}{1000}$.
2. 63. **8.** $\frac{720}{1728}$, $\frac{60}{1728}$, $\frac{5}{1728}$. **14.** $\frac{40}{72}$, $\frac{9}{72}$, $\frac{66}{72}$.
3. 12. **9.** $\frac{7}{112}$, $\frac{48}{112}$, $\frac{28}{112}$. **15.** $\frac{70}{84}$, $\frac{28}{84}$, $\frac{27}{84}$.
4. $\frac{15}{120}$, $\frac{8}{120}$, $\frac{90}{120}$. **10.** $\frac{33}{220}$, $\frac{25}{220}$, $\frac{40}{220}$. **16.** $\frac{520}{1248}$, $\frac{143}{1248}$, $\frac{84}{1248}$.
5. $\frac{32}{48}$, $\frac{30}{48}$, $\frac{27}{48}$. **11.** $\frac{24}{140}$, $\frac{14}{140}$, $\frac{13}{140}$. **17.** $\frac{210}{273}$, $\frac{14}{273}$, $\frac{18}{273}$.
6. $\frac{3}{40}$, $\frac{38}{40}$, $\frac{36}{40}$. **12.** $\frac{33}{1320}$, $\frac{22}{1320}$, $\frac{40}{1320}$. **18.** $\frac{28}{420}$, $\frac{30}{420}$, $\frac{180}{420}$.

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1. $3\frac{3}{5}$. **6.** $5\frac{4}{17}$. **11.** $6\frac{1}{20}$. **16.** $18\frac{9}{40}$. **21.** $\frac{53}{9}$. **26.** $\frac{208}{5}$. **31.** $\frac{118}{15}$.
2. $1\frac{11}{15}$. **7.** $4\frac{1}{2}$. **12.** $10\frac{13}{25}$. **17.** $10\frac{74}{75}$. **22.** $\frac{63}{4}$. **27.** $\frac{56}{3}$. **32.** $\frac{203}{25}$.
3. $6\frac{7}{13}$. **8.** $5\frac{5}{19}$. **13.** $16\frac{4}{5}$. **18.** $17\frac{29}{35}$. **23.** $\frac{53}{2}$. **28.** $\frac{367}{8}$. **33.** $\frac{3609}{50}$.
4. $3\frac{1}{25}$. **9.** $2\frac{14}{17}$. **14.** $2\frac{7}{33}$. **19.** $6\frac{11}{25}$. **24.** $\frac{589}{8}$. **29.** $\frac{157}{8}$. **34.** $\frac{6454}{75}$.
5. $2\frac{6}{13}$. **10.** $7\frac{2}{13}$. **15.** $12\frac{9}{16}$. **20.** $10\frac{10}{11}$. **25.** $\frac{135}{7}$. **30.** $\frac{469}{10}$. **35.** $\frac{423}{35}$.

1. $\frac{2}{5}$. **4.** $\frac{15}{34}$. **7.** $\frac{33}{46}$. **10.** $\frac{5}{32}$ **13.** $\frac{5}{24}$. **16.** 0. **19.** $1\frac{9}{80}$. **22.** $\frac{331}{576}$.
2. $\frac{33}{34}$. **5.** $\frac{8}{55}$. **8.** $\frac{1}{48}$. **11.** $\frac{28}{45}$. **14.** $\frac{4}{15}$. **17.** $1\frac{9}{80}$. **20.** $\frac{19}{120}$. **23.** $\frac{2}{15}$.
3. $\frac{37}{64}$. **6.** $\frac{29}{32}$. **9.** $\frac{15}{46}$. **12.** $\frac{139}{180}$. **15.** $\frac{19}{64}$. **18.** $\frac{493}{1008}$. **21.** $1\frac{29}{36}$. **24.** $2\frac{17}{144}$.

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25. $\frac{29}{30}$.
26. $3\frac{7}{24}$ mi.
1. Improper fraction; $\frac{59}{8}$.
2. $23\frac{3}{8}$.

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3. $1\frac{4}{5}$.
4. $\frac{3}{5}$.
5. $1\frac{17}{45}$.
6. $\frac{2}{5}$.
7. $23\frac{1}{3}$.
8. $163\frac{4}{33}$.
9. $58\frac{1}{6}$.
10. $97\frac{1}{12}$.
11. $3\frac{17}{25}$.
12. $3\frac{3}{25}$.
13. $2\frac{5}{48}$.
14. $\frac{5}{48}$.
15. $40\frac{7}{16}$.
16. $77\frac{41}{58}$.
17. $76\frac{44}{45}$.
18. $128\frac{1}{16}$.
19. $2\frac{23}{32}$.
20. $2\frac{5}{32}$.
21. $\frac{95}{108}$.
22. $\frac{25}{108}$.
23. $74\frac{28}{45}$.
24. $72\frac{29}{36}$.
25. $91\frac{43}{58}$.
26. $132\frac{67}{72}$.
27. $140\frac{5}{18}$.
28. $1,100\frac{3}{4}$.
29. $23\frac{7}{18}$.
30. $22\frac{5}{9}$.
31. $578\frac{1}{2}$.
32. $31\frac{11}{28}$.
33. $158\frac{19}{30}$.
34. $51\frac{31}{50}$.
35. $4.38\frac{31}{32}$.
36. $8\frac{1}{40}$ yd.
37. $1\frac{1}{30}$ yd.
38. $202\frac{3}{5}$ T.
39. Tuesday; Monday, $\frac{3}{4}$¢ higher; Wednesday, $3\frac{3}{8}$¢ higher; Thursday, $5\frac{1}{4}$¢ higher; Friday, $3\frac{5}{8}$¢ higher; Saturday, $2\frac{1}{2}$¢ higher.
40. $1\frac{5}{24}$ ft.

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1. 18.
2. $3\frac{1}{3}$.
3. 36.
4. $\frac{3}{10}$.
5. 4.
6. $\frac{5}{12}$.
7. $\frac{63}{128}$.
8. $40\frac{4}{25}$.

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9. $\frac{10}{21}$.
10. $\frac{21}{40}$.
11. $\frac{1}{12}$.
12. $2\frac{5}{8}$.
13. $\frac{3}{16}$.
14. $36\frac{4}{7}$.
15. $68\frac{1}{16}$.
16. $14\frac{71}{81}$.
17. $\frac{9}{140}$.
18. $57\frac{3}{4}$.
19. $389\frac{71}{140}$.
20. $17\frac{1}{2}$.
21. $24\frac{3}{10}$.
22. $42,013\frac{8}{9}$.
23. $\frac{1}{10}$.
24. $\frac{7}{12}$ yd.
25. $\frac{1}{4}$.
26. 341 ft.
27. $3.92.
28. $84\frac{4}{9}$ cu. yd.
29. 584 T.
30. $9,932.40.
31. $94\frac{62}{81}$ sq. yd.
32. $380\frac{81}{256}$ mi.
33. $3\frac{51}{64}$ ft.; $2\frac{27}{32}$ ft.; $2\frac{29}{80}$ ft.; $1\frac{73}{80}$ ft.; $1\frac{47}{64}$ ft.; $3\frac{25}{64}$ ft.

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34. 180 lb.; $28.35.
35. $41,040.
36. Mexico, $45,360; Costa Rica, $41,040; Guatemala, $41,760; Venezuela, $38,880; Colombia, $38,880; Brazil, $26,640.
37. 1,460,250,000 lb.
38. $135,073,125.
39. 63,675 lb.
40. $13,365,000.

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1. $1\frac{1}{2}$.
2. $18\frac{2}{7}$.
3. 225.
4. 128.
5. 2.
6. $\frac{8}{75}$.
7. $1\frac{3}{10}$.
8. 30.
9. $1\frac{1}{8}$.
10. $2,666\frac{2}{3}$.
11. $1\frac{11}{29}$.
12. $3\frac{55}{63}$.
13. 28.
14. 40.
15. $4\frac{4}{9}$.
16. $\frac{1}{3}$.
17. $1\frac{1}{5}$.
18. $\frac{2}{13}$.
19. $\frac{24}{125}$.
20. 4.
21. $\frac{57}{98}$.
22. $3\frac{101}{127}$.
23. 192.
24. $68\frac{4}{7}$.
25. 2.
26. 240.
27. $\frac{1}{8}$.
28. $3\frac{3}{8}$.
29. $6\frac{3}{7}$.
30. 1000.
31. $2\frac{8}{9}$.
32. $\frac{151}{315}$.
33. $34\frac{2}{3}$.
34. 20.
35. $27\frac{1}{5}$.

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1. $\frac{16}{53}$.
2. $4\frac{4}{7}$.
3. $\frac{220}{6783}$.
4. $\frac{56}{89}$.
5. $\frac{2}{3}$.
6. $2\frac{112}{403}$.
7. $2\frac{3768}{3805}$.
8. $4\frac{53}{70}$.
9. $26\frac{28}{55}$.

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1. $3\frac{7}{12}$.
2. $6\frac{7}{16}$.
3. $1\frac{5}{24}$.
4. $1\frac{3}{20}$.
5. $\frac{7}{8}$.
6. $4\frac{7}{24}$.
7. $9\frac{1}{6}$.
8. $6\frac{7}{15}$.
9. $12\frac{4}{15}$.
10. $1\frac{29}{108}$.

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11. $\frac{21}{32}$.
12. $\frac{5}{18}$.
13. $47\frac{43}{48}$.
14. $112\frac{41}{120}$.
15. $\frac{7}{64}$.
16. $\frac{91}{240}$.
17. $423\frac{191}{512}$.
18. $895\frac{31}{75}$.
19. $\frac{133}{288}$.
20. $\frac{24}{125}$.
21. $107\frac{187}{288}$.
22. $309\frac{41}{512}$.
23. $1\frac{7}{8}$.
24. $\frac{2}{9}$.
25. $1\frac{233}{472}$.
26. $1\frac{7}{18}$.
27. $\frac{1}{4}$.
28. $4\frac{21}{115}$.
29. 96.
30. $1\frac{1}{2}$.
31. $4\frac{2}{243}$.
32. $3\frac{29}{108}$.
33. $\frac{10}{11}$.
34. $5\frac{13}{15}$.
35. $\frac{64}{279}$.
36. $53\frac{1}{3}$ sq. ft.
37. $354.60.
38. $525 ; $360.94.
39. $421.10.
40. 116.1 gr.; 12.9 gr.
41. 19.29 gr. of nickel ; 57.87 gr. of copper.

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42. Dining room, $49\frac{1}{2}$ ft.; living room, $60\frac{5}{12}$ ft.; parlor, $61\frac{1}{6}$ ft.; library, 49 ft.; kitchen, $44\frac{1}{2}$ ft.
43. $12\frac{1}{6}$ ft.
44. 44 ft.
46. 371.25 gr.
47. 420 sq. in.; $11\frac{19}{21}$¢; 17.14\frac{2}{7}$.
48. $42,857.14.
49. Length, 3 ft. $6\frac{7}{16}$ in.; width, $8\frac{1}{12}$ in.
50. Length, $18\frac{3}{16}$ in.; width, $15\frac{5}{32}$ in.

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52. $67\frac{1}{2}$ ft.; $4\frac{1}{2}$ ft.
53. 19 ft. 1 in.; 15 ft.
54. 12 hr.; $1\frac{1}{2}$ da.
55. $.56.
56. $25\frac{1}{6}$ ft.; $8\frac{7}{18}$ yd.
57. $42\frac{3}{4}$ ft.

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1. 163,208 oz.
2. 143 pt.
3. 982 units.
4. 9,660 cu. in.
5. 2,346,629 cu. in.
6. 7,922 ft.
7. 504,390 sec.
8. 145,245″.
9. 415 qt.
10. 2,132 in.
11. 2,418 sheets.
12. 14,152 sq. in.

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1. 15 gal. 2 qt. 1 pt.
2. 2,065 gal. 3 qt.
3. 10 bu. 7 qt.
4. 109 gal. 1 qt. 1 pt.
5. 15 bu. 1 pk. 5 qt.
6. 2 T. 956 lb.
7. 3 da. 7 hr. 23 min.
8. 1 hr. 34 min. 50 sec.
9. 2 lb. 4 oz. 10 gr.
10. 21 rd. 13 ft. 11 in.
11. 1 mi. 5,098 ft.
12. 4 sq. yd. 4 sq. ft. 130 sq. in.
13. 9 cu. ft. 211 cu. in.

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1. 1 hr. 18 min.; 2 hr. 42 min.; 4 hr. 54 min.; 12 hr. 36 min.; 30 min.
2. 22 hr. To add without reducing is shorter.
3. 7 yd. 2 ft. 4.8 in.; 5 yd. 9 in.
4. 272 sq. rd.
5. 83.25 sq. ft.
6. $\frac{5}{48}$ yd.
7. 198 oz.
8. $5.66\frac{2}{3}$ ft.; $9.41\frac{2}{3}$ ft.
9. 2.875 gal.; 5.375 gal.
10. 6.5 sq. ft.; $15.66\frac{2}{3}$ sq. ft.
11. 1.90+ sq. yd.
12. 4.336+ cu. yd.
13. 6.75 ft.; 2.8 ft.; 7.65 ft.; 8.92 ft.
14. $26\frac{3}{25}$ ft.; 26.12 ft. Adding with decimals is shorter.

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1. 23 gal. 3 qt.
2. 16 pk. 6 qt.
3. 45 mi. 202 rd.
4. 33 yd. 1 ft. 8 in.

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5. 31 da. 7 hr. 1 min.
6. 69 bu. 3 pk. 6 qt.
7. 29 cu. yd. 23 cu. ft. 1,110 cu. in.
8. 101 A. 20 sq. rd. 13 sq. yd.
9. 33 wk. 1 da. 18 hr.
10. 65 sq. yd. 5 sq. ft. 21 sq. in.
11. 10 wk. 5 da. 16 hr.
12. 9 mi. 260 rd. 5 ft.
13. 8 yd. 4 in.
14. 2 cu. yd. 20 cu. ft. 828 cu. in.
15. 25 bu. 1 pk. 7 qt.
16. 14 sq. yd. 7 sq. ft. 69 sq. in.
17. 38 yd. 1 ft. 3 in.
18. 5 hr. 48 min. 26 sec.
19. 17 min. 45 sec.
20. 4 gal. 2 qt. 1 pt.
21. 2 min. $32\frac{2}{5}$ sec.

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1. 24 gal. 1 qt. 1 pt.
2. 59 pk. 4 qt. .5 pt.
3. 35 bu. 1 pk. 7 qt.
4. 162 mi. 19 rd. 10.5 ft.
5. 201 sq. rd. 129 sq. ft. 60 sq. in.
6. 12 cu. yd. 4 cu. ft. 471 cu. in.
7. 4 da. 4 hr. 50 min. 40 sec.
8. 3 sq. mi. 564 A. 114 sq. rd. 7.5 sq. yd.
9. 23 yd. 1 ft. 4 in.; $70\frac{1}{3}$ ft.
10. $376\frac{1}{2}$ ft.

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11. 4.75 bu.; 2.25 bu.; $6.12\frac{1}{2}$ bu.; $8.37\frac{1}{2}$ bu.
12. 47 bu. 2 pk.; 22 bu. 2 pk.; 61 bu. 1 pk.; 83 bu. 3 pk.
13. 47.5 bu.; 22.5 bu.; 61.25 bu.; 83.75 bu. Multiplying the decimal equivalents is shorter.
14. 4.875 gal.; 24.375 gal.
15. 10.128+ mi.; 162.048+ mi.

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1. 41 lb. 8 oz.; 920 lb.; 1 bu. 4 qt.
2. 2 bu. 2 pk. $\frac{2}{5}$ qt.
3. 7 gal. 1 qt.
4. 245 rd. 9 in.
5. 7 cu. yd. 22 cu. ft.; 672 cu. in.
6. 64 sq. ft. 20 sq. in.

7. $407\frac{1}{2}$ lb.
8. 1 hr. 22 min. 1 sec.
9. 5 hr. 50 min.
10. 1 bu. 2 pk. $1\frac{7}{15}$ pt.
11. $104\frac{19}{96}$.
12. 2 yd. $1\frac{7}{10}$ in.
13. $8\frac{34}{39}$.
14. 5 rd. 3 ft. 7 in.
15. 7 yd. 1 ft. 2 in.
16. $45\frac{3}{5}$.
17. $625.09\frac{3}{8}$ bu.
18. 80.
19. $12\frac{12}{17}$.
20. $100\frac{27}{32}$.
21. $5\frac{5}{7}$.
22. 7.

Page 291

1. 1,728 cu. in.; $7\frac{37}{77}$ gal.
2. $53\frac{17}{36}$ cu. ft.
3. $2692\frac{76}{77}$ gal.
4. $\frac{385}{576}$ cu. ft.
5. 114 bbl., nearly.

1. 10,752.1 cu. in.
2. 1,612.815 cu. in.
3. 4.99+ bu.
4. 1.24+ cu. ft.
5. 774.19+ bu.
6. 128,000 bu.

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7. 67.20 cu. in.; 57.75 cu. in. The dry quart is 9.45 cu. in. larger.

1. $\frac{96}{175}$.
2. $\frac{144}{175}$.
3. 9.504 gr.; .0217+.
4. $437\frac{1}{2}$ gr.
5. $\frac{175}{192}$.

Page 294

1. 19° 44′ 13.5″; 171° 15′ 31.5″; 11° 3′ 28.5″; 126° 18′ 46.5″.

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1. 43 min. $22\frac{2}{3}$ sec.
2. 21 min. 13 sec.
3. 32 min. $39\frac{13}{15}$ sec.
4. 57 min. 23 sec.
5. 6 hr. $3\frac{1}{3}$ sec.
6. 50 min. $\frac{2}{3}$ sec.
7. 1 hr. 40 min. $49\frac{2}{3}$ sec.
8. 1 hr. 8 min. $1\frac{4}{15}$ sec.
9. 12 hr. 2 min. 40 sec., or 11 hr. 57 min. 20 sec. in the opposite direction.
10. 125° 3′ 45″.
11. 91° 10′.
12. 88° 44′.
13. 150° 11′ 15″.
14. 181° 2′ 30″.
15. 350° 4′, or 9° 56′.

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1. 133 pt.
2. 31,018 in.
3. 933 in.
4. 6,868 sq. in.
5. 9,765 cu. in.
6. 1 T. 507 lb. 13 oz.
7. 1 mi. 5 in.
8. 160 cu. yd. 7 cu. ft.
9. 16.875 gal.
10. $5.95\frac{5}{16}$ bu.
11. 12 wk. 3 da. 13 hr.
12. 44 gal. 1 qt. 1 pt.
13. 31 bu. 2 pk. 4 qt.
14. 91 sq. yd. 2 sq. ft.
15. 2 gal. 2 qt.
16. 1 mi. 319 rd. 14.5 ft.
17. 64 T.
18. 44 cu. yd. 12.5 cu. ft.
19. 24 yd. 2 ft. $2\frac{14}{17}$ in.
20. 20 bu. 1 qt. $\frac{5}{41}$ pt.
21. 159.58 cu. in.
22. $1\frac{31}{144}$ lb. troy.

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23. 358 lb. 8 oz.
24. $10\frac{1}{2}$ lb.
25. 8 ft.
26. 194 ft. 10 in.
27. 8 gal. 1 pt.
28. 2 wk. 4 da. 4 hr.
29. 3 gal. $\frac{1}{2}$ pt.
30. 122 A. 80 sq. rd.
31. 40° 30′ 59″; 84° 55′ 43″.
32. 60″; 150″; 345″.
33. 90′; 285′.
34. 76° 18′ 40″.

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35. 1 P.M.; 10 A.M.; 11 A.M.; 7 P.M.; 6 P.M.; 1 A.M. of the following day.
36. 9 min. $33\frac{1}{5}$ sec. past noon; noon; 9 min. $33\frac{1}{5}$ sec.
37. 960 rd.
38. $(16\frac{1}{2})^2$; 43,560 sq. ft.
39. $30\frac{1}{4}$ hr.
40. $87.50.
41. $224.50.
42. $123.68.

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1. 32¢.
2. $3.25.
3. 7,000.
4. 2,072

Page 301

1. 60 yd.
2. 4 hr. 32 min.
3. $52.
4. 480 pages.

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5. $\frac{980}{8}$ yd.; $\frac{980}{16 \times 8}$ yd.; $\frac{20 \times 980}{16 \times 8}$ yd.; $\frac{28 \times 20 \times 980}{16 \times 8}$ yd. $= 4{,}287\frac{1}{2}$ yd.

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1. 9
2. 10.
3. 8.
4. 62.

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5. 5.
6. 23.
7. 10.
8. $1,250.
9. 40 mi.
10. 285 mi.
11. 945 mi.
12. 297 mi.
13. 6.
14. 2.
15. John, 11 yr.; James, 15 yr.
16. $5\frac{1}{4}$ ft.

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17. $2\frac{1}{3}$ ft.; $1\frac{5}{9}$ ft. **18.** Tower, 80 ft.; staff, 20 ft. **19.** House, $3,000; lot, $500.
20 Part out of water, 27 ft.; part under water, 18 ft.
21. $139\frac{1}{2}$ lb.
22. 63 lb.
23. $1\frac{7}{68}$ lb. starchy foods. $\frac{3}{17}$ lb. fats. $\frac{15}{68}$ lb. albuminous foods.
24. 5 bu.
25. 656.
26. 73 lb.

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1. 175 mi.
2. $153.60.
3. 5,400.

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1. A, $\$272\frac{8}{11}$ ($272.73). B, $\$727\frac{3}{11}$ ($727.27). C $1,000.
2. A, $200; B, $600; C, $300; D, $1,300.
3. 1st, $20; 2d, $40; 3d, $80.
4. 362.88 cu. in. of oxygen. 1,365.12 cu. in. of nitrogen.

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1. 60 lb., 70 lb.
2. 9 yr.
3. $1000.
4. $900.
5. $3,125; $1,875.
6. $7,951.50; $9,718.50.
7. $12,600; $2,400
8. 960 A. $15\frac{2}{3}$%.
9. 5 yr.

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10. $1.60.
11. 590,884 sq. mi.
12. 3,435 sq. mi.
13. 115,026 sq. mi.
14. 6,736 sq. mi.
15. 209,358 sq. mi.
16. 4,277,170 sq. mi.
17. 11,286,344 sq. mi.
18. 208,830 sq. mi.
19. 1st, $2,400; 2d, $3,600; 3d, $6,000.

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1. 80.475.
2. 45.7875.
3. 258.
4. 28.152.
5. 14.11.
6. 2.375.
7. 59.67.
8. 148.8.
9. 70.
10. 150.41.
11. 1.207.
12. 2.66.
13. 9.666.
14. .00153.
15. 1.59185
16. 24.72$\frac{4}{9}$.
17. 756.6.
18. 1.76625.

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19. $42; $661.50; $4,100.73.
20. $492.08; $1,485.75; $808.21; $950.48.
21. 136.08 A.

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1. 57; 19.
2. 20; 5.
3. 80 sq. ft.; 300 sq. ft.; 100 sq. ft.
4. 30 min.; 1,600 cu. ft.; 4,000 cu. ft.; 8,000 cu. ft.
5. 115; 172; $20.04; $21.16; 316.
6. 121; 230; $60.22; $120.30; $101.21.
7. 32; $41.02; $42.10; $21.06; 18.
8. 60; 135; $150.30; $375.15; $213.15.
9. 600; $570.30; $750.60; $903.45; 675.
10. 140; 315; 350.70; 875.35; 497.35.
11. 242; 460; $120.44; $240.60; $202.42.
12. 729; 418.5; 776.7; $90.18; $904.50.
13. 19.63; 180.5; 1.725; $4.09; 176.2.
14. 30.66\frac{2}{3}$; $4.60; 4.48$\frac{1}{3}$; 677$\frac{1}{3}$; 153$\frac{1}{3}$.

Page 315

1. 4$\frac{1}{6}$%; 4%; 3$\frac{4}{7}$%.
2. 4$\frac{6}{11}$%; 16$\frac{2}{3}$%; 12$\frac{1}{2}$%; 900%.
3. 10$\frac{5}{7}$%; 1$\frac{11}{19}$%; 3$\frac{8}{19}$%.
4. 90$\frac{10}{11}$%; 9$\frac{1}{11}$%.
5. 84$\frac{8}{13}$%; 15$\frac{5}{13}$%.
6. 2%; 98%.
7. 2,083$\frac{1}{3}$; 2$\frac{1}{12}$; .2$\frac{1}{12}$; .2$\frac{1}{12}$%.
8. 85%.

Page 316

1. $500.
2. 1000 sq. ft.; 1,775 sq. ft.
3. 170.
4. 262$\frac{1}{2}$.
5. 600.
6. 560.
7. 1,066$\frac{2}{3}$
8. 1,728.
9. 310.

Page 317

1. $\frac{100}{100}$; 100%.
2. $\frac{3}{10}$; 30%.
3. $\frac{1}{9}$; 11$\frac{1}{9}$%.
4. $\frac{1}{5}$; 20%.
5. 18¢; $\frac{1}{5}$.
6. 24¢; $\frac{1}{12}$.
7. 12¢; 16$\frac{2}{3}$%.
8. 9¢; 33$\frac{1}{3}$%.
9. 17$\frac{1}{2}$¢; 14$\frac{2}{7}$%.
10. $70.
11. $500.
12. $200.
13. $300.
14. $200.
15. $1000.

Page 318

1. $109.38.
2. $32.81.
3. $26.13.
4. $52.
5. $46.
6. $22.80.
7. $2.81.
8. $15.99.
9. $12.958.
10. $1.25.
11. $18.75; 5%.
12. $425.

Page 320

1. 42.9+%.
2. $32\frac{1}{2}$%; 50%; $48\frac{7}{10}$%.
3. The prices are identical, $166.32; the discount in each case equaling $49\frac{3}{5}$%.
4. $376.32.
5. (1) $34.56. (2) $9.22. (3) $69.84. (4) $169.75. (5) $121.25.
(6) $33.10. (7) $11.06. (8) $93.36. (9) $73.72.

Page 321

1. $31,600.
2. $31\frac{51}{79}$%; $5\frac{30}{79}$%
4. $1,580,000.

Page 323

1. $412\frac{1}{2}$ gr.
2. $46\frac{2}{3}$%; 45 lb.
3. $15,000.
4. $45.
5. 2%.
6. .3%.
7. 9.8%.
8. 24%.

Page 324

11. Decrease of 9.1%.
12. Decrease of 12.5%; decrease of 5.4%; decrease of 3.8%.
14. $2,356.06.
15. $395.83.
16. $377.34.
17. $3,010.
18. $2,000.
19. $1,500.

Page 325

20. 54+%. **21.** 93+%. **22.** 29,069. **23.** 174,400. **24.** 22,464 sq. mi. **25.** $\frac{1}{2}$%
26. 1st, $475; 2d, $250; 3d, $50; 4th, $25; 5th, $5.
27. 1st, $2,375; 2d, $2,500; 3d, $1,250; 4th, $1,250; 5th, $125.
28. 1st, $479.75; 2d, $252.50; 3d, $50.50; 4th, $25.25; 5th, $5.05.
29. $1,600; $14.

Page 327

1. $81.
2. $103.20.
3. $79.13.
4. $257.60.
5. $349.
6. $104.26.
7. $18.39.
8. $178.89.
9. $251.92.
10. $186.92.
11. $3,825.
12. $7.72.
13. $38.51.
14. $133.58.
15. $130.87.
16. $144.66.
17. $279.29.
18. $276.11.
19. $87.92.
20. $1,433.33.
21. $100.44.
22. $389.70.

Page 328

1. $28.67. 3. $104.93. 5. $55.07. 7. $69.19.
2. $245.33. 4. $80.06. 6. $233.56. 8. $64.80.

1. $70.83. 2. $91. 3. $221.67. 4. $256. 5. $78.75. 6. $135.

Page 329

1. $101.59. 2. $7.34. 3. $42.73. 4. $29.50.

Page 330

1. $5.25. 2. $4.52. 3. $6. 4. $1.50. 5. $4.66. 6. $5.04.

Page 332

1. $3.17. 3. $.97. 5. $2.76. 7. $15.23.
2. $4.33. 4. $10.77. 6. $13.12. 8. $1.56.

1. 1 yr. 6 mo. 3. 3 yr. 5. 3 yr. 7. 1 yr. 4 mo.
2. $4\frac{1}{11}$ yr. 4. 5 yr. 4 mo. 6. 3 yr. 4 mo.

Page 333

1. $5\frac{2}{5}$%. 2. 8%. 3. 6%. 4. 3%. 5. 4%. 6. 6%. 7. 8%. 8. 2%.

Page 334

1. $500. 3. $600. 5. $450. 7. $250.
2. $600. 4. $400. 6. $631.17. 8. $150.

Page 336

6. $20. 7. $9.

Page 337

8. $108. 9. $230; $220; $210; $60.
10. $1,776.50; $1,751; $1,725.50.
11. $1,800; $1,760; $1,720; $1,680; $1,640. 12. $2,113; $1.50 less.

Page 338

1. $1,040. 2. $10; $250; $100. 3. $212. 4. $128.13.

Page 339

5. $73.16. 6. $46.32. 7. $252.83. 8. $53.

1. $55; $54.88; $54.75; $54.63; $300; $5,700. 2. $53.50; $53.38.

Page 340

3. $52; $51.88; $3,000; $40; $39.88. 4. 20 yr.; $25.13.
6. $11.80; $11.75. 7. $11.20; 3 yr.
1. $147.70. 2. $344.50.

Page 341

3. $670.65. 4. $608.26.
1. $139.52. 3. $491. 5. $1,244.32. 7. $1,615.17.
2. $49.27; $90.25. 4. $845.59. 6. $1,070.94.

Page 343

3. $2.08.
4. $1.08.
5. $1.33.
6. $.69.
7. $151.25.
8. $228.38.
9. $122.60.
10. $452.81.
11. $672.51.
12. $1,505.63.
13. $2,515.63.
14. $2,841.67.

Page 344

1. $993.67. 2. $3,464.51. 3. $7,965.33.

Page 345

1. $1,001.94; $3,507.82; $8,031.71. 2. $.56; $250.69. 3. $1.52; $351.69.
4. $2.11; $402.39. 6. $9.10; $614.90. 8. $9.69; $377.81. 10. $3.19; $228.56.
5. $3.13; $430.37. 7. $10.68; $523.07. 9. $6.08; $243.92. 11. $7.43; $883.61.

Page 346

1. $1,159.27.
2. $562.43.
3. $703.58.
4. $457.68.
5. $838.88.
6. $30; $2,030.
7. $30.45; $2,060.45; $2,186.89.

Page 347

1. $316.12.

Page 348

2. $.56.
3. $144.24.
4. $78.86.
5. $104.21.
6. $207.72.

Page 349

1. $203.48.

Page 350

2. $35.86.
3. $214.22.
4. $14.81.
5. $477.93.
6. $787.04.

Page 352

1. $ 145.23.
2. $ 267.05.
3. $ 65.59.
4. $ 51.47.
5. $ 28.56.
6. $ 99.51.
7. $ 126.12.
8. $ 233.55.
9. $ 13.90.
10. $ 5.21.
11. $1\frac{1}{2}$ yr.
12. $3\frac{1}{2}$%.
13. $ 500.07 ($ 500).
14. $ 1,608.75.
15. $ 151,980.
16. (*a*) $ 526,825; (*b*) $ 674,100; (*c*) $ 361,620; (*d*) $ 897,975; (*e*) $ 120,220.
17. $1\frac{19}{81}$ yr.

Page 353

19. (1) $ 3.75; (2) $ .60; (3) $ 3.67.
20. (1) $ 247.90; (2) $ 372.87; (3) $ 1,948.10.
21. $ 258.15.
22. $ 12.55.
23. $ 562.43.
24. $ 10.15.
25. $3\frac{3}{4}$ yr.
26. $ 11,980.

Page 354

1. $ 30.45.
2. $ 31.36.

Page 355

3. $ 43.40.
1. $ 62.12.

Page 356

2. $ 57.39.
1. $ 10.58.

Page 357

2. $ 50.84.
3. $ 3.44.
6. $ 132.
7. $ 771.45.
8. $ 1,586.

Page 358

1. $ 237.90.
2. $ 90.92.
3. $ 181.91.

Page 359

4. $ 28.12.
5. $ 266.50
6. $ 222.60.
7. $ 3,377.47.

Page 364

1. (*a*) $ 1,503.75.
   (*b*) $ 1,629.06.
   (*c*) $ 3,508.75.
   (*d*) $ 86.72.
   (*e*) $ 10,676.63.
   (*f*) $ 195.99.
   (*g*) $ 1,228.06.
   (*h*) $ 40.40.

Page 365

1. $ 2.95.
2. $ 323.37.
3. $ 4,488.75.
4. $ 1,485.

Page 366

1. $ 19.50; $ 39.
2. $ 13.50; $ 40.50.
3. $ 309.
4. $ 213.75.
5. $ 15,500.

Page 367

6. $2\frac{1}{2}$%.
7. $ 1,422.

Page 368

1. $ 151.20.
2. $ 868.

Page 369

1. $\frac{5}{26}$ of 1%; .1%.
2. $ 6.25; $ 5,768.75.
3. $ 160,000.

Page 372

1. $ 1,787.13.
2. $ 19,218.75.
3. $ 256,250.
4. $ 126,562.50.
5. $ 208,125.
6. $ 1,782.88; $ 19,181.25; $ 255,750; $ 126,187.50; $ 206,875.

Page 373

1. 7½%.
2. 5%.
3. 9.5%.

Page 376

1. $ 15,768.75.
2. $ 5,418.75; $ 10,625; $ 18,800; $ 6,915.
3. $ 250; $ 600; $ 800; $ 300.
4. $ 6.25; $ 12.50; $ 25; $ 7.50.

Page 378

2. $ 36.25.
3. $ 13,012.50; $ 12,987.50.
4. $ 1,190.
5. $ 1,125.
6. 5%.
7. 3½%.
8. $ 55,000.
9. $ 45.

Page 380

1. 225.
2. $\frac{81}{225}$.
3. 121.
4. 625.
5. 64.
6. $\frac{9}{16}$.
7. 6.25.
8. .01.
9. 64 cu. in.
10. 125 cu. ft.
11. 1.728 cu. in.
12. 27 cu. ft.
13. .216 cu. in.
14. 729 cu. ft.
15. 64 cu. yd.
16. 343 cu. ft.

1. 12.
2. 20.
3. 16.
4. 31.
5. 17.
6. 99.
7. 25.
8. 21.
9. 14.
10. 51.
11. 18.
12. 60.
13. 24.
14. 13.
15. 30.

Page 384

1. 27.
2. 8.5.
3. 1.3.
4. $\frac{4}{7}$.
5. 35.
6. 83.
7. 19.
8. 29.
9. $\frac{4}{5}$.
10. 75.
11. 2.1.
12. 25.
13. $\frac{11}{25}$.
14. 65.
15. 150.
16. 28.
17. 4.5.
18. $\frac{7}{12}$.
19. 85.
20. 1.5.
21. 45.
22. $\frac{7}{45}$.
23. 38.
24. 1.6.
25. 41.
26. 55.
27. $\frac{13}{17}$.
28. 105.
29. 12.5.
30. 56.
31. .45.
32. 94.
33. 9.8.
34. 44.
35. 2.5.
36. 35 yd.
37. 320 rd.; 192 rd.
38. 164 rd.
39. 502.
40. 85 ft.
41. 1.414.
42. 3.316.
43. 3 605.
44. 3.872.
45. 1.732.
46. 1.581.
47. .547.
48. .447.
49. 2.236.
50. .141.
51. .223.
52. 1.224.
53. 2.645.
54. 1.024.
55. 8.660.
56. .866.
57. 12.64 rd.

Page 386

1. 15 in. **2.** 13 in. **3.** 20 in. **4.** Same in each case.
1. 3. **2.** 5. **3.** 5. **4.** 8. **5.** 10. **6.** 16.97 + ft. **7.** $238.

Page 387

8. $84. **10.** 45 ft. **12.** 50 ft. **14.** 50 yd.
9. 48 ft. **11.** 40 ft. **13.** 5 mi. **15.** 3.6 ft.

Page 389

1. 18.84 in. 3.14 ft. 37.68 in. 18.84 yd. **3.** 1 ft. **5.** 720.6.
2. 3.14. **4.** 1 in.; 1$\frac{1}{2}$ yd.

Page 390

1. 25.12 in.; 50.24 sq. in.
2. 113.04 sq. in.; 3.14 sq. ft.; 12.56 sq. in.; 12.56 sq. ft.; 314 sq. ft.; 314 sq. in.
3. Diameter = 4 rd. **6.** 9.81$\frac{1}{4}$ sq. ft. **9.** $\frac{1}{4}$; 9.616 sq. ft.
4. $62.17. **7.** 1,616.75 sq. ft. **10.** 7.74 sq. in.
5. 3 yd. **8.** 12$\frac{1}{4}$ sq. ft.

Page 391

11. 2.15 sq. in.
12. 452.16 sq. ft.; 907.46 sq. ft.; 455.30 sq. ft. **13.** $401.92.
14. 4.5 ft.; 15.896 sq. ft. **15.** $\frac{1}{16}$; 2d ring, $\frac{3}{16}$; 3d ring, $\frac{5}{16}$; 4th ring, $\frac{7}{16}$.
16. $3,694.18. **17.** $21.98. **18.** $212. **19.** $\frac{157}{200}$.
20. 25,120 mi.; 1,046$\frac{2}{3}$ mi.; 17.44 mi.

Page 394

1. 125.6 sq. in. **3.** 4 in. **5.** $314. **7.** $305.21.
2. 5 in. **4.** 5,652 sq. ft. **6.** 6,104.16 sq. ft.

Page 395

1. 40 cu. ft.; 80 cu. ft.; 200 cu. ft. **2.** 113.04 sq. ft.; 1,808.64 cu. ft.
3. 13,564.8 gal. **6.** 4 in. **9.** $137.51. **12.** 1,130.4 gal.
4. 4,239 gal. **7.** 16 ft. **10.** 542.2 cu. yd. **13.** 109,795.5 gal.
5. 56.52 cu. in. **8.** 678.24 sq. in. **11.** $46.89.

Page 396

14. 431.75 cu. ft. **15.** 11,550 cu. ft. ($\pi = \frac{22}{7}$). **16.** 28,875 cu. ft.
17. 48,000 gal.; 336,000 gal.; 17,520,000 gal.
18. 117,750 gal. **20.** 3,679.68 + gal.; $331.17 +; $478.36.
19. 432,000 gal.; 157,680,000 gal.; 13.6 + da. **21.** 865.46 gal.

Page 398

1. 96 cu. in. **2.** 556,516 sq. ft.; 89,042,560 cu. ft. **3.** 907,136 sq. ft.
4. $66\frac{2}{3}$ cu. ft.; 324 sq. yd.; $37\frac{1}{2}$ in.; $29\frac{1}{3}$ cu. in.; 1,275 cu. ft.
5. 62,500 cu. ft. **6.** 9,918.75 cu. ft. **7.** 7 ft.

Page 400

1. 72 sq. in.
2. 301.44 sq. ft.
3. $33\frac{1}{3}$ sq. yd.; $ 4.61.
4. 84.78 sq. in.; 75.36 sq. in.

Page 401

1. 12 cu. in. **2.** 80 cu. ft. **3.** 36 ft. **4.** 96 sq. yd.

Page 403

1. 36π sq. in.; 144π sq. in.; 64π sq. ft.; 4π sq. yd.
2. 113.04 sq. in.; 452.16 sq. in.; 200.96 sq. ft.; 12.56 sq. yd.
3. 282.6 ft. **4.** $ 254.34.

1. 2,144.6656. **2.** 1,436.7584 cu. in.
3. 113.0976 cu. ft.; 4.1888 cu. yd.; 33.5104 cu. in.; 268.0832 cu. in.; 523.6 cu. ft.

Page 404

4. $r=1$; $d=2$; $v=\frac{4}{3}\pi$.
5. $d=4$; $a=16\pi$; $v=\frac{32}{3}\pi$.
6. $r=3$; $d=6$; $v=36\pi$.
7. $r=4$; $a=64\pi$; $v=\frac{256}{3}\pi$.
8. $d=\frac{2}{3}$; $a=\frac{4}{9}\pi$; $v=\frac{4}{81}\pi$.
9. $r=\frac{2}{7}$; $a=\frac{16}{49}\pi$; $v=\frac{32}{1029}\pi$.
10. $d=2\frac{2}{5}$; $a=\frac{144}{25}\pi$; $v=\frac{288}{125}\pi$.
11. $r=\frac{3}{10}$; $a=\frac{9}{25}\pi$; $v=\frac{9}{250}\pi$.
12. 268,190,476,190.47 + cu. mi.
13. 6 ft.
14. $42{,}782\frac{1}{7}$ sq. ft.
15. $314\frac{2}{7}$ sq. yd.
16. $ 19.80.
17. $5{,}749\frac{1}{3}$ cu. in.; 25 gal. nearly.
18. $ 15.09.
19. $\frac{2}{3}\pi r^3$; $\frac{1}{6}\pi r^3$.
20. $\frac{1}{6}\pi d^3$; $1{,}767\frac{6}{7}$ cu. in.

Page 405

1. 25 sq. ft.; 100 sq. ft.; 735 sq. ft.; 2,940 sq. ft.; 3,065 sq. ft.
2. 366 sq. in. or $2\frac{13}{24}$ sq. ft. **3.** $1\frac{3}{4}$ yd. **4.** 8 $(7\frac{1049}{1368})$.

Page 406

1. 418 sq. in.
2. 90.2 gal.
3. $21\frac{25}{36}$ sq. ft.
4. 6,225.81 cu. ft.
5. $ 103.12.
6. $193\frac{17}{21}$ cu. ft.

Page 407

1. 12.56 sq. in. **2.** 3.44 sq. in. **3.** 141.3 ft.; 125.6 ft.
4. 3,179.25 sq. ft.; 2,512 sq. ft.; 667.25 sq. ft.
5. $8\frac{1}{2}$ in.; 53.38 in.; 226.87 sq. in. **6.** 7 ft.; 21.98 ft.; 38.47 sq. ft.

7. 2 ft.; 1 ft.; 6.28 ft.
8. 6 ft.; 3 ft.; 28.26 sq. ft.
9. 376.8 sq. in.; 533.8 sq. in.; 942 cu. in.
10. 8 in.; 207.24 sq. in.; 226.08 cu. in.
11. 6 ft.; 150.72 sq. ft.; 207.24 sq. ft.
12. 2 in.; 69.08 sq. in.
13. $ 17.42.

Page 408

14. 6,941.76 cu. in.
15. 1 ft.
16. 183.1248 T.
17. $53\frac{1}{3}$ sq. yd.
18. 80.
19. 20.
20. 180 sq. in.
21. 20 ft.
22. $\frac{4}{7}$.
23. 180.
24. 12.
25. $\frac{2}{5}$.
26. 5.
27. $1\frac{1}{2}$.
28. 4 *m.*
29. 420.
30. 20.
31. 40.
32. 10.
33. $83\frac{1}{3}$.
34. 14.35.
35. 10 *d.*

Page 409

2. 9×10^6; 19×10^9.
3. 4,436,156.
4. 51,099,941.
5. 3,045.582.
6. 294,881.
7. 63,084.
8. 81.63998.
9. 23,045,055.
10. 40,776,810.
11. 6,356.28572.
12. $1,018\frac{25}{89}$.
13. $426\frac{361}{1287}$.
14. $10\frac{4866}{8815}$.
15. $2\frac{37}{72}$.
16. $11\frac{11}{20}$.
17. $1\frac{9}{20}$.
18. $23\frac{3}{8}$.
19. $2\frac{1}{21}$.
20. $42\frac{28}{45}$.
21. $\frac{3}{34}$.
22. $4\frac{5}{8}$.
23. $3\frac{1}{2}$.
24. $2\frac{15}{16}$.
25. $\frac{3}{25}$.
26. $10\frac{17}{20}$.

Page 410

27. $\frac{25}{48}$.
28. $7\frac{1}{8}$.
29. $\frac{39}{49}$.
30. $37\frac{1}{3}$.
31. $\frac{1}{3}$.
32. $416\frac{2}{3}$.
33. $1\frac{5}{7}$.
34. $1\frac{1}{5}$.
35. $\frac{8}{9}$.
36. $2\frac{16}{23}$.
37. $\frac{2}{3}$.
38. $1\frac{43}{46}$.
39. 201 da. 11 hr.
40. 493 mi. 940 ft.
41. 128 bu. 1 pk.
42. 33 gal. 2 qt.
43. 5 yd.
44. 27 cu. yd. 16 cu. ft.
45. 86 A. 40 sq. rd.
46. 688 gro. 9 doz.
47. 62 mi. 2,760 ft.
48. 2 hr. $38\frac{1}{3}$ min.
49. 15 yd. $\frac{4}{15}$ ft.
50. 3 ft. $3\frac{90}{121}$ in.
51. $ 12.42.

Page 411

52. $ 13.18.
53. 159 qt.; 333 qt.
54. 1,296 sq. in.; 720 sq. in.; 14,400 sq. in.
55. 45 yd.; 55 yd. 1 ft. 8 in.
56. 5 da. 17 hr. 36 min.; 8 da. 8 hr.
57. .625.
58. .112.
59. .583.
60. .281.
61. .172.
62. 1.444.
63. .115.
64. 9.812.
65. 3.819 da.
66. 5.046 mi.
67. 7 gal. 2 qt. 1.2 pt.
68. 45 da.
69. $ 5.94.; $ 11.88; $ 13.93.

Page 412

70. 400 sq. ft.; $ 100.
71. 18 ¢.
72. $ 1.80.
73. $ 1.42.
74. $ 7.22.
75. $ 58.67.
76. $ 12.20.
77. $ 24.63.
78. $ 2.21.
79. $ 7.28.
80. $ 13.50.
81. $ 7.
82. 19.
83. 28.
84. Draw lines: 14 in. long, $11\frac{1}{2}$ in. long, 11 in. long, 11 in. long, 9 in. long, 9 in. long, 8 in. long, 7 in. long, 6 in. long, $5\frac{1}{2}$ in. long.

Page 413

85. $443,923,200.66.
86. $6,002,400.
87. .8427 mi.; 4,449.456 ft.
88. 26.
89. $27.
90. 12 ft.
91. 11,000 sq. ft.
92. 57.63 mi.
93. $139\frac{1}{2}$ ft.
94. $39\frac{2}{3}$ ft.
95. $64.75.

Page 414

96. 244 bbl.
97. $12\frac{1}{2}$%; $16\frac{2}{3}$%; $33\frac{1}{3}$%; 4%.
98. 50; 100.
99. $200,000.
100. $60,000.
101. 6 gal. .88 pt.
102. $166\frac{2}{3}$ lb.
103. 66.59%.
104. 95.4%.
105. 2,975 lb. fish; 4,900 lb. salt meat; 1,500 qt. ice cream; 25 T. potatoes.
106. $302{,}034{,}715\frac{15}{119}$ T.
107. $\$365{,}355{,}603\frac{43}{69}$.

Page 415

108. $60,000,000.
109. 200.
110. 141,213 bu.; 350,398 bu.
111. $105,909.75; $262,798.50.
112. $132.39; $328.50.
113. $10; $48.75.
114. $10; $102.50.
115. $18,731.25.
116. $8,259.375.
117. $2,896.88.

Page 416

118. $\$400;\ 3\frac{7}{11}\%$.
119. 2,035.
120. $9,095.64.
121. $120; $360.
122. $180; $144; $120; $102.
123. 334,400,000 bu.
124. $117{,}197{,}105\frac{5}{19}$ bu.
125. $331{,}318{,}690\frac{30}{113}$ bu.
126. 70,500,000 bu.
127. $6,500,000.

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128. 15.6 bu.
129. 8,000,000 bu.
130. 252,000 bu.
131. 12; 1,350.
132. $37\frac{1}{27}$.
133. Great Britain, 60,000,000 bu.; Australasia, 40,000,000 bu.; Egypt, 12,000,000 bu.
134. 1,125 T.
135. $\$3\frac{1}{3}$.
136. 300; $18,000.
137. 375,000 lb.

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138. 268,000,000 bu.; 402,000,000 bu.
139. 3,076,000,000 bu.; 1,538,000,000 bu.
140. 2,475,000,000 bu. Yes.
141. $504{,}266\frac{2}{3}$ sq. mi.
142. $337\frac{1}{2}$ bales.
143. $50; $16,875.
144. .3842 bales; 192 lb.
145. $645,480,000.
146. 4,275,151; 39.7%.

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147. 105,000,000 gal.
148. $\frac{1}{16}$.
149. $8\frac{1}{3}$¢.
150. 724+%.
151. 250,000,000 gal.
152. 10,980,000 bbl.
153. $3,111,000.
154. $42,000.
155. 7,825,000 T.
156. $156,600,000; 149+%.

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157. 189,350 sq. mi.
158. Appalachian field,
167,941.8 T.;
Central field,
38,755.8 T.;
Western field,
31,004.64 T.;
Other fields,
20,669.76 T.

159. $24.
160. $29.46.
161. $71.875.
162. $167.33.
163. $119.60.
164. $171.28.
165. 192.
166. 500.
167. 420.

168. 10.
169. 40.
170. 32.
171. 108 sq. ft.
172. 8 in.
173. 1,125 cu. ft.
174. 162 cu. ft.
175. 376.8 sq. in.
176. 113.04 sq. in.

Page 423

1. $12.60.
2. $3.22.
3. $9.60.

4. 30 sq. m.; 24.84 sq. m.
5. 536 cm.
6. 1,600 cm.; 6 cm.

8. 30.
9. 7 m.
10. 36 m.

11. 10 dm.; 100 cm.
12. 10 mm.; 100 mm.; meter; meter; meter.
14. 3.937 in.; .03937 in.

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15. 11.811 in.; 7.874 in.; 3.937 in.
16. 75.692 cm.; 76.454 cm.; 72.644 cm.

17. 1,625 m.
18. 84 mi. (1,625 km. = 1 mi.).

1. 200 sq. cm.; 40,000 sq. cm.; 1,000,000 sq. mm.

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2. 100 sq. m.; 200 ares.
3. 100 ha.
4. $3,000.

5. 74.1 A.; 123.5 A.
6. 100,000 sq. m.
7. 4,800.

8. $540 gain.
9. 60,500.
10. 181.5 ha.

Page 426

1. 346.5 metric tons.
2. 57.2 cu. m.

3. 63.
4. 6,000 cu. cm.

5. 26.25 steres; 7.25 cords.
6. $3,071.25.

7. 50 cu. m.; 1,768.5 cu. ft.
8. 52,296 cu. cm. in excess; 52 ¢.
9. 3.1543 m.; 1.06155 m.; .8189 m.; 2.74 + cu. m.
10. 3.62 steres.

Page 427

1. 405 l. **2.** 1,000 l. **3.** 400 hl. **4.** 50%. **5.** $250. **6.** 1,100 l.; 1,166 qt.

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7. 11,000,000 cu. cm.; 11 kl.; 2,915 gal.
8. 5,000.

9. 2,190.5 l.; 2,321.93 qt.
10. $\frac{1}{5}$; $\frac{1}{350}$; $\frac{1}{2500}$.

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1. 500 g.
2. 1000 g.; 1 kilo.
3. 830 g., .83 kilo, 1.826 lb.
4. 5 metric tons; 5.5 English tons.
5. 1000 l.; 1000 kilos; 2,200 lb.
6. 184 kilos; 404.8 lb.
7. 1,540 lb.
8. .05 g.; .3 g.; .35 g.
9. .643 kilos.
10. .9508 hectograms.
11. 28,125 g.; 28.125 kilos.
12. 2,000 cu. ft.; 125,000 lb.; 56,250 kilos.

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2. $1\frac{25}{144}$ sq. yd.
3. $ 1,706.25.
4. 11,000 lb.
5. $7\frac{7}{32}$.
6. 984 ft.
7. 196,020 lb.
8. 8.8 kilos.
9. 30 sq. m.; 3,000 tiles.
10. .09051 g.
11. 84,780 kilos.
12. 185.62 m.

Page 433

1. $976. **2.** £205 2*s.* 7*d.* **3.** $6,930.91. **4.** $2,350,000. **5.** £13,555,000.
6. $492,480,000; $65,960,000; $16,950,000; $67,775,000; $643,165,000.
7. $479,330,784; $64,198,868; $16,497,435; $65,965,407.50; $625,992,494.50.
8. Denmark, $75.80; Switzerland, $63.32; Germany, $36.64; Norway, $36.16; Australia, $35.62; United States, $30.39; Austria-Hungary, $26.36; France, $21.57; Great Britain, $20.07; Canada, $11.01.

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	Imports	Exports
9. British Possessions . .	$ 483 895,342 00	$ 405 996,332.41
United States	613 481 477 31	71.617,793.72
France	250 122 493.13	66,701,446 16
Germany	138 861.484.77	109,622,715.74
Others	802.736 576 95	481.708,088 71
Total	$ 2,289,097,374.16	$ 1,135,646,376.74

1. 171.39 fr. **2.** 84.58 fr. **3.** 76.71 fr. **4.** 2,575 fr. **5.** $388.07.

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6. 79 fr. **7.** $9.20. **8.** $87.91. **9.** 49.21 fr.
10. United States: Imports, 10.4+%; exports, 6.2+%.
Others: Imports, 2,868 million francs; exports, 2 126 million francs.

	Imports	Exports
11. British Isles	$ 129.117.000	$ 239.127,000
United States	88.780,000	49 022,000
Germany	79,516.000	88.780,000
Others	553,524.000	410,318,000
Total	$ 850,937,000	$ 787,247,000

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1. 3.87 M.

2. 9.425 M.

3. 63.36 M.

4. 1.98 M.

5. (1 M. = 25 ¢.) (1) $.1125; $.967 +. (2) $.1625; $2.356 +. (3) $1.32; $15.84. (4) $.825; $.495.

6. $4,775.

7. 5,013.05 M.

8. (1 M. = 25 ¢.) $166,462,644; $43,051,993.75; $88,977,512.50; $19,981,402.50; $146,589,602.

9. Army, $159,596,059.94; navy, $41,276,099; postal and telegraph, $85,307,190.11; railways, $19,157,169.65; administration, $140,542,780.92.

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1. Hats, $1.44; gloves, $.74; suit, $16.25; overcoat, $12.63; necktie, $.36.

2. £18 14*s.* 2*d.*; 1st item, $11.56; 2d item, $25.35; 3d item, $41; 4th item, $15.63.

3. $91.41.

4. 7*s.* 6*d.*

5. $50.

6. $3.10.

7. $13.40.

8. $325.

9. $60; $57.12; $79.97.

10. $20,000.

11. 175.07 + M.

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1. $49.653.

2. $49.09.

3. $49.46.

4. £102 10*d.*; 2,562.50 fr.; 2,062.50 M.

5. $49.76.

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1. $11a + 14b$.

2. $a + m + b + 7c$.

3. $32a + 8b$.

4. $10a + 12b$.

5. $6a + b + 2c$.

6. $9a + 3b + c$.

7. $3m + 6n + 10p$.

8. $50m + 5n + 2p$.

9. $10m + 10n + 10p$.

10. $4a + 5b$.

11. $4a + 3b$.

12. a.

13. $8b$.

14. $a + 4b + 3c$.

15. $a + 2c$.

16. $5a + 5b$.

17. $20 + 6p$.

18. $x + 6y + 7z$.

19. $40m + 2n + 10p$.

20. $4a + b + 2c$.

21. $8m + 5n$.

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22. $(2a + 3b + 4c)$ bu.

23. $(4a + b + 3c)$ bu.; $(6a + 4b + 7c)$ bu.

24. $(125a + 25b + 22c)$ ft.

25. $(2m + 5n)$ bu.

1. 3^2.

2. 2^3.

3. 3^4.

4. 2^3a^2.

5. 5^2.

6. 2^4.

7. b^2.

8. 2^4b^3.

9. 7^2.

10. s^5.

11. c^3.

12. 3^3.

13. $2^2 \times 5^2$.

14. a^2y^3.

15. $2^2 \times 5^2a^2b^2$.

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16. 7^2.

17. 12^2.

18. 12^2.

19. 60^2.

20. 12^3.

21. 100^2.

22. $12a^2$.

23. $40m^2$.

24. $10b^2$.

25. $18b^4$.

26. $24c^3$.

27. $8x^3$.

28. $8x^5$.

29. $120s^3$.

1. $3a^2 + ax + 3ab + bx$.

2. $x^2 + 2x + 1$.

3. $s^2 + 9s + 18$.

4. $4ax + 5a + 4x^2 + 5x$.

5. $n^2 + 4n + 4$.

6. $2 + 2m + m^2 + m^3$.

7. $2b + ab + 2x + ax$.

8. $z^2 + 10z + 25$.

9. $ab + bx + ay + xy$.

10. $3y^2 + 11y + 6$.

11. $x^2 + 2xy + y^2$.

12. $x^3 + x^2 + 2x + 2$.

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14. $a^2 + 2ar + r^2$.
15. 32.
16. 1,849.

Page 445

1. $c + d$.
2. $m + p + q$.
3. $m + p + q$.
4. $2a + b$.
5. $a + 2b$.
6. $a + 2b$.
7. $x + 3y$.
8. $m + 2n$.

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1. 25; 105; $32b^4$.
2. $16a^2b^7$; $55a$; $13a^2b^3$.
3. $25mp^2$.
4. $35a^3b^3$.
5. $16xy$.
6. $19p^4q^4$.
7. $12s^4t^3$.
8. $14m^3n^2$.
9. $65mn^5$.
10. $21xy^6$.

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1. 7.
2. $2,000.
3. $171.43.
4. $50.
5. $500.
6. $400.
7. $1,200.
8. 30 ft.

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9. $2,000.
10. $1,500.
11. 5.
12. $11,000.
13. $4,000.
14. 27⅜ A.
15. 79½ ¢.
16. 21 ¢; 25½ ¢.
17. 3(2c — $100) = h.
18. $1,800.

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1. $2a$.
2. 10.
3. y.
4. $15x$.
5. $3ay^2$.
6. $85a + 3$.
7. $66z^2$.
8. 20.
9. $8a + 3b$.
10. $48y^2 + 3y$.
11. $2a + 3b$.
12. $11a + 11$.
13. $5x + 18y$.
14. $2a + 17b$.
15. $3 + 6a$.
16. $9m + 15$.
17. $8q$.
18. $x + 9q$.
19. $7b + 5c$.
20. $(5a)^2$.
21. $(3x)^3$.
22. $(16)^3a$.
23. $(36)^2x$.
24. $6x^2 + 11x + 4$.
25. $30a^2 + 21ab + 3b^2$.
26. $a^2 + 2am + m^2$.
27. $2a^2 - 3ac + c^2$.
28. $56x$.
29. $168m$.
30. $96a$.
31. a^2x^4.
32. 6.
33. 5.
34. 2.
35. 4.
36. 20.
37. 5.

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1. 8.
2. 12.
3. 15.
4. 24.
5. 54.
6. 64.
7. 84.
8. 105.

Page 453

1. 3.
2. 1.
3. 4.
4. 7.

Page 454

1. 16.
2. 23.
3. 51.
4. 23.5.
5. 57.
6. 65.3.
7. 14.3.
8. 23.7.
9. 183.
10. 215 ft.
11. 28.3 in.
12. 28 ft.
13. 37 ft.